Economic Policy Analysis and Modelling

Course Code	**DEC562**
Credit Hours	**3**
Classification	**Core**
Instructor	**Arega Sh.(PhD)**
Semester	**Year 1, Semester II**

Prerequisites: Advanced Microeconomics, Macroeconomics and Econometrics

Course outline

- **Chapter 1: Introduction to CGE and SAM**
- General equilibrium theory:
- CGE Analysis Vs. Partial Analysis
- Benefit and Drawbacks of CGE
- Steps in CGE Modelling and Approaches
- Blocks, Assumptions and Closures of CGE Model
- SAM for a closed and open economy
- **Chapter 2: Microeconomic Theories of Consumers and Producers**
- **Producer theory:** Demand for factors of production (Theory of production functions, Cobb Douglas production function, Leontief production function, CES production function), Demand for Production Factors, Supply of Goods, Exercises using GAMS
- **Consumer Theory:** Demand for commodities: (Cobb Douglas utility function, Stone Geary utility function (linear expenditure system) and CES utility function), Demand for Goods, Exercise Two using GAMS
- Open Market case and Current Account

Course outline

- **Chapter 3: A Simple General Equilibrium Model**
- Specification of Products Market Equations and Factor Market
- Identification of Endogenous and Exogenous Variables and Application of Walras' Law
- Calibration of Numerical vales of Parameters
- Numerical Experiments
- Shock models: Impulse response models
- **Chapter 4: Input-Output Table, SAM and Multipliers Analysis**
- Input-Output Table and Multiplier Analysis
- Social Accounting Matrix and Multiplier Analysis
- Python Software Application and SimSIP SAM Application
- **Chapter 5: A Standard and Advanced General Equilibrium Model**
- Types of Models: IFPRI model, PEP model, STAGE model, GTAP model, Dual- Dual Model, MAMS model etc
- Social Accounting Matrix and an overview of the Standard CGE model
- Mathematical Model Statement
- The Standard Model in GAMS
- Exercises using GAMS

Introduction

- **Policy** is a technique/action made by the government targeting to achieve a **predefined objective**.
- **Policy analysis** is the process of **examining** and **evaluating** the different options to implement the policy measure by the government.
- **Improper economic policy** would result in **economic** and **social chaos**, requiring years to readjustment even at a **very high cost**.
- Unlike natural sciences in which laboratories test is possible on, **economists** have to rely on **natural experiments (data**).
- Rapid development of technologies allow economists to examine ideas by simulation model before they put into practice.
- **Simulation** and **economic modeling** has become a major field in applied economics.
- In the past decades, different models have been developed and policy makers use them.
- The **central objective** of **development** was **GDP growth**, and **per capita**.

Chapter One

- **Introduction to CGE and SAM**

- **General equilibrium theory**
- **CGE Analysis Vs. Partial Analysis**
- **Benefit and Drawbacks of CGE**
- **Steps in CGE Modelling and Approaches**
- **Blocks, Assumptions and Closures of CGE Model**
- **SAM for a closed and open economy**

General equilibrium theory

- **Partial equilibrium model:-** all prices other than the price of the good being studied are assumed to remain fixed.
- **General equilibrium model:-** all prices are variable and equilibrium requires that all markets clear (all of the interactions between markets are taken into account).
- **General Equilibrium Theory** (GET) was founded by **Walras** in 1874-77.
- **Walras** never fully formalized the dynamical system and consequently never solved it analytically: yet, this deficiency was immaterial for Walras, since – according to him – the solution is directly "found" by the market.
- The **Pareto Optimality** …….
- The **Edgeworth–Bowley box**
- In 1970s Walrasian general-equilibrium converted from an abstract representation into realistic economic models to conduct policy evaluations by specifying production and demand functions through real world data.

General equilibrium theory

- In a general equilibrium system of the Walrasian type there are as many markets as there are commodities and factors of production.
- For each market there are three types of functions demand functions, supply functions and a ‘clearing-the-market’ equation, which stipulates that the quantities demanded be equal to the quantities supplied.
- In a commodity market the number of demand functions is equal to the number of consumers, and the number of the supply functions is equal to the number of firms which produce the commodity.
- In each factor market the number of demand functions is equal to the number of firms multiplied by the number of commodities they produce.
- The number of supply functions is equal to the number of consumers who own (ex-hypothesis) the factors of production.
- Since the number of equations is equal to the number of unknowns, one should think that a general equilibrium solution exists.

General equilibrium theory

- A necessary (but not sufficient) condition for the existence of a general equilibrium is that there must be in the system as many independent equations as the number of unknowns.
- Thus the first task (in establishing the existence of a general equilibrium) is to describe the economy by means of a system of equations, defining how many equations are required to complete (and solve) the system.
- For example, assume that an economy consists of two consumers, A and B, who own two factors of production K and L.
- These factors are used by a firm to produce X and Y.
- It is assumed that each firm produces one commodity, and each consumer buys some quantity of both.
- It is assumed that both consumers own some quantity of both factors (but the distribution of ownership of factors is exogenously determined).

General equilibrium theory

The number of equations

demand functions of consumers	$2 \times 2 = 4$
supply functions of factors	$2 \times 2 = 4$
demand functions for factors	$2 \times 2 = 4$
supply functions of commodities	2
clearing-the-market of commodities	2
clearing-the-market of factors	2
Total number of equations	18

The number of Unknowns

quantities demanded of X and Y by consumers	$2 \times 2 = 4$
quantities supplied of K and L by consumers	$2 \times 2 = 4$
quantities demanded of K and L by firms	$2 \times 2 = 4$
quantities of Y and X supplied by firms	2
prices of commodities Y and X	2
prices of factors K and L	2
Total number of 'unknowns'	18

General Equilibrium

- In households' and firms' choice, we assumed that the **real wage** was beyond the control of any individual household or firm.
- In a competitive economy, individuals are small relative to the entire economy, so that individual decisions could not **influence market prices**.
- Prices ultimately determined by **aggregate supply** and **demand** conditions in the **market place**.
- So, households and firms should come together to appraise how they interact in the **market place**.
- The outcome of this interaction is called **a general equilibrium**.
- The **economy's general equilibrium** is defined as an allocation (c^*, y^*, n^*, l^*) and a price system (w^*).
- Where: - c= consumption, y= income, n=labor, w= wage,

  l = leisure, z = profit, n^S =labor supply, n^D = labor demand

General equilibrium theory

- Given the previous information the following could be true:-

1. Given w^*, the allocation (c^*, n^*, l^*) **maximizes utility** subject to the budget constraint.
2. Given (w^*, z), the allocation (y^*, n^*) **maximizes profit**.
3. The price system (w^*) clears the **market** $[n^S(w^*) = n^D(w^*, z)$ or $c^D(w^*) = y^S(w^*, z)]$.

- The general equilibrium concept is interpreting the world as a situation wherein all of its actors are trying to **do the best they can** (subject to the constraints) in the **competitive markets**.
- Observed prices (**equilibrium prices**) are likewise interpreted to be prices that are consistent with the **optimizing actions** of **all individuals together**.
- The pattern of exchanges in the general equilibrium occur in two periods; and each period is divided into **two stages**.
- Firstly, workers supply their labor (n) to firms in exchange for coupons (M) redeemable for y units of output.

General Equilibrium Theory

The real GDI at this stage is given by y.

- Secondly, (after production has occurred), households take their coupons (M) and redeem them for output (y).
- Since M is a claim against y, the real GDE at this stage is given by y.
- Thus, firms actually produce y, the real GDP is given by y as well.
- The theory makes a prediction over various real quantities (c^*, y^*, n^*, l^*) and the real wage (w^*).
- These 'starred' variables are the model's **endogenous variables**; i.e., these are the objects that the **theory is designed to explain**.
- Notice that the theory here contains variables that have **no explanation**, '**God-given**' and are **exogenous variables**.
- In the above theory, the **key exogenous variables** are **technology** and **preferences**.
- Thus, the **theory** explains how its **endogenous variables** react to any particular (**unexplained**) change in the set of **exogenous variables**.

CGE Analysis Vs. Partial Analysis

- Computable general equilibrium models are usually multi-sectoral, economy-wide models, calibrated to a Social Accounting Matrix (SAM).
- These models may be static or dynamic with short run coverage of one to three years, three to seven for the medium run and long-term models that extends beyond a decade Gunning and Keyzer (1995).
- Static models compare two points in time without explicit attention to the path connecting these points while dynamic models connect a locus of points with explicit stock-adjustment processes.
- The models may exhibit a wide range of adjustment mechanisms, from closed, purely competitive, Walrasian models to macro structural models.
- CGEs can be used to estimate shadow prices as discussed below.

CGE Analysis Vs. Partial Analysis

- CGEs in theory can be extended to address a **range of related policy problems**, such as environmental blocks.
- So long as stable contaminant coefficients can be found and linked to production and consumption levels, the models can generate an endogenously determined estimate of environmental quality along with its forecasts for production, consumption, investment and international trade.
- There are several important problems of implementation, however, the first of which is that contaminant levels can vary significantly between two industries that have been aggregated into a larger category and even within an industry pollution levels can vary between two firms.
- Existing CGEs could adequately capture a command and control system that targeted output levels, but would fail to capture technical change that was in part inspired by trends in environmental protection.
- Moreover, models that do not include a feedback loop from the toxic contaminants to price or output levels would also fail to capture reality.
- These models are currently in their infancy, however, and while environmentally augmented CGEs have been employed in a small subset of developing countries, they are far from reaching their full potential.

CGE Analysis Vs. Partial Analysis

- The **G**eneral **A**lgebraic **M**odeling **S**ystem (**GAMS**) is a high-level modeling system for mathematical programming problems.
- What is **CGE**? **C**omputable: **S**olved numerically

  General: **E**conomy wide

  Equilibrium: **Optimizing agents** have found their **best solutions** subject to their **budget constraints.**

- **Thus**, **Quantities demanded** = **Supplied** in factor and commodity markets.
- **Macroeconomic balance …….**
- **The model** is applicable at the **country level**; however, only minimal changes are needed before it can be applied to a **region within a country** involved in production and consumption activities.
- Over the previous 25 years, **CGE models** have become a standard tool of empirical economic analysis.

CGE Analysis Vs. Partial Analysis

- The **model incorporates** features developed over recent years.
- These features are important in **developing countries:-**include **household consumption** of non-marketed ("**home**") commodities, **explicit treatment** of **transaction costs** for commodities in the **market.**
- Separation between **production** and **commodities** that permits to produce **multiple commodities** and produced by **multiple activities**.
- **Partial economic sector models** (PEM) have a focus on a **certain sector of the economy**, than multi-sectoral general economy models.
- **Sector models** work on the simplifying assumption that major feedbacks between the specific sector and the economy as a whole, e.g. effects on employment and growth, can be neglected.
- Taking **macroeconomic conditions** and **certain prices** as given, the **allocation** and **distribution effects** within the sector can therefore be looked at more **realistically**.

Applications of the CGE methodology

- tax reforms
- trade liberalization
- economic integration
- change in world prices
- economic growth, dynamic model
- changes in public expenditure
- energy and environment policies

Equilibrium in CGE models

- **An equilibrium:** can be defined as a set of signals such that the resulting decisions of all agents **jointly satisfy** the system constraints.
- In a perfectly competitive **CGE model** the assumption that **excess demand equals zero** in all markets is a system constraint that defines the nature of equilibrium.

Components in CGE models

- **Institutional structure** of the **model economy**, which are the **rules of the game** by which various **agents interact**.

  E.G, **perfect competition** implies that each agent is a price taker and prices are flexible.

- A set of **explicit definitions** of **equilibrium conditions**, which are "**system constraints**" that must be satisfied for the **whole economy**.
- A **set of economic agents** such as firms, households and government whose behavior is to be analyzed.
- **Behavioral rules** for these agents that reflect their assumed motivation such as **profit maximization** for firms and **utility maximization** for consumers.
- A set of **signals** observed by these agents on which they make their economic decisions, such as **market prices** or **government rationing quotas**.

Knowledge required for CGE analysis

- Basic Micro and Macroeconomics
- Knowledge of **general equilibrium theory**
- Knowledge of **real world data**.
- Be able to manipulate the data into a **model admissible** form
- Knowledge of **computer programming**. Be able to implement the model in computer.
- Knowledge of **policy issues** and **institutional structure**.

Mathematical structure of CGE

- The GAMS is commonly used to solve CGE.
- **Assume**:
- an economy that produce one item using **two factors**: Labor and Capital.
- one agent receives all his income from supplying **factors** of **production** and
- the agent uses the money to purchase the **item produced**.
- Finally, all markets (**commodity** and **factor**) are clear.
- The technology of production is expressed in Cobb-Douglas function.

$$Y = A\,KD^{\alpha}\,LD^{1-\alpha}$$

- Where: Y= National production (Supply) α = elasticity

  A = scale parameter KD = Capital demand

  LD= Labor demand

Cont’d

- The labor and capital demand could be derived from the cost minimization function subject to the Cobb-Douglas Production function.
- $LD = ((1-\alpha)\ P.Y)/W$ = Share of the value of production (P.Y) divided by price of labor
- $KD = (\alpha.\ P.Y)/\ R$ = Share of the value of production (P.Y) divided by price of capital
- Where:- P= Price of the commodity
- W = Remuneration for labor
- R= Remuneration for labor
- Both LD and KD increases with price of commodity and inversely related to the respective remuneration.
- National income and total consumption are given by:-

$INC = W.\ LS + R.KS$

Cont'd

- C= INC/P where:- C= national consumption(Demand) and INC =National income.
- Assume that each market is equilibrium:
- C = Y (Consumption=output)
- KD = KS (it is exogenous = 3000)
- LD = LS (it is exogenous =7000)
- Thus, we have 8 equations and endogenous variables
- 1. $Y = A\,KD^{\alpha}\,LD^{1-\alpha}$ 1. Y 7. KD = KS 7. R
- 2. LD = ((1-α) P.Y)/W 2. LD 8. LD = LS 8. W
- 3. KD = (α. P.Y)/ R 3. KD
- 4. INC= W. LS + R.KS 4. INC
- 5. C= INC/P 5. C
- 6. C = Y 6. P

Cont'd

- For demonstration let us have three equations, and let us assume that the two are satisfied, then the third one will be satisfied based on 'Wlaras law'
- From equation 5 :- CP= INC
- If the commodity market is in equilibrium, C=Y (From Equa.6)
- Therefore, the above equation can be transformed => PY= INC
- Then combine equation 4 and the above one;

INC= W. LS + R.KS = PY

- Now from equation 2 (LD = ((1-α) P .Y)/W) and 3 (KD = (α.P.Y)/R)

W. LD + R.KD= (1-α) P.Y + α.P.Y) => P.Y - α.P.Y+ α.P.Y= **P.Y**

- Thus, **P.Y=** W. LD + R.KD = W. LS + R.KS
- If we have an equilibrium in the labor market, LD=LS
- P.Y= ~~W.LS~~ + R.KD= ~~W.LS~~ +RKS

Cont’d

- Therefore; R.KS= R.KD
- Finally, KS=KD, proof of the Walrasian law.
- Thus, in a system of equations specified in a model:
- If the commodity market is in equilibrium and the labor market is in equilibrium, then the capital market will be in equilibrium.
- We have eight equations, but one of them is dependent, which means 7 of them are independent.
- To solve linear simultaneous equations, they must be independent to have **unique solutions**.
- If we withdraw one equation, then we will have **7 equations** and 8 **endogenous variables**.
- The model will **not be square**….
- Let us remove equation **capital market**….**7 equations** will remain.
- You can withdraw any of the equations…
- Now let us express **one price**, P, that in terms of which all other prices will be expressed, this is what we call ***numeraire***

Putting on GAMS

- Typically, the program can be divided into four parts

Calibration

Model

Initialization

Resolution

- In the calibration process, define the *SETS*, *PARAMETERS* and introduce the values known beforehand.
- Further calibrate the behavior of parameters as a benchmark values and the algebraic relationship defined in the system of equations.
- Thus, they could be consistent with the known values
- The data represents the solution values of the equation.

Steps in CGE Modelling and Approaches

1. Define the issue to be studied
2. Construct a consistent mathematical model
3. Data collection – construct the benchmark that will be used for calibration
4. Code the model
5. Replicate the benchmark – consistency
6. Conduct policy experiments
7. Analysis of results – compare the counterfactual solution with the benchmark

- The data requirements used to construct a CGE model are small when compared to the number of model parameters -- calibration.

Programming to solve CGE models

- Single country models: GAMS, GEMPACK
- Global models: GTAP (GEMPACK)

Start with defining the parameters

- Let's start with defining the parameters, as we do not have any sets in our simple (one commodity) model.

```
PARAMETERS
* Behaviour parameter
 alpha    Cobb-Douglas (elasticity)
 A        Cobb-Douglas (scale parameter)

* Benchmark levels for variables
 C0       National consumption (demand)
 INC0     National income
 KD0      Demand for capital
 KS0      Capital supply
 LD0      Demand for labour
 LS0      Labour supply
 P0       Price of the commodity
 R0       Remuneration of capital
 W0       Remuneration of labour
 Y0       National production in volume (supply)
;
```

- Let's start with defining the parameters, as we do not have any sets in our simple (one commodity) model.

```
PARAMETERS
* Behaviour parameter
 alpha    Cobb-Douglas (elasticity)
 A        Cobb-Douglas (scale parameter)

* Benchmark
 C0       Nat
 INC0     Nat
 KD0      Dem
 KS0      Cap
 LD0      Dem
 LS0      Lab
 P0       Pri
 R0       Rem
 W0       Rem
 Y0       Nat                                    y)
;
```

Note that these are our variables to which we added an '0' at the end to distinguish the benchmark values (here on the left) from the variables of the model (defined further in the program).

Calibration

- In our example, the only a priori information we have is that total labor supply equals 7,000 and that capital supply equals 3,000. We labeled this as "exogenous" previously.
- As discussed previously, there is an infinite set of prices that can solve the model. Prices are seen as indices and we can set them initially at any value.

➢ For simplicity, let's set prices (of one commodity and two factors) equal to one.

Calibration

Signed a value of unity to all prices

- In GAMS language:

```
* Calibration process
 LS0      = 7000;
 KS0      = 3000;

* Arbitrary prices
 P0       = 1;
 R0       = 1;
 W0       = 1;
```

Calibration

- In GAMS language:

```
* Calibration process
LS0      = 7000;
KS0      = 3000;

* Arbitrary prices
P0       = 1;
R0       = 1;
W0       = 1;
```

```
PARAMETERS
* Behaviour parameter
alpha   Cobb-Douglas (elasticity)
A       Cobb-Douglas (scale parameter)

* Benchmark levels for variables
C0      National consumption (demand)
INC0    National income
KD0     Demand for capital
KS0     Capital supply
LD0     Demand for labour
LS0     Labour supply
P0      Price of the commodity
R0      Remuneration of capital
W0      Remuneration of labour
Y0      National production in volume (supply)
;
```

Calibration

We all a signed values to all these variables

- In GAMS language:

```
* Calibration process
LS0        = 7000;
KS0        = 3000;

* Arbitrary prices
P0         = 1;
R0         = 1;
W0         = 1;
```

PARAMETERS
* Behaviour parameter
alpha   Cobb-Douglas (elasticity)
A   Cobb-Douglas (scale parameter)

* Benchmark levels for variables
C0   National consumption (demand)
INC0   National income
KD0   Demand for capital
~~KS0   Capital supply~~
LD0   Demand for labour
~~LS0   Labour supply~~
~~P0   Price of the commodity~~
~~R0   Remuneration of capital~~
~~W0   Remuneration of labour~~
Y0   National production in volume (supply)
;

Calibration

We still assign values to other benchmark variables α and A

- We still have to assign values to the other benchmark variables and to parameters.

Given equations 7 and 8:

7. $KD = KS$

8. $LD = LS$

we can easily calibrate KD0 and LD0.

```
KD0        = KS0;
LD0        = LS0;
```

```
PARAMETERS
* Behaviour parameter
 alpha    Cobb-Douglas (elasticity)
 A        Cobb-Douglas (scale parameter)

* Benchmark levels for variables
 C0       National consumption (demand)
 INC0     National income
 KD0      Demand for capital
 KS0      Capital supply            [struck through]
 LD0      Demand for labour
 LS0      Labour supply             [struck through]
 P0       Price of the commodity    [struck through]
 R0       Remuneration of capital   [struck through]
 W0       Remuneration of labour    [struck through]
 Y0       National production in volume (supply)
;
```

Calibration

Thus calibrate KD0 and LD0

- We still have to assign values to the other benchmark variables and to parameters.

Given equations 7 and 8:

7. KD = KS

8. LD = LS

we can easily calibrate KD0 and LD0.

```
KD0        = KS0;
LD0        = LS0;
```

PARAMETERS

* *Behaviour parameter*

alpha   Cobb-Douglas (elasticity)

A   Cobb-Douglas (scale parameter)

* *Benchmark levels for variables*

C0   National consumption (demand)

INC0   National income

~~KD0   Demand for capital~~

~~KS0   Capital supply~~

~~LD0   Demand for labour~~

~~LS0   Labour supply~~

~~P0   Price of the commodity~~

~~R0   Remuneration of capital~~

~~W0   Remuneration of labour~~

Y0   National production in volume (supply)

;

Calibration

Compute total income from equa. 4

- We still have to assign values to the other benchmark variables and to parameters.

Next, given equation 4:

$$4.\quad INC = W \cdot LS + R \cdot KS$$

we can calibrate INC0:

```
INC0      = W0*LS0 + R0*KS0;
```

PARAMETERS

** Behaviour parameter*

alpha   Cobb-Douglas (elasticity)
A   Cobb-Douglas (scale parameter)

** Benchmark levels for variables*

C0   National consumption (demand)
INC0   National income
~~KD0   Demand for capital~~
~~KS0   Capital supply~~
~~LD0   Demand for labour~~
~~LS0   Labour supply~~
~~P0   Price of the commodity~~
~~R0   Remuneration of capital~~
~~W0   Remuneration of labour~~
Y0   National production in volume (supply)
;

Calibration

Calibrate consumption from equa.5

- We still have to assign values to the other benchmark variables and to parameters.

Given equation 5:

$$5.\quad C = \frac{INC}{P}$$

we can calibrate C0:

```
C0          = INC0/P0;
```

```
PARAMETERS
* Behaviour parameter
 alpha    Cobb-Douglas (elasticity)
 A        Cobb-Douglas (scale parameter)

* Benchmark levels for variables
 C0       National consumption (demand)
 INC0     National income
 KD0      Demand for capital
 KS0      Capital supply
 LD0      Demand for labour
 LS0      Labour supply
 P0       Price of the commodity
 R0       Remuneration of capital
 W0       Remuneration of labour
 Y0       National production in volume (supply)
;
```

(The lines INC0 through W0 are struck through on the slide.)

Calibration

Equilibrium in goods market, Y0 = C0

- We still have to assign values to the other benchmark variables and to parameters.

And finally, given equation 6:

6. $C = Y$

we can calibrate Y0:

```
Y0          = C0;
```

```
PARAMETERS
* Behaviour parameter
 alpha     Cobb-Douglas (elasticity)
 A         Cobb-Douglas (scale parameter)

* Benchmark levels for variables
 C0        National consumption (demand)
 INC0      National income
 KD0       Demand for capital
 KS0       Capital supply
 LD0       Demand for labour
 LS0       Labour supply
 P0        Price of the commodity
 R0        Remuneration of capital
 W0        Remuneration of labour
 Y0        National production in volume (supply)
;
```

(Lines C0 through W0 are struck through.)

Calibration

Order of calibration is important

- Note that the order in which the calibration is done is very important:

```
* Calibration process
LS0      = 7000;
KS0      = 3000;

* Arbitrary prices
P0       = 1;
R0       = 1;
W0       = 1;

* Calibration of other parameters
KD0      = KS0;
LD0      = LS0;
INC0     = W0*LS0 + R0*KS0;
C0       = INC0/P0;
Y0       = C0;
```

- Given the way KD0 and LD0 are calibrated, KS0 and LS0 must be defined before.

Calibration

KD0 and LD0 must be defined first

- Note that the order in which the calibration is done is very important:

```
* Calibration process
LS0        = 7000;
KS0        = 3000;

* Arbitrary prices
P0         = 1;
R0         = 1;
W0         = 1;

* Calibration of other parameters
KD0        = KS0;
LD0        = LS0;
INC0       = W0*LS0 + R0*KS0;
C0         = INC0/P0;
Y0         = C0;
```

- Given the way KD0 and LD0 are calibrated, KS0 and LS0 must be defined before.

Calibration

All the parameters on the right hand side of INC must be defined befor

- Note that the order in which the calibration is done is very important:

```
* Calibration process
LS0       = 7000;
KS0       = 3000;

* Arbitrary prices
P0        = 1;
R0        = 1;
W0        = 1;

* Calibration of other parameters
KD0       = KS0;
LD0       = LS0;
INC0      = W0*LS0 + R0*KS0;
C0        = INC0/P0;
Y0        = C0;
```

- Given the way KD0 and LD0 are calibrated, KS0 and LS0 must be defined before.
- The same goes for INC0. All parameters on the right hand side must be defined before INC0.

Calibration

In the same way for C0

- Note that the order in which the calibration is done is very important:

```
* Calibration process
LS0      = 7000;
KS0      = 3000;

* Arbitrary prices
P0       = 1;
R0       = 1;
W0       = 1;

* Calibration of other parameters
KD0      = KS0;
LD0      = LS0;
INC0     = W0*LS0 + R0*KS0;
C0       = INC0/P0;
Y0       = C0;
```

- Given the way KD0 and LD0 are calibrated, KS0 and LS0 must be defined before.
- The same goes for INC0. All parameters on the right hand side must be defined before INC0.
- In the same way, INC0 and P0 must be calibrated before we can calibrate C0.

Calibration

Similarly Y0= C0

- Note that the order in which the calibration is done is very important:

```
* Calibration process
LS0       = 7000;
KS0       = 3000;

* Arbitrary prices
P0        = 1;
R0        = 1;
W0        = 1;

* Calibration of other parameters
KD0       = KS0;
LD0       = LS0;
INC0      = W0*LS0 + R0*KS0;
C0        = INC0/P0;
Y0        = C0;
```

- Given the way KD0 and LD0 are calibrated, KS0 and LS0 must be defined before.
- The same goes for INC0. All parameters on the right hand side must be defined before INC0.
- In the same way, INC0 and P0 must be calibrated before we can calibrate C0.
- Finally, C0 must be defined prior to Y0.

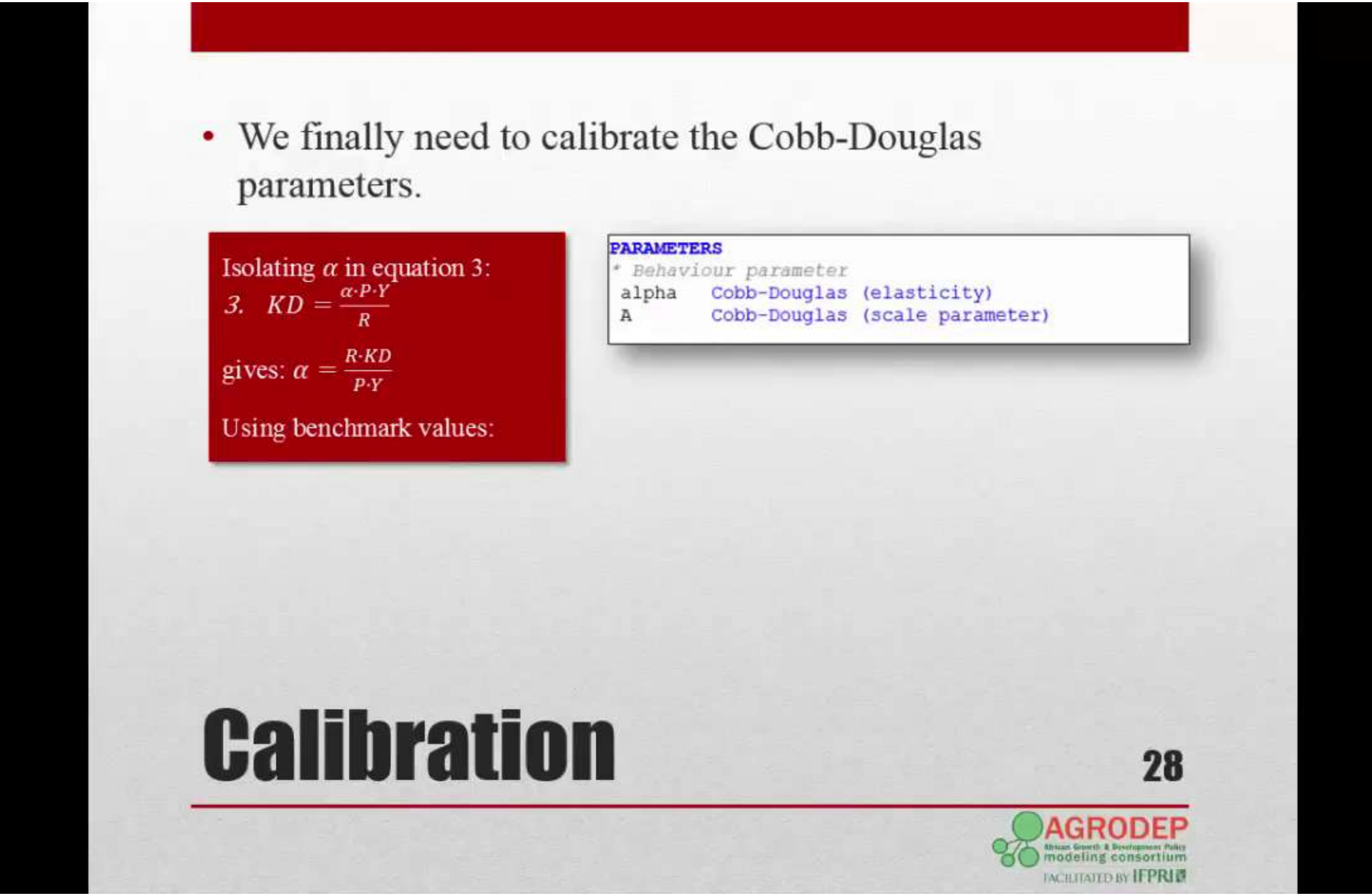
Calibration

Calibrate the Cobb-Douglas parameters

- We finally need to calibrate the Cobb-Douglas parameters.

```
PARAMETERS
* Behaviour parameter
 alpha    Cobb-Douglas (elasticity)
 A        Cobb-Douglas (scale parameter)
```

Solve equation 3 for α



Use bench mark values of R, KD, P &Y to compute α

- We finally need to calibrate the Cobb-Douglas parameters.

Isolating α in equation 3:

$$3.\quad KD = \frac{\alpha \cdot P \cdot Y}{R}$$

gives: $\alpha = \frac{R \cdot KD}{P \cdot Y}$

Using benchmark values:

```
alpha    = (R0*KD0)/(P0*Y0);
```

```
PARAMETERS
* Behaviour parameter
~~alpha      Cobb-Douglas (elasticity)~~
A          Cobb-Douglas (scale parameter)
```

Calibration

Use value of Y0, KD0, α & LD0 to compute A

- We finally need to calibrate the Cobb-Douglas parameters.

Similarly, isolating A in equation 1:

1. $Y = A \cdot KD^{\alpha} \cdot LD^{1-\alpha}$

gives: $A = \frac{Y}{KD^{\alpha} \cdot LD^{1-\alpha}}$

Using benchmark values:

```
PARAMETERS
* Behaviour parameter
alpha     Cobb-Douglas (elasticity)
A         Cobb-Douglas (scale parameter)
```

```
A          = Y0/((KD0**alpha)*(LD0**(1-alpha)));
```

Calibration

Order is important

- Again, the order in which the calibration is done is very important:

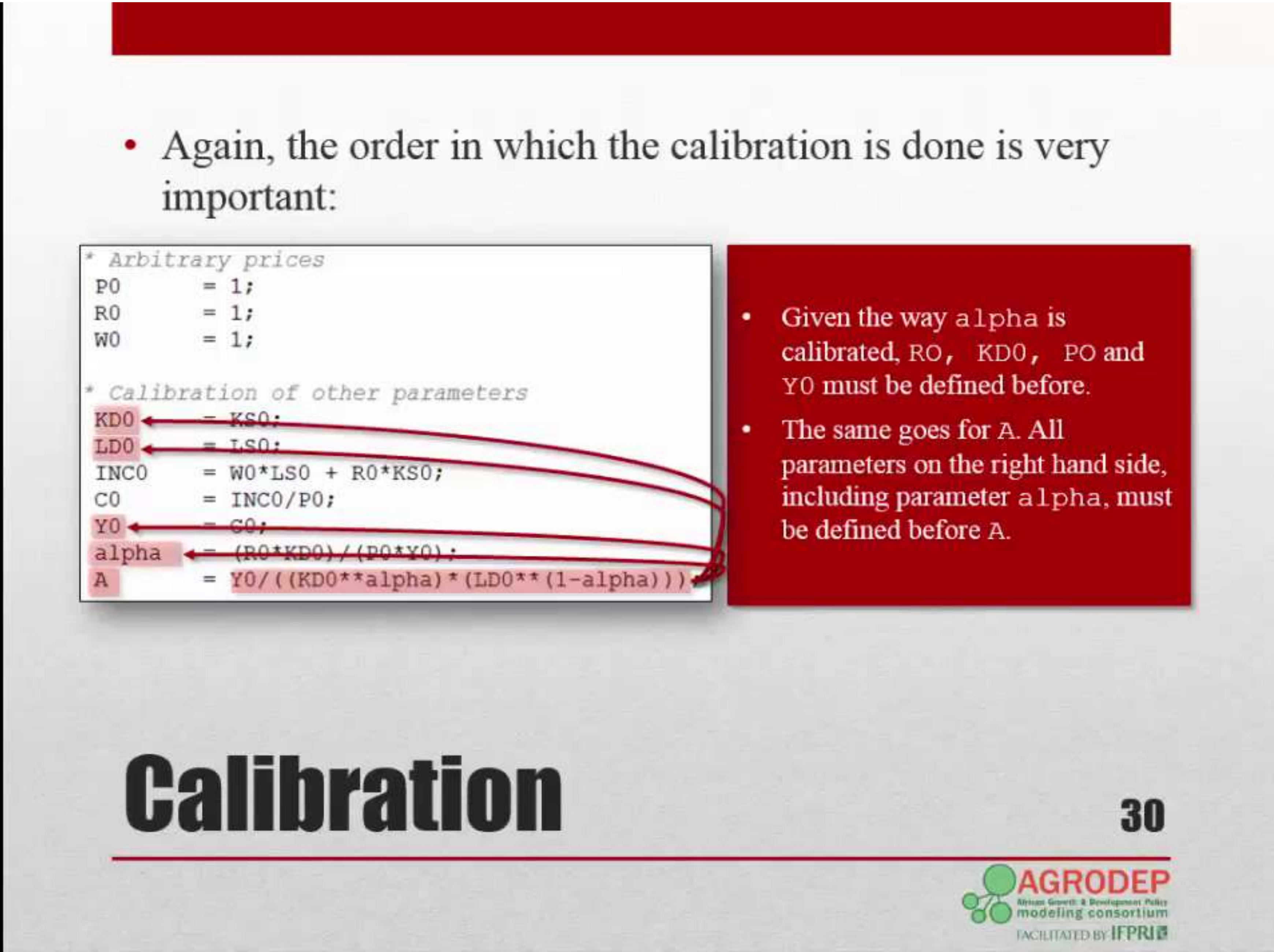
```
* Arbitrary prices
P0        = 1;
R0        = 1;
W0        = 1;

* Calibration of other parameters
KD0       = KS0;
LD0       = LS0;
INC0      = W0*LS0 + R0*KS0;
C0        = INC0/P0;
Y0        = C0;
alpha     = (R0*KD0)/(P0*Y0);
A         = Y0/((KD0**alpha)*(LD0**(1-alpha)));
```

- Given the way `alpha` is calibrated, `RO, KD0, PO` and `Y0` must be defined before.

Calibration

Do the same for A



Definition of variables

- After completing the calibration process, define the model per se.
- This part of the GAMS code typically includes two main parts.
- Definition of variables in the model
- Definition of equations in the model

In this case we do have 10 variables

- Consistent with the mathematical structure, we have 10 variables to define.

```
VARIABLES
 C        National consumption (demand)
 INC      National income
 KD       Demand for capital
 KS       Capital supply
 LD       Demand for labour
 LS       Labour supply
 P        Price of the commodity
 R        Remuneration of capital
 W        Remuneration of labour
 Y        National production in volume (supply)
 leon     Walras
;
```

Model

Add *leon*

- Add the above variable in reference to *Leon Walras*
- It is an extra variable
- It verifies consistency of the model
- It could verify the market equilibrium for which the equation is redundant or dependent is satisfied.

We have 7 endogenous variables, we must define equations

- As we have 7 endogenous variables, we must define 7 equations.

```
EQUATIONS
 Y_EQ     Production
 LD_EQ    Labour demand
 KD_EQ    Capital in GDP
 INC_EQ   Income
 C_EQ     Consumption
 P_EQ     Equilibrium on goods market
 W_EQ     Equilibrium on labour market
 leon_EQ Verification of the Walras law
;

 Y_EQ..         Y =e= A*[KD**alpha]*[LD**(1-alpha)] ;
 LD_EQ..        LD =e= [(1-alpha)*P*Y]/W;
 KD_EQ..        KD =e= [alpha*P*Y]/R ;
 INC_EQ..       INC =e= W*LS + R*KS;
 C_EQ..         C =e= INC/P;
 P_EQ..         Y =e= C;
 W_EQ..         LD =e= LS;
 leon_EQ..      leon =e= KD - KS;
```

Model

Each equation reflects the corresponding mathematical structure of the model

- As we have 7 endogenous variables, we must define 7 equations.
- Again, given Walras' law, we define an extra equation, `leon_EQ`, that will compute the difference between capital supply and demand in order to verify that the equilibrium on this market is indeed respected.

```
EQUATIONS
 Y_EQ    Production
 LD_EQ   Labour demand
 KD_EQ   Capital in GDP
 INC_EQ  Income
 C_EQ    Consumption
 P_EQ    Equilibrium on goods market
 W_EQ    Equilibrium on labour market
 leon_EQ Verification of the Walras law
;

 Y_EQ..        Y =e= A*[KD**alpha]*[LD**(1-alpha)] ;
 LD_EQ..       LD =e= [(1-alpha)*P*Y]/W;
 KD_EQ..       KD =e= [alpha*P*Y]/R ;
 INC_EQ..      INC =e= W*LS + R*KS;
 C_EQ..        C =e= INC/P;
 P_EQ..        Y =e= C;
 W_EQ..        LD =e= LS;
 leon_EQ..     leon =e= KD - KS;
```

Model

Initialization

- If the capital market is in equilibrium, then leon will be zero.
- If we say SOLVE after we have done the model GAMS will look for a solution, but it will use 0 as starting value for every variable.
- In other words GAMS will start the process of finding a solution with values of variables very far from the solution value, this will make the solver harder and inefficient
- It is a common practice in CGE modeling to initialize each variable at its benchmark value.
- In this way, if the calibration was properly done, the solver will automatically find the solution, which will simply replicate the benchmark data.
- The solver do not work hard to solve the model

The initialization process

- The initialization process consists in assigning a starting value for every variable, using the .L suffix for endogenous variables.
- Similarly, use .FX suffix for exogenous variables.
- Formulate the equality equation with the benchmark values computed previously.

Look the following table

- The initialization process consists in assigning a starting value for every variable, using the `.L` suffix for endogenous variables:

```
* Initialisation
C.l      = C0;
KD.l     = KD0;
LD.l     = LD0;
INC.l    = INC0;
R.l      = R0;
W.l      = W0;
Y.l      = Y0;
```

These are the calibrated benchmark values computed previously.

- and the `.FX` suffix for exogenous variables:

```
* Closure
KS.fx    = KS0;
LS.fx    = LS0;
* Numeraire
P.fx = P0 ;
```

labor and capital supply are fixed at their benchmark values.

The price of commodity Y is the *numeraire* of the model.

Initialization

The model resolution

- It is the end point of the GAMS, code
- Remember that we first to define the model to be solved using the model command; CGE
- It should include the word all, which implies that all equations.
- Then include the solve command
- ***See the following slide***

The ending command

- Finally, the GAMS code ends with the resolution commands. Remember from previous lessons that we need first to define the model to be solved, using the MODEL command:

```
MODEL CGE Model lesson 8 /all/ ;
```

We called our model CGE.

It includes all equations.

- Then, we can include the SOLVE command:

```
SOLVE CGE using cns ;
```

cns stands for "*Constrained Nonlinear System*". Note that this solution procedure requires the model to be square. In other words, there needs to be the same number of endogenous variables as there are independent equations.

Resolution

Running the GAMS

- In order to solve the equation run the GAMS.
- The can be done either by pressing the red arrow icon on GAMS or F9 of you keyboard.

The process window

- If no error is encountered, the process window should look like this:

```
   C O N O P T 3   version 3.15M
   Copyright (C)   ARKI Consulting and Development A/S
                   Bagsvaerdvej 246 A
                   DK-2880 Bagsvaerd, Denmark


  Iter Phase Ninf   Infeasibility   RGmax    NSB   Step InItr MX OK
     0   0        0.0000000000E+00 (Input point)
                               Pre-triangular equations:       1
                               Post-triangular equations:      1
     1   0        0.0000000000E+00 (After pre-processing)
     2   0        0.0000000000E+00 (After scaling)
     2            0.0000000000E+00

 ** Feasible solution to a square system.

--- Restarting execution
--- Lesson8.gms(93) 2 Mb
--- Reading solution for model CGE
*** Status: Normal completion
```

Resolution

Note the following

- The infeasibility at the input point should be equal to 0.
- It represents the largest gap between the left and right hand side; given the values assigned to the model variables and parameters.
- The values assigned to the parameters and model are solutions to the systems of the equations.
- This confirms that the calibration was properly done
- The calibration was consistent with the model structure
- The equations are correctly written
- All the variables are correctly initialized

- If so, then the solution of the model simply replicates the benchmark values for all variables. Indeed, in the LST file, you should have:

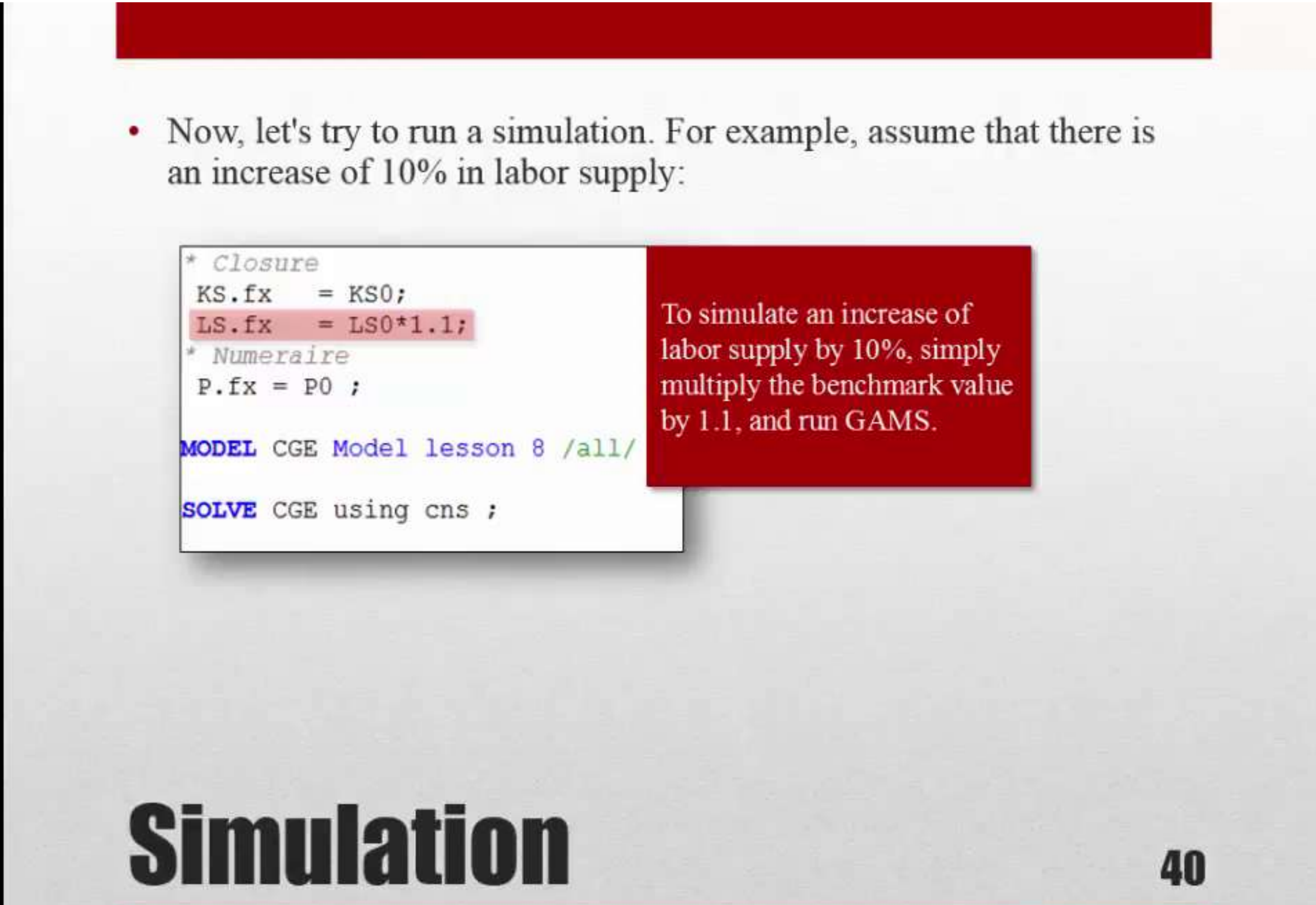
	LOWER	LEVEL	UPPER
---- VAR C	-INF	10000.000	+
---- VAR INC	-INF	10000.000	+
---- VAR KD	-INF	3000.000	+
---- VAR KS	3000.000	3000.000	3000
---- VAR LD	-INF	7000.000	+
---- VAR LS	7000.000	7000.000	7000
---- VAR P	1.000	1.000	1
---- VAR R	-INF	1.000	+
---- VAR W	-INF	1.000	+INF
---- VAR Y	-INF	10000.000	+INF
---- VAR leon	-INF	.	+INF

These are the values, at benchmark, for all of the variables in the model.

Resolution

Simulation

Assume that there is 10% increment in the labor supply.



Solution window

- If no error is encountered, the process window should look like this:

```
  Iter Phase Ninf   Infeasibility   RGmax    NSB   Step InItr MX OK
     0   0        1.4000000000E+03 (Input point)
                               Pre-triangular equatio
                               Post-triangular equatio
     1   0        2.0899304492E+03 (After pre-processi
     2   0        3.4056768178E-01 (After scaling)
    10   0      0 5.4751484369E-07                1.0E
    15            5.3033066827E-10

 ** Feasible solution to a square system.

--- Restarting execution
--- Lesson8.gms(93) 2 Mb
--- Reading solution for model CGE
*** Status: Normal completion
```

Note that the infeasibility at the input point is not equal to zero anymore, as we introduced a shock.

Nevertheless, you must verify that GAMS found a solution.

Simulation

After the shock

- The solution of the model will now show values that are different from the benchmark values. Indeed, in the LST file, you should have:

	LOWER	LEVEL	UPPER
---- VAR C	-INF	10689.930	+INF
---- VAR INC	-INF	10689.930	+INF
---- VAR KD	-INF	3000.000	+INF
---- VAR KS	3000.000	3000.000	3000.000
---- VAR LD	-INF	7700.000	+INF
---- VAR LS	7700.000	7700.000	7700.000
---- VAR P	1.000	1.000	1.000
---- VAR R	-INF	1.069	+INF
---- VAR W	-INF	0.972	+INF
---- VAR Y	-INF	10689.930	+INF
---- VAR leon	-INF	-1.632E-6	+INF

Simulation

The new solutions

- The solution of the model will now show values that are different from the benchmark values. Indeed, in the LST file, you should have:

	LOWER	LEVEL	UPPER
---- VAR C	-INF	10689.930	+INF
---- VAR INC	-INF	10689.930	+INF
---- VAR KD	-INF	3000.000	+INF
---- VAR KS	3000.000	3000.000	3000.000
---- VAR LD	-INF	7700.000	+INF
---- VAR LS	7700.000	7700.000	7700.000
---- VAR P	1.000	1.000	1.000
---- VAR R	-INF	1.069	+INF
---- VAR W	-INF	0.972	+INF
---- VAR Y	-INF	10689.930	+INF
---- VAR leon	-INF	-1.632E-6	+INF

These are the values, after simulation, for all of the variables in the model.

Verify that the value of `leon` is very close to 0 (it is the case here). If not, you must verify that your model is correctly specified and that there is no error in writing them.

Simulation

Conclusion

- GAMS is widely used for CGE model
- Write the mathematical structure and verify that you same number of endogenous variables as you have equations.
- Keep attention for Walras law.
- The model should be square, but should consist of independent equations only.
- Calibrate every parameter and benchmark variable based on your data source and consistently with your set of equations.
- Make sure your model replicates your data
- Verify this under simulation, all markets are in equilibrium through an additional variable, *leon*.
- Check the coefficient for *leon* is very close to 0

Benefit and Drawbacks of CGE

Social **A**ccounting **M**atrix for Closed & Open Economy

- A SAM is a particular representation of the macro and meso economic accounts of an economic system.
- It captures the transactions and transfers between all economic agents in the system.
- SAM is a comprehensive, economy-wide database that contains information about the flow of resources and associated transactions between economic agents in a given period of time.
- It is not only about productive activities in the economy, but also other non-productive institutions and markets like factor, capital, households, government, and the rest of the world.
- A SAM can be described as a "***comprehensive, economy-wide data framework***"
- SAM identify all monetary flows from sources to recipients, within a disaggregated national account.

Social **A**ccounting **M**atrix

- Additionally, it has **two principle objectives**:-

  1. It **organize information** about the economic and social structure of an economy in a given period (usually one year).

  2. It provides the statistical basis for the creation of plausible **economic models**.

- In **organizing data** accounts are included in the SAM to represent agents that are involved in **economic transactions**.
- Transactions are captured in the relevant accounts of the SAM, showing the values and direction of the flow of resources.
- A SAM thus forms a **complete database** of all transactions that take place between agents in a given period.
- It presents a '**static image**' or '**snapshot picture**' of the structure of an economy for that period.

Social **A**ccounting **M**atrix

- SAM accounts are created for all economic agents, including producers, households, government and the rest of the world.
- Accounts also exist for commodity, factor and capital markets.
- It incorporate any type of transactions such as the **purchasing of intermediate goods** and **hiring of factors**.
- It also includes **current account transactions** of institutions (households, enterprises and government) like inter-institutional transfers, consumption expenditure and the payment of various taxes.
- It further includes **capital account transactions** of institutions, such as **savings** and **investments**.
- Finally, it can include any transaction that takes place across **international borders**, such as foreign direct investment and international trade transactions.

Social **A**ccounting **M**atrix

- Important property of SAM is transaction values should be reported in a **consistent manner**.
- Each account clearly shows the **value of the transaction** and the **accounts** that are **debited** and **credited**.
- "Economic accounting is based on a fundamental principle of economics: For every **income** or **receipt** there is a corresponding **expenditure** or **outlay**. This principle makes up the macroeconomic accounts of any country."
- A SAM is a square matrix where its column represents a specific account in the economy, and the exact/same accounts also appear in the rows.
- **Incomes** or **receipts** are shown in the rows of the SAM, *i.e.* if households receive **1000** the amount should appear in one of the cells along the household row account.

Social **A**ccounting **M**atrix

- Expenditures or outlays are shown in the column, *i.e*. if government spends 1000 the amount should appear on the column.

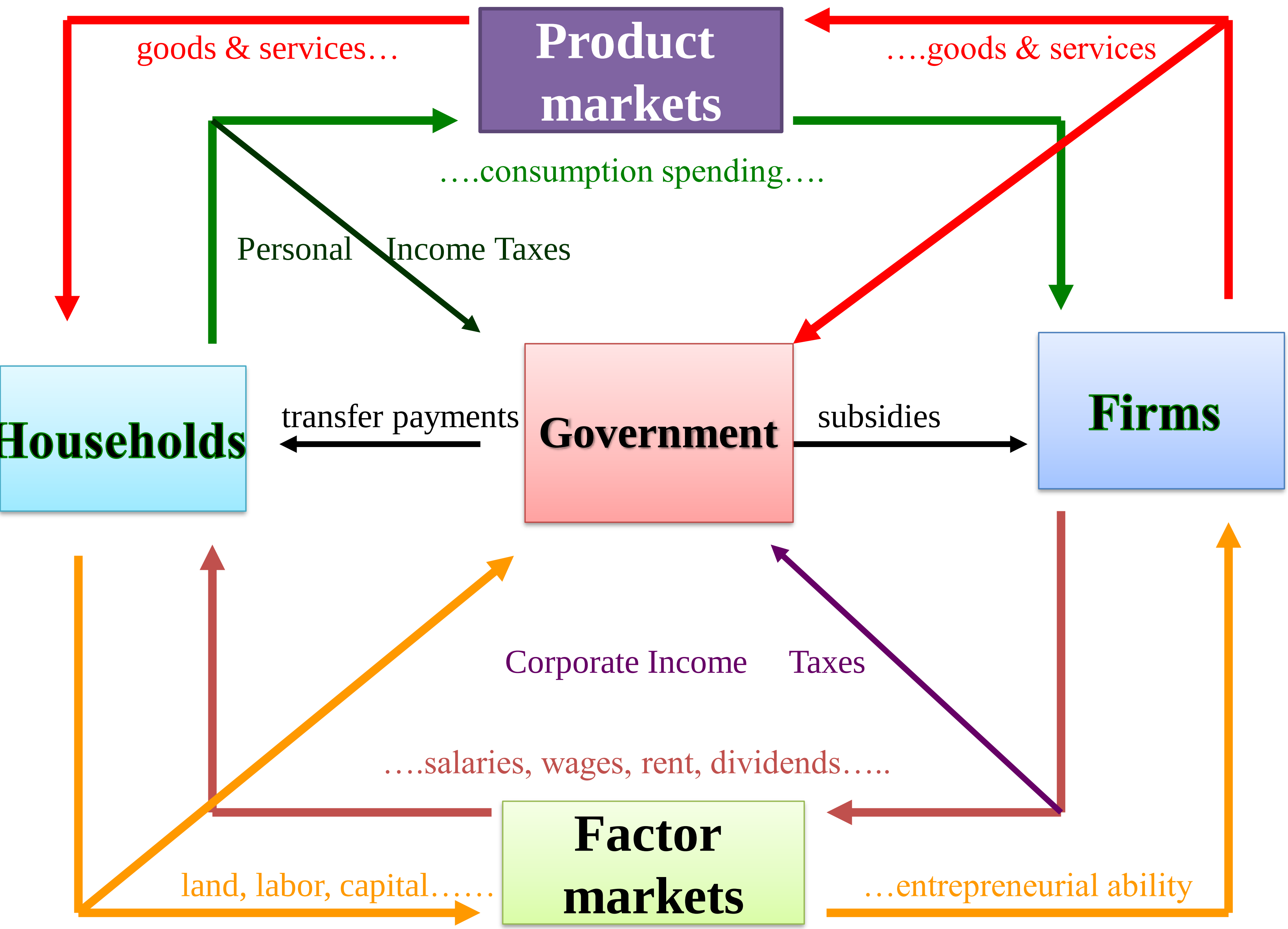
	Households	**Government**	**Firms**	**RoW**
Household		**1000**		
Government				
Firms				
RoW				

- In a SAM, the income of each account (**row total**) must equal the expenditure of each account (**column total**).
- A SAM ensures that all accounts are simultaneously balanced.
- SAM can be described as complete and consistent.

Social **A**ccounting **M**atrix

Resource flows in a multi-sector economy

- **Government**, **households** and **business enterprises** are all regarded as **institutions**.
- All the **produces** may be flow to each account.
- **Inter-institutional transfers** can easily be identified in the more detailed institutional breakdown typically used in a SAM.
- **All income** not spent by **households**, **enterprises** and **government** is saved.
- Government savings can be **negative** (**budget deficit**) or **positive** (**budget surplus**).
- ***See the following simple figure for resource flow***


Product markets
goods & services…
….goods & services
….consumption spending….
Personal Income Taxes
Households
transfer payments
Government
subsidies
Firms
Corporate Income Taxes
….salaries, wages, rent, dividends…..
Factor markets
land, labor, capital……
…entrepreneurial ability

Social **A**ccounting **M**atrix

The accounts of a SAM

- SAMs are usually constructed with **six types of accounts**.
- Institutional accounts are disaggregated into separate accounts for **households**, **business enterprises** and **government**.
- Each account is represented by a **row** and **column** in SAM, where **row** entries represents **flows into the account**, while **column** entries represent **flows from the account**.
- Each account must be **balanced**, *i.e.* flows **into** the account must match flows **from** the account.

SAM

- **Structure of a SAM for a Basic Model of an Open Economy**

	Activity	Commodity	Household	Rest of World	Total
Activity		Domestic Sales		Exports	Total Sales
Commodity	Intermediate Consumptio n		Household Consumption		Total Absorption
Household	Payments to Factors of Production			Balance of Trade	Total Household Income
Rest of World		Imports			Total Earnings of Rest of the World
Total	Total Factor Payments	Total Supply of Consumption Goods	Total Household Expenditure	Total Expenditure by Rest of World	

- A macroeconomic SAM.docx

SAM

- An accounting framework that reflects the circular flow of economic activity.
- A square matrix: dimension based on the number of sectors and agents considered.
- Each entry represents a payment to a row-account by a column-account.
- Consistency implies that row total must equal corresponding column total.
- Also, if all but one accounts balance, the last one must balance as well (Walras’ law).

SAM
Hypothetical example

	Commodity	Factor	Enterprise	Gov.	RoW.	Total
Commodity		120		40	20	180
Factor	110		15	35		160
Enterprise		30	150		20	200
Gov.	50				30	80
RoW.	20	10	35	5	5	75
Total	180	160	200	80	75	

- The values in each row and column should be equal.
- The total sum should be equal in the two sides (row and column).

Chapter Two

- **Microeconomic Theories of Consumers and Producers**
- **Producer theory:** Demand for factors of production (Theory of production functions)

  Cobb Douglas production functions

  Leontief production functions

  CES production function

  Demand for Production Factors, Supply of Goods, Exercises using GAMS
- **Consumer Theory:** Demand for commodities:

  Cobb Douglas utility function

  Stone Geary utility function (linear expenditure system)

  CES utility function

  Demand for Goods

  Exercise Two using GAMS

  Open Market case and Current Account

Producer theory

- **Demand for factors of production**

  (Theory of production functions)
- A production function tells us the maximum output a firm can produce (in a given period) given available inputs.
- It is the economist's way of describing technology or engineering relationships.
- $Q = f(K, L)$

Production Jargon

- *Factors of production*: Inputs used in production (for example, *K* and *L*).
- *Production set*: The set of points (combinations of inputs and outputs) that are feasible according to the production function.
- *Technically inefficient*: feasible production points that yield less than the maximum possible for given amounts of inputs.
- *Technically efficient*: feasible production points that yield the maximum possible output for given amounts of inputs.

Production Theory

- Factor Markets
- Factors of Production: Land, Labour, Capital, Entrepreneurship
- Focus on Labour

Derived Demand

- The demand for a factor (labour) is a derived demand for the product that the factor produces.
- Recall the income and substitution effects of a wage increase (*i.e*. labour-leisure trade-off)

Labour Demand

Maximise output subject to $wl + rK = \overline{C}$

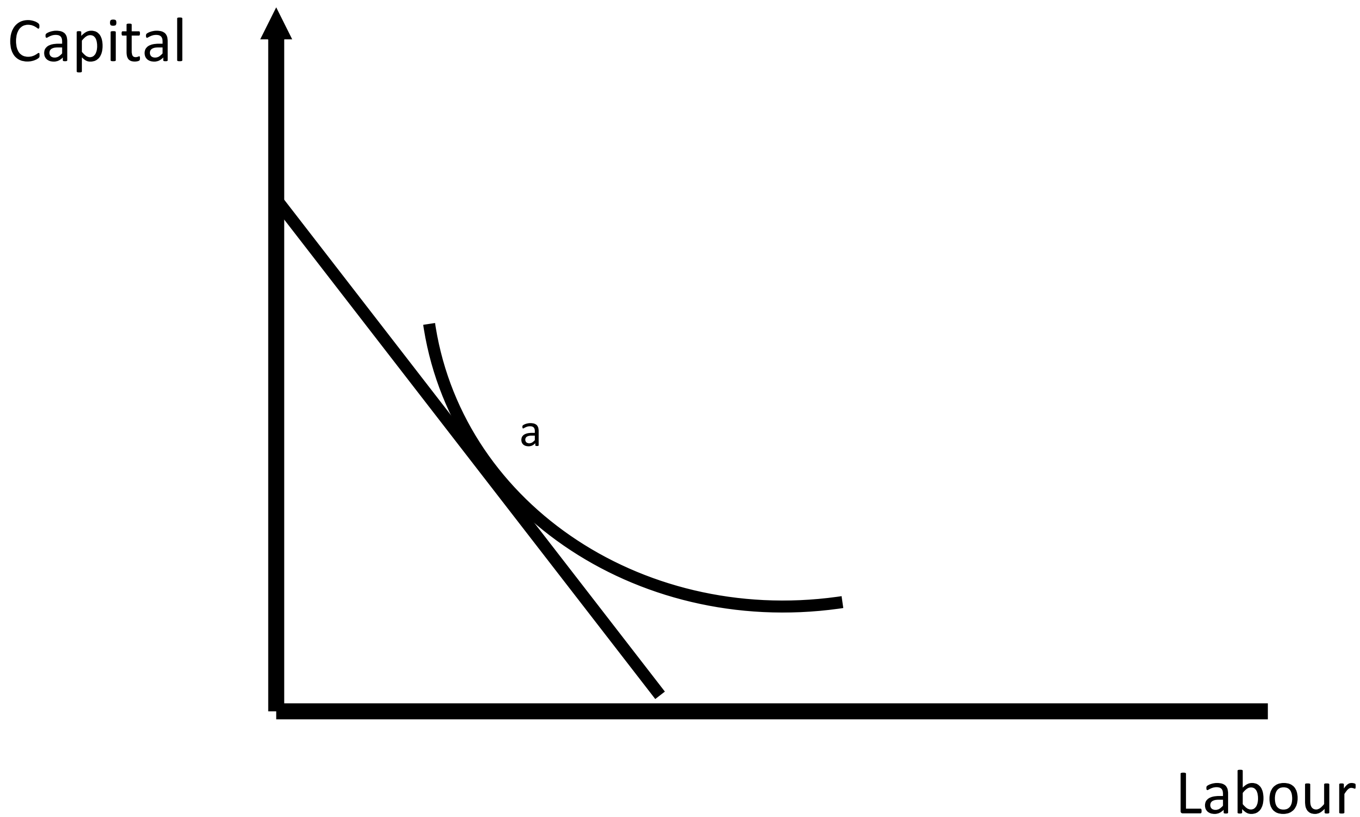
$$\rightarrow \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{w}} \quad \textbf{?}$$

Minimise cost subject to $\mathrm{F}(\mathrm{K}, \mathrm{L}) = \overline{Y}$

$$\rightarrow \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{w}} \quad \textbf{?}$$

Labour Demand

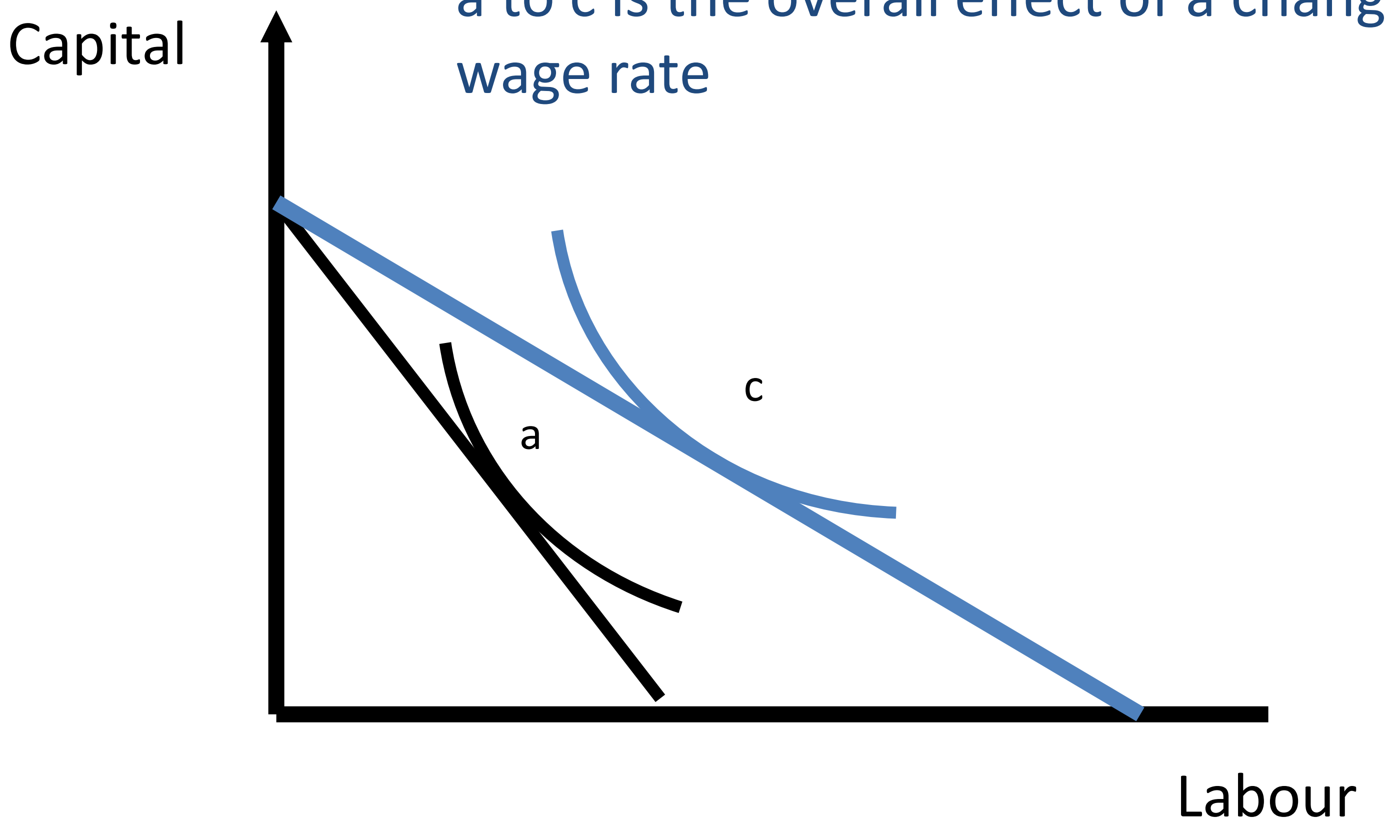
Maximize output subject to fixed amount of costs



Labour Demand

Maximise output subject to fixed amount of costs

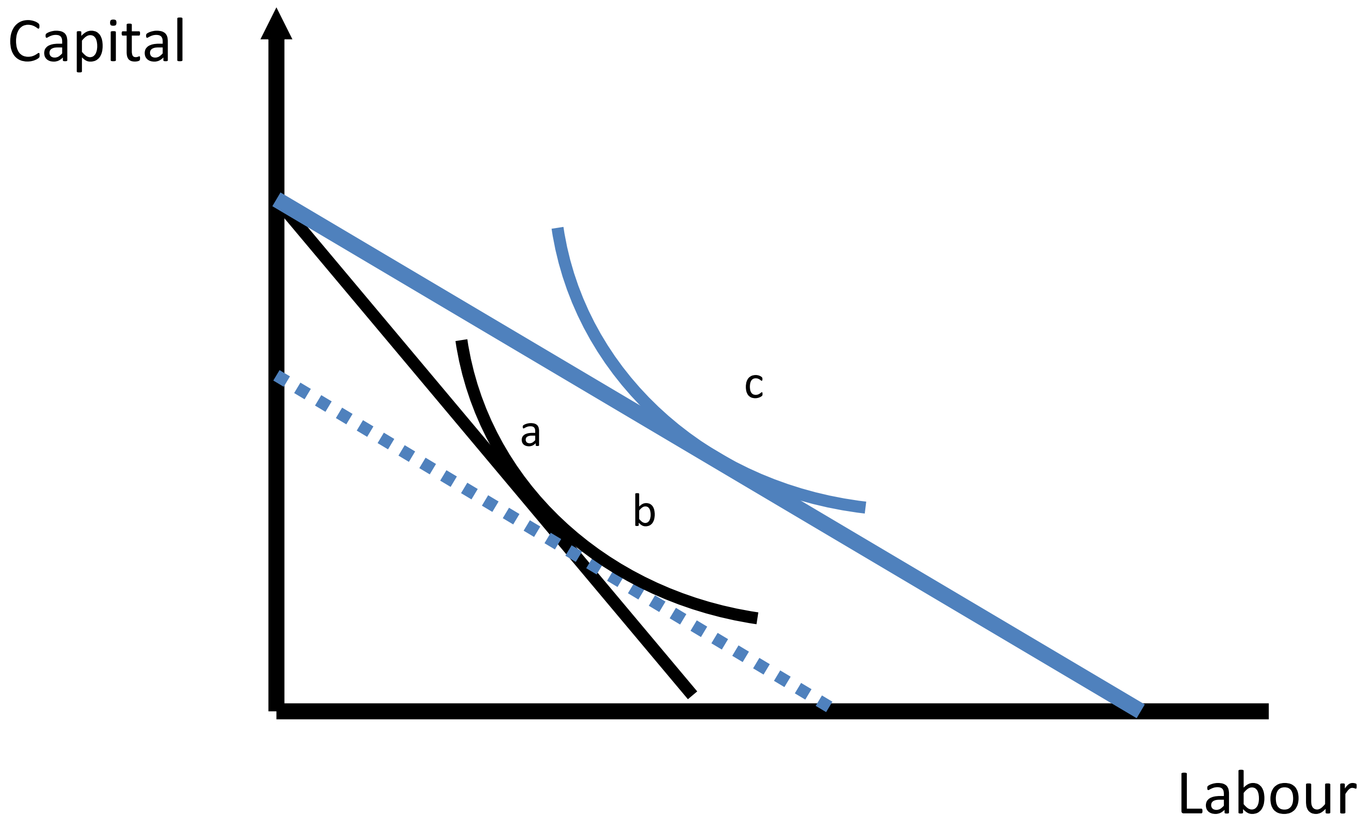
a to c is the overall effect of a change (fall) in the wage rate



Labour Demand

Maximise output subject to fixed amount of costs

The substitution effect is a to b (b to c is the output effect)



Labour Demand

Minimise costs subject to output constraint

$wL + rK = C$

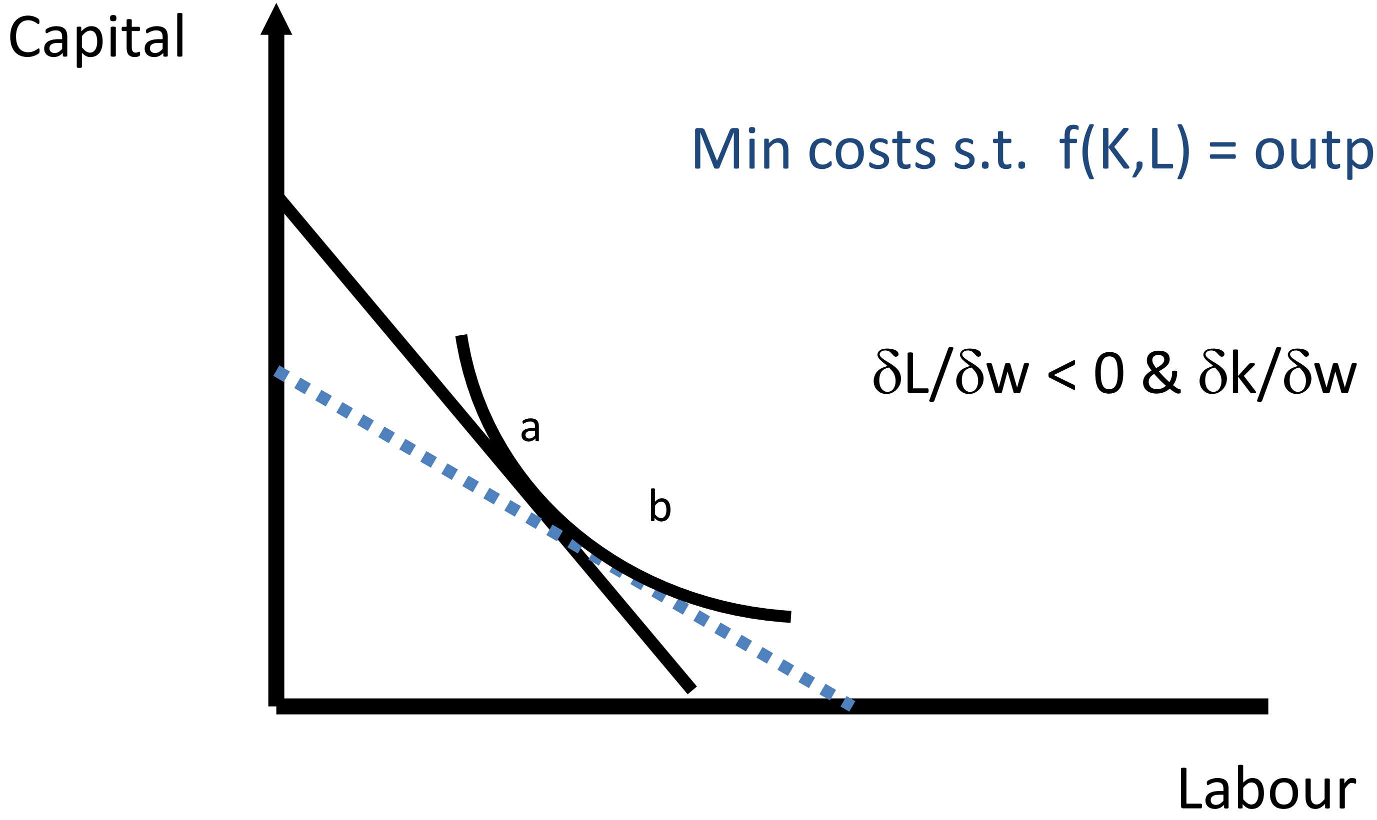
$Y = f(K,L)$

Labour Demand

The substitution effect is a to b

Min costs s.t. f(K,L) = output (fixed level)

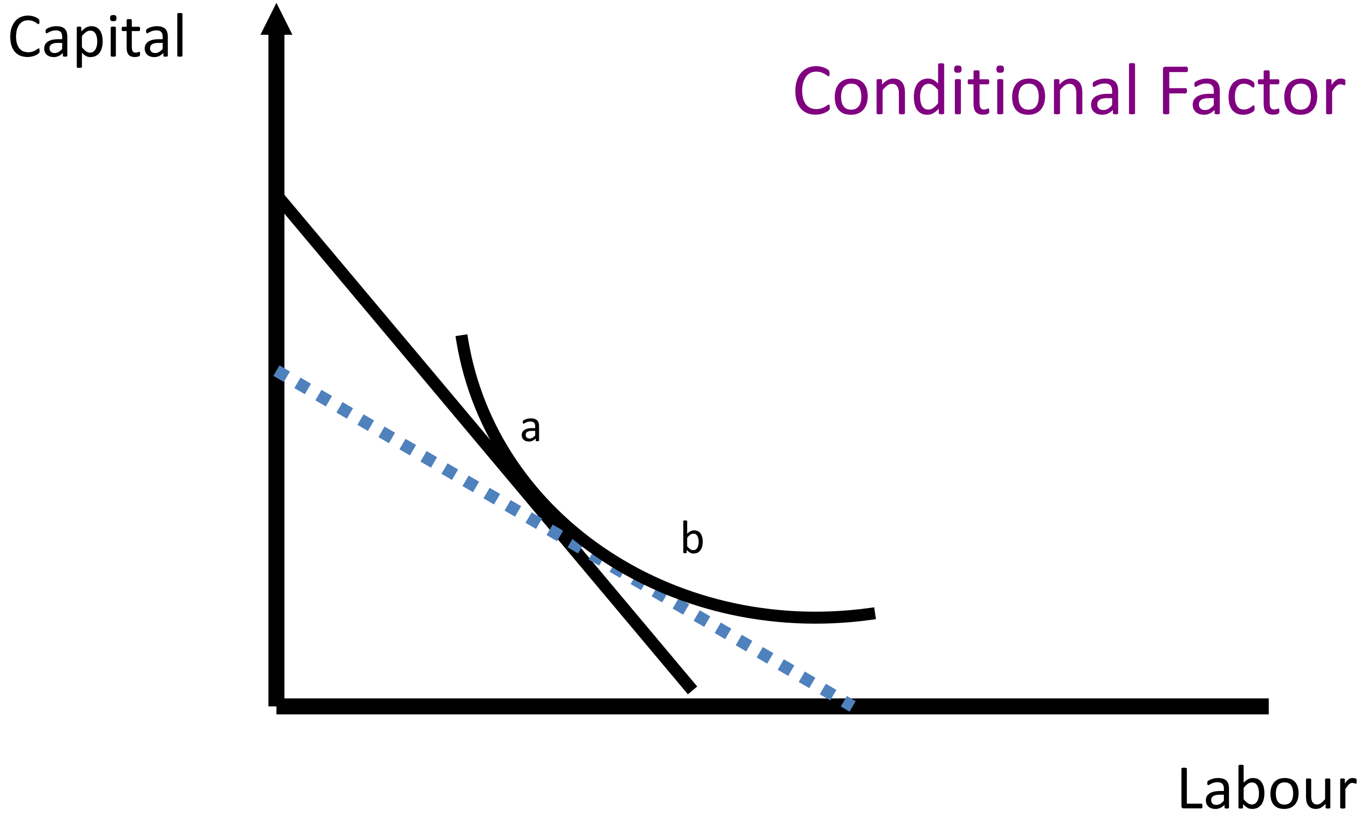
$\delta L/\delta w < 0$ & $\delta k/\delta w > 0$



Labour Demand

Min costs s.t. F(K,L) = output (fixed level)

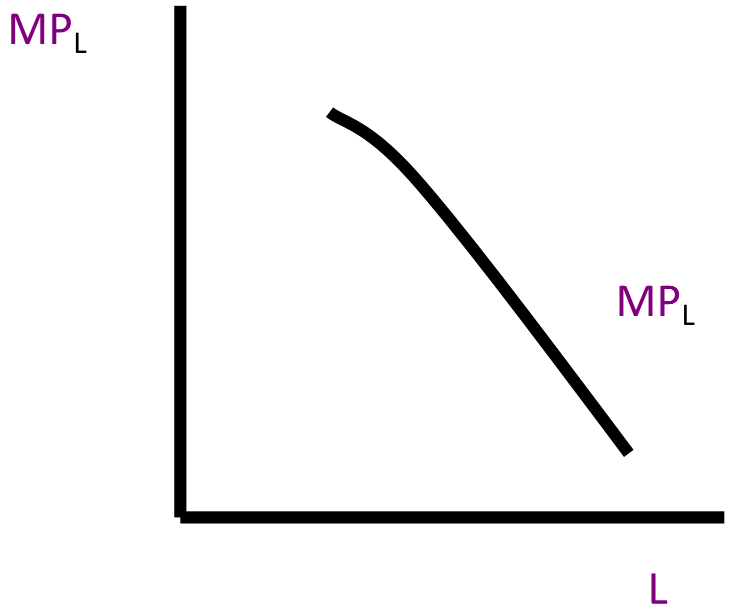
Conditional Factor Demand



Short Run: Firm's Demand for Labour

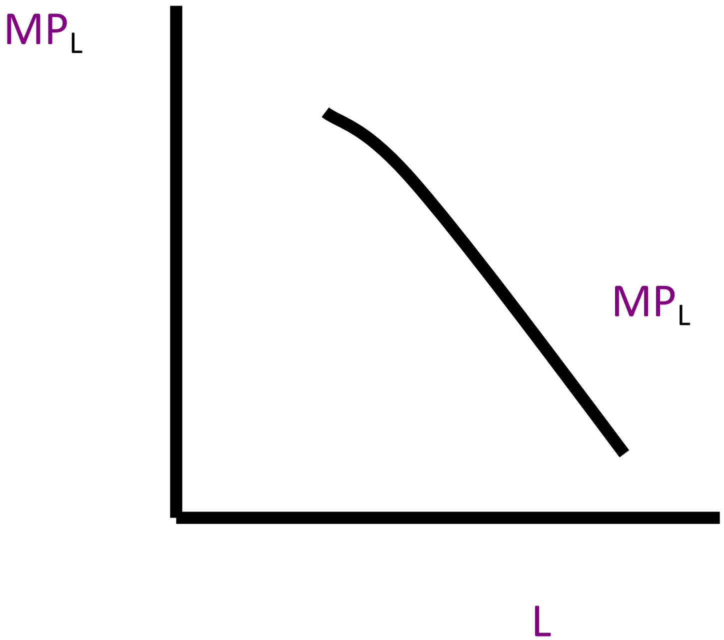
- Restrictions: Product and factor markets are assumed to be perfectly competitive.
- Capital is fixed, Labour is variable.
- As labour is increased the extra output resulting from the additional unit of labour declines, i.e. MP_L declines.
- The additional revenue from employing an extra unit of labour is referred to as the Value of the Marginal Product (VMP_L) = $P.MP_L$.

Short Run: Firm's Demand for Labour



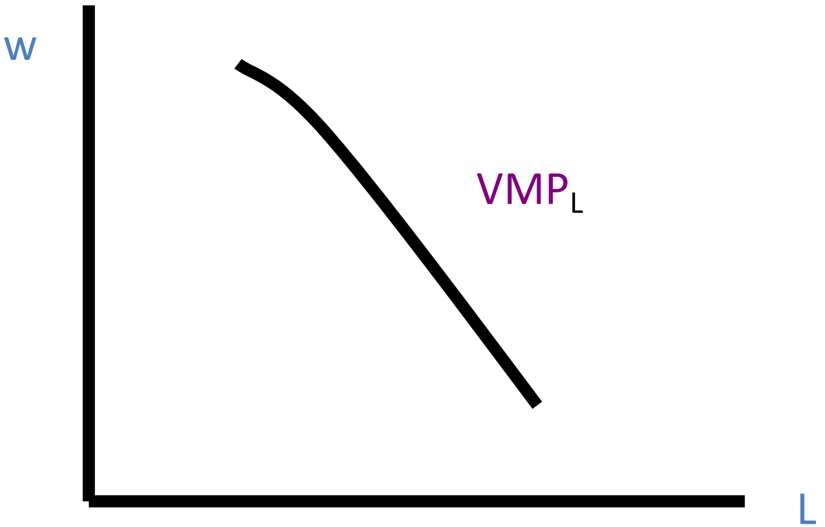
Diminishing Marginal Returns

Short Run: Firm's Demand for Labour

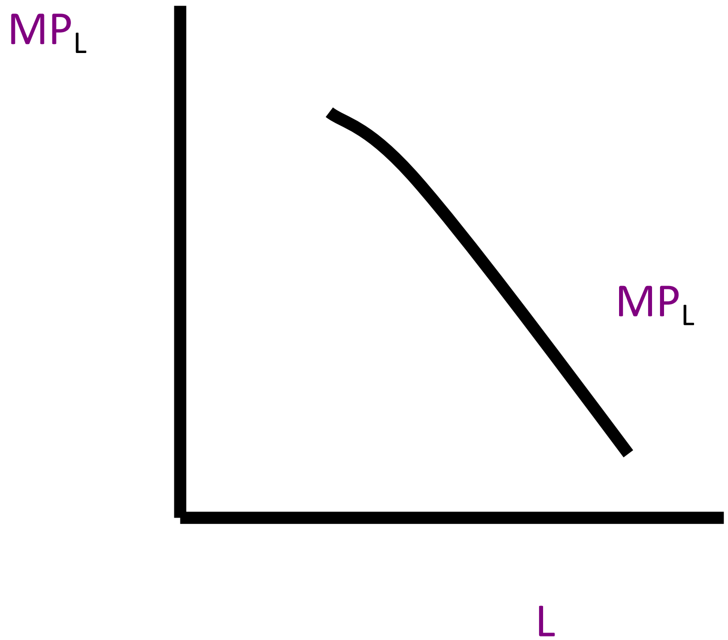


Diminishing Marginal Returns

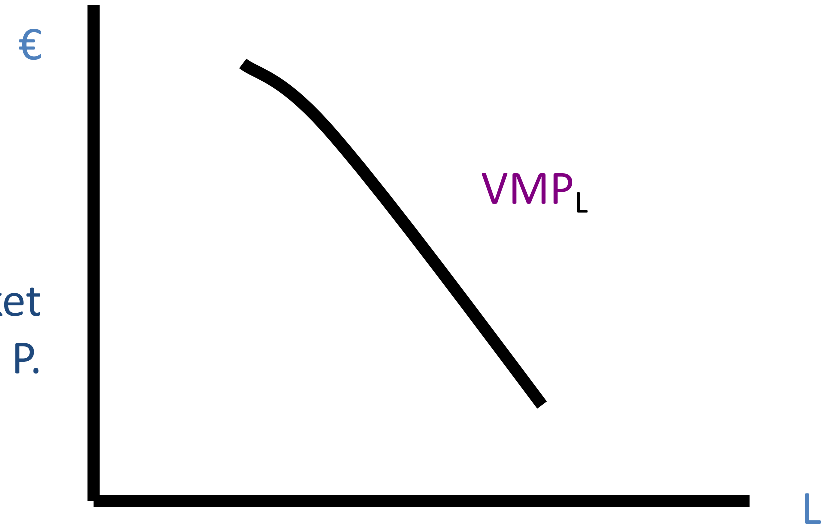
$VMP_L = P.MP_L$



Short Run: Firm's Demand for Labour



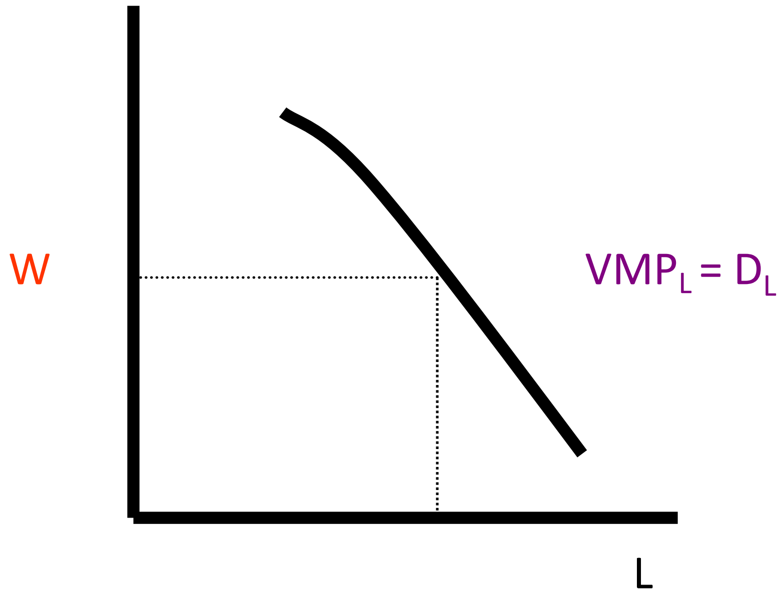
Diminishing Marginal Returns



$VMP_L = P.MP_L$

Perfect competition in product market assumed, i.e. firm cannot change P.

Short Run: Firm's Demand for Labour



$W = VMP_L = P.MP_L$

Demand for labour by the firm

The wage is the marginal cost of an extra unit of labour and VMP_L is the marginal benefit of an extra unit of labour.

Short Run: Market Demand for Labour

As wage ↓

⇒L_d ↑ for each firm

⇒Output ↑

⇒Price (P) ↓

⇒VMP_L shifts inwards

⇒Market demand for labour curve is less elastic than the firm demand for labour curve

Exercise for yourself

Long Run: Firm's Demand for Labour

As wage $\downarrow$

$\Rightarrow$ L $\uparrow$ and usually K $\uparrow$

$\Rightarrow$ If K $\uparrow$ then MP_L $\uparrow$

$\Rightarrow$ L $\uparrow$ again (i.e. an extra "kick")

$\Rightarrow$ The long run demand for labour curve is more elastic than the short run demand for labour curve

Exercise for yourself

Long Run: Market Demand for Labour

As wage ↓

$\Rightarrow$Ld ↑ for each firm

$\Rightarrow$Output ↑

$\Rightarrow$Price (P) ↓

$\Rightarrow VMP_L$ shifts inwards

$\Rightarrow$Long run market demand for labour curve is less elastic than the short run market demand for labour curve

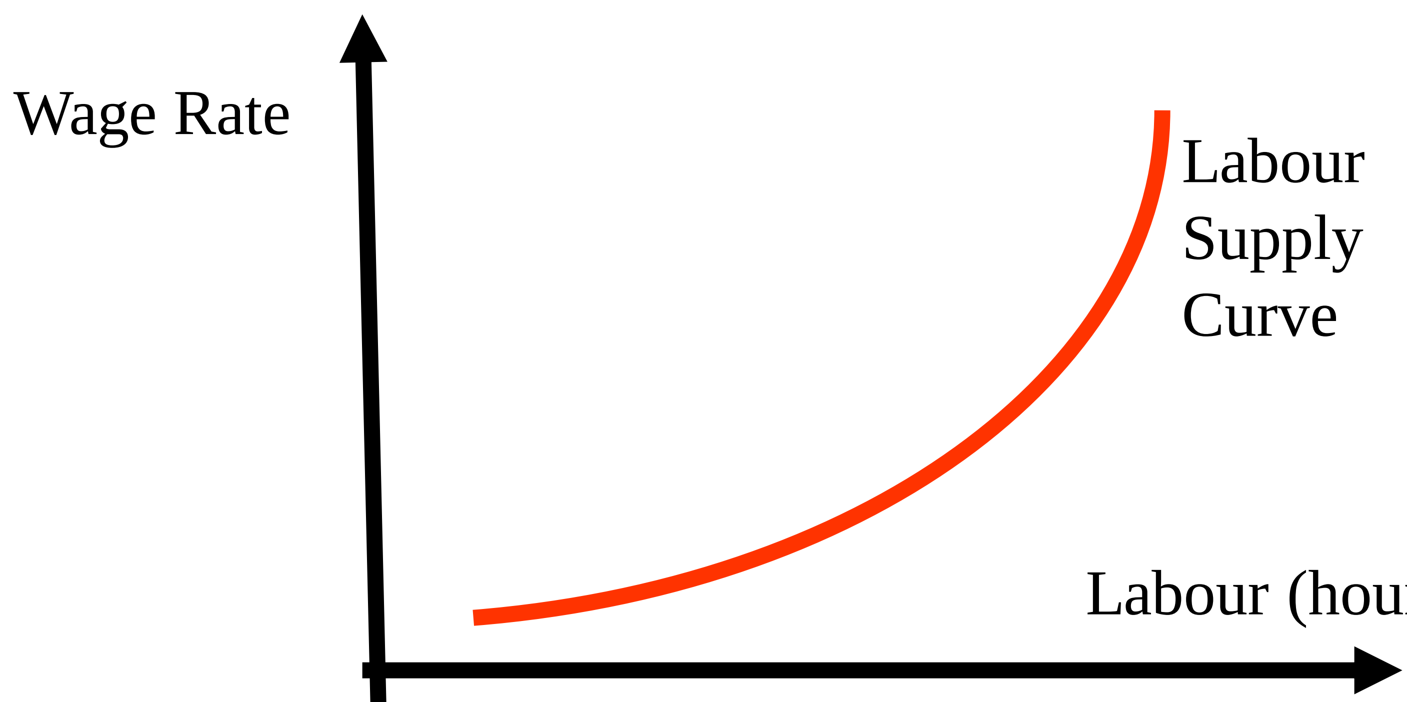
Exercise for yourself

Individual Supply of Labour

Substitution Effect: w ↑ ⇒ Price of Leisure ↑ ⇒ Demand for leisure ↓ ⇒ Supply of Labour ↑

Income Effect: w ↑ ⇒ Real Income ↑ ⇒ Demand for leisure ↑ (↓) if leisure normal (inferior) ⇒ Supply of Labour ↓ (↑)

Market Supply of Labour (Review)



Production Function

- The Production Function measures the maximum possible output that the firm can produce from a given amount of inputs (such as labor, capital, land, raw materials, etc) by given technology..

General Notation : $q = f(x_1, x_2, \ldots, x_n)$

where f(....) is production function, x_i's are inputs and q is output.

We will mostly work on production functions with two inputs :

Capital (K) and Labor (L)

Specific Notation : $q = f(L, K)$

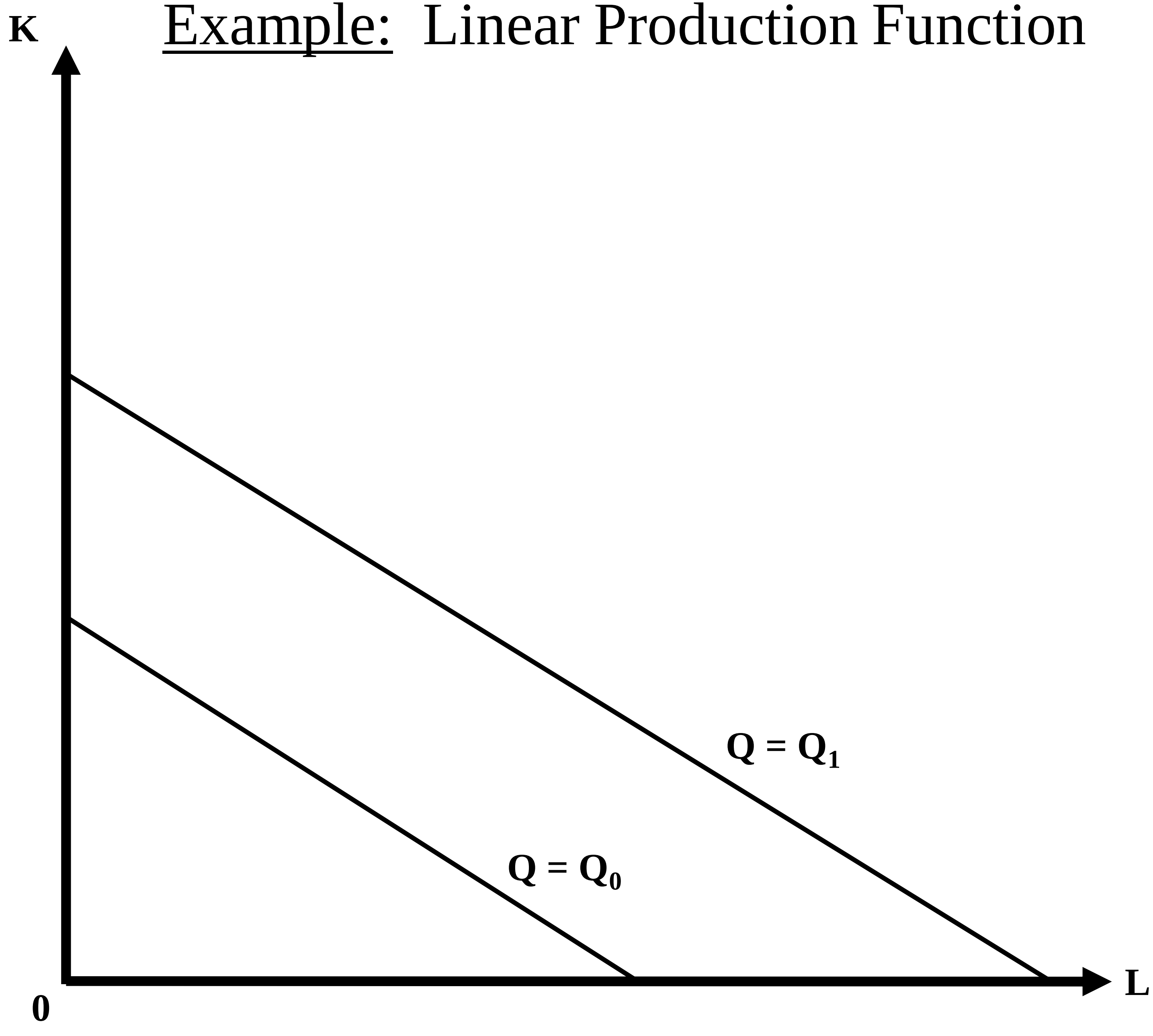
where $f(\ldots)$ is production function, K is capital amount,

L is labor amount and q is output.

Special Production Functions

- 1. **Linear Production Function (**(CES). An special form of homogeneous production function is the one which has **C**onstant **E**lasticity of **S**ubstitution.
    - $Q = aL + bK$
    - MRTS constant
    - Constant returns to scale
    - Inputs are **PERFECT SUBSTITUTES**

Example: Linear Production Function



Special Production Functions

- 2. Fixed proportion production function(**Leontief production function**)

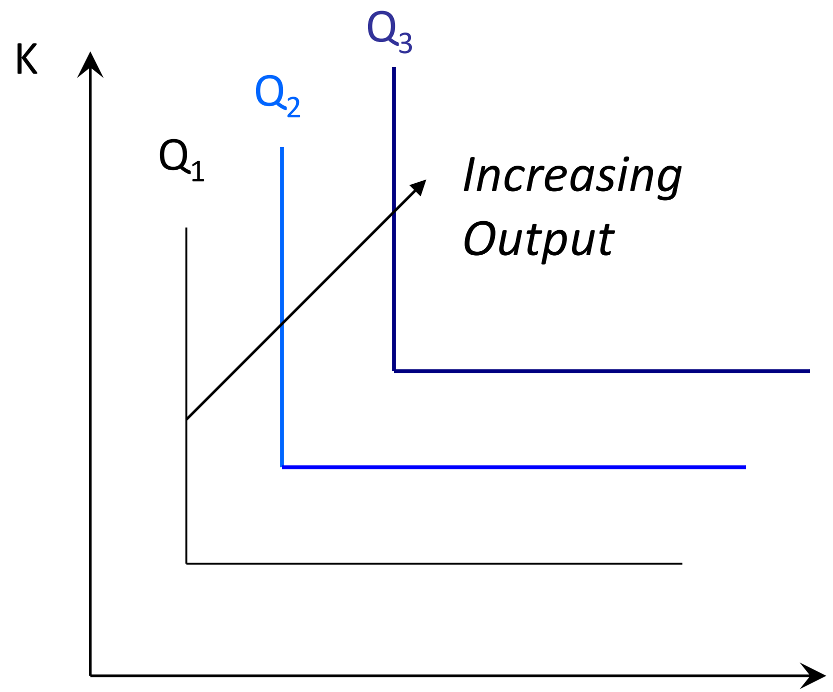
  Q= min(aL, bk)

  L-shaped isoquants

  MRTS varies (0, infinity and undefined)

Special Production Functions

- Q = min{aK, bL}
- Capital and labor are perfect complements and cannot be substituted (no MRTS <=> no slope)
- Capital and labor are used in fixed-proportions
- Both inputs needed to produce output



Special Production Functions

3. Cobb-Douglas Production Function:

- $$Q = aL^{\alpha}K^{\beta}$$

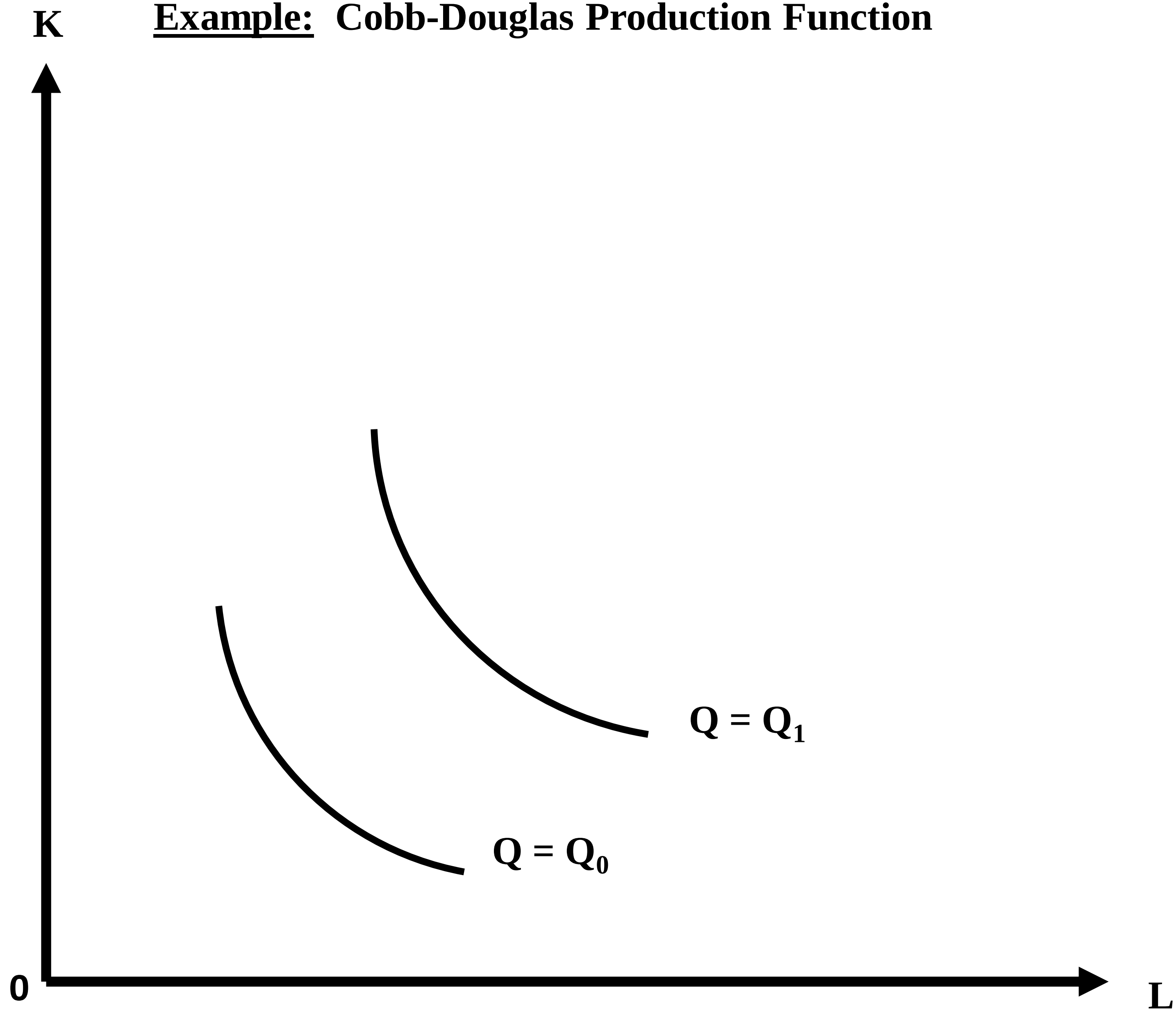
⇨ if $\alpha + \beta > 1$ then IRTS

⇨ if $\alpha + \beta = 1$ then CRTS

⇨ if $\alpha + \beta < 1$ then DRTS

- smooth isoquants
- MRTS varies along isoquants

Example: **Cobb-Douglas Production Function**



Demand for Production Factors, Supply of Goods, Exercises using GAMS

- Use production functions in GAMS

Consumer Theory: Demand for commodities

- Indifference curves of an individual are given by his/ her preferences.
- The demand and supply curves depend upon the preferences.
- Preferences may be:-

  Perfect Substitutes

  Perfect Complements

  Cobb-Douglas

  Stone-Geary

UTILITY FUNCTIONS

Cobb-Douglas Utility Function

$$U(x_1, x_2) = x_1^a x_2^b \quad (a > 0, b > 0)$$

Perfect Substitutes Utility Function

$$U(x_1, x_2) = ax_1 + bx_2$$

Note: MRS = (-)a/b

Perfect Complements Utility Function

$$U(x_1, x_2) = \min(x_1, x_2)$$

Note: MRS = ?

Perfect substitutes

- An indifference curve is given by:

$$q_1 + q_2 = constant$$

- Hence a utility function which represents these preferences is

$$U(q_1 , q_2) = q_1 + q_2$$

Or

$$U(q_1 , q_2) = f(q_1 + q_2) \text{ for any } f(.)$$

- **An other example**

An indifference curve is given by:

$$q_1 + q_2/2 = constant$$

Hence a utility function which represents these preferences is

$$U(q_1 , q_2) = q_1 + q_2/2$$

Or

$$U(q_1 , q_2) = f(q_1 + q_2/2) \text{ for any } f(.)$$

Perfect complements

- An indifference curve is given by:

  $$min(q_1, q_2) = constant$$

  Hence a utility function which represents these preferences is:-

  $$U(q_1 , q_2) = min(q_1, q_2)$$

  Or

  $U(q_1 , q_2) = f[min(q_1, q_2)]$ for any $f(.)$

- **An other example**

  An indifference curve is given by:-

  $$min(q_1, q_2/2) = constant$$

  Hence a utility function which represents these preferences is

  $$U(q_1, q_2) = min(q_1, q_2/2)$$

  Or

  $U(q_1, q_2) = f[min(q_1, q_2/2)]$ for any $f(.)$

Cobb-Douglas with parameter *a*

- An indifference curve is given by:-

  $q_1^a \, q_2^{(1-a)} = constanta$

  Or by:

  $a \ln(q_1) + (1-a) \ln(q_2) = constant$

  Hence a utility function which represents these preferences is

  $U(q_1, q_2) = q_1^a \, q_2^{(1-a)}$

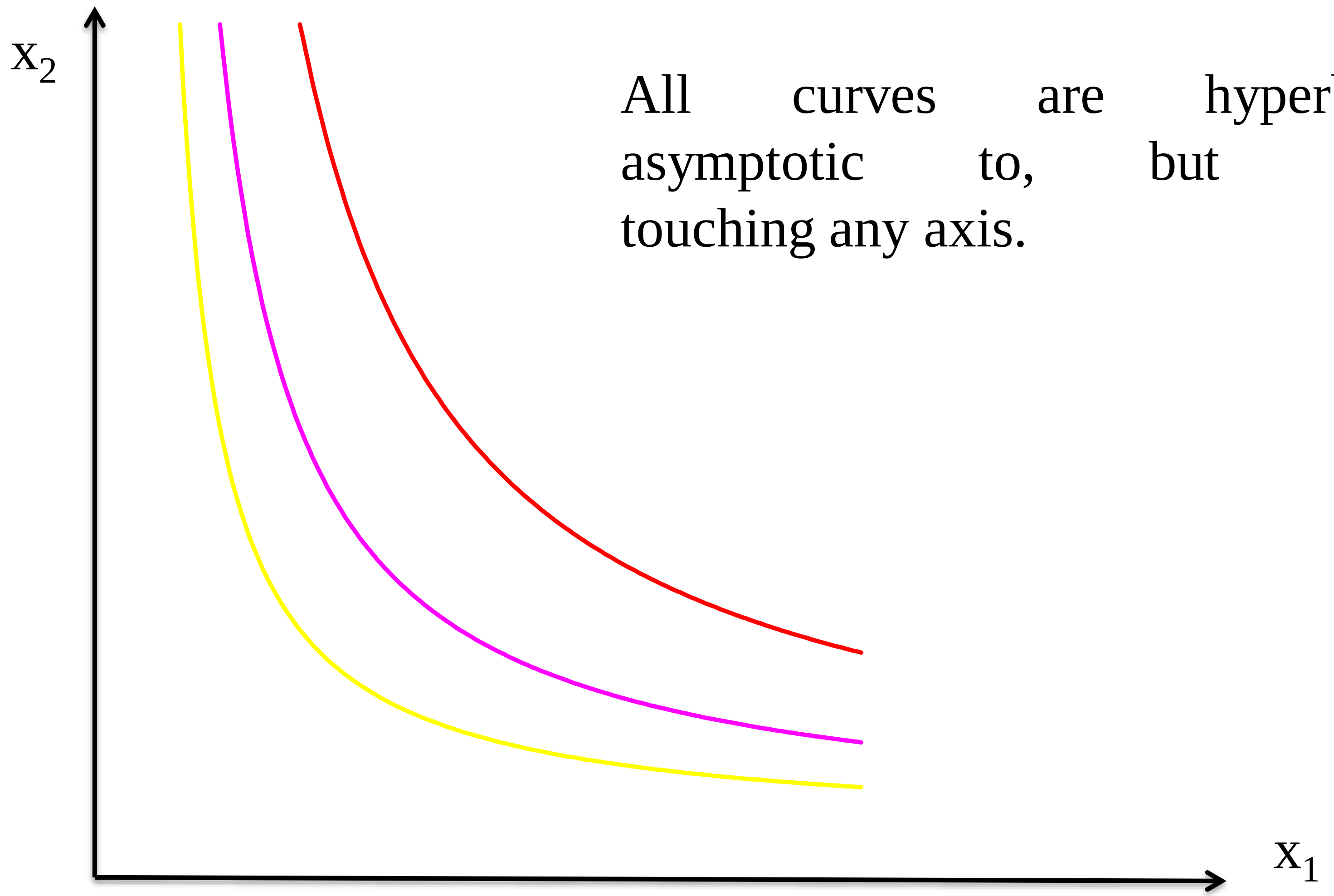
  or

  $U(q_1, q_2) = a \ln(q_1) + (1-a) \ln(q_2)$

  or

  $U(q_1, q_2) = f(q_1^a \, q_2^{(1-a)})$ for any $f(.)$

Cobb-Douglas Indifference Curves



All curves are hyperbolic, asymptotic to, but never touching any axis.

Stone-Geary function

- The Stone-Geary function is often used to model problems involving subsistence levels of consumption.
- In these cases, a certain minimal level of some good has to be consumed, irrespective of its price or the consumer's income.
- The Stone-Geary uses the natural log function to model utility.
- The sum of all the proportions of the goods consumed must equal 1.
- In the problem below, the subsistence levels of A and B are α and β. The term I is income, and p_k $\{k=a,b\}$ are the prices of A and B.

Stone-Geary function

- The Lagrangean and the First-Order conditions are:

$$L = \gamma \ln(A - \alpha) + (1 - \gamma)\ln(B - \beta) + \lambda(I - p_a A - p_b B)$$

$$L_A = \frac{\gamma}{A - \alpha} - p_a \lambda = 0$$

$$L_B = \frac{1 - \lambda}{B - \beta} - p_b \lambda = 0$$

$$L_\lambda = I - p_a A - p_b B = 0$$

Stone-Geary function

- Use the first 2, FO conditions to eliminate the Lagrangean Multiplier.

$$\frac{\gamma/(A-\alpha)}{(1-\gamma)/(B-\beta)} = \frac{p_a\lambda}{p_b\lambda}$$

$$\Rightarrow \frac{\gamma}{1-\gamma}\frac{B-\beta}{A-\alpha} = \frac{p_a}{p_b}$$

$$\Rightarrow p_a(1-\gamma)(A-\alpha) = p_b\gamma(B-\beta)$$

$$\Rightarrow A-\alpha = \frac{p_b\gamma(B-\beta)}{p_a(1-\gamma)}$$

$$\Rightarrow A = \frac{p_b}{p_a}\frac{\gamma}{1-\gamma}(B-\beta)+\alpha \qquad B = \frac{p_a}{p_b}\frac{1-\gamma}{\gamma}(A-\alpha)+\beta$$

Stone-Geary function

- Substitute into the third, FO condition.

$$I = p_a A + p_b \left(\frac{p_a}{p_b} \frac{1-\gamma}{\gamma} (A-\alpha) + \beta \right)$$

$$\Rightarrow I = p_a A + \frac{p_b}{p_b} \frac{1-\gamma}{\gamma} p_a (A-\alpha) + p_b \beta$$

$$\Rightarrow I - p_b \beta = p_a A + \left(\frac{1-\gamma}{\gamma} p_a (A-\alpha) \right)$$

$$\Rightarrow I - p_b \beta = p_a A + (1-\gamma) \left(\frac{p_a A}{\gamma} - \frac{p_a \alpha}{\gamma} \right)$$

$$\Rightarrow I - p_b \beta = p_a A + \frac{p_a A}{\gamma} - \frac{p_a \alpha}{\gamma} - p_a A + p_a A$$

Stone-Geary function

- Multiply the last equation above through by γ/p_a.

$$\frac{\gamma}{p_a}\left(I - p_a\alpha - p_b\beta\right) = A - \alpha$$

$$\Rightarrow A^* = \alpha + \frac{\gamma}{p_a}\left(I - p_a\alpha - p_b\beta\right)$$

$$\Rightarrow B^* = \beta + \frac{1-\gamma}{p_b}\left(I - p_a\alpha - p_b\beta\right)$$

Notes on Stone-Geary function

- Each function (A* and B*) are the **Marshallian** demand functions for the *Stone-Geary* utility.
- The first term on the right-hand-side of the equality, is the **subsistence** consumption. A consumer will always consume this amount irrespective of their **budgets** or the **price**.
- The term $I - p_a\alpha - p_b\beta$ is the income the consumer has left over, after the subsistences are met. It is in effect, the **residual income**.
- Amount of A and B bought by the **residual income** is influenced *negatively* and *positively* by price & good's importance, respectively.
- For instance, if γ increases, it implies that good A is relatively more important than B.
- Thus, as to this demand functions, consumers will purchase less of B and more of A, all other things equal.

Stone-Geary with parameters a, s_1 and s_2

- An indifference curve is given by:

$$(q_1 - s_1)^a (q_2 - s_2)^{(1-a)} = constant$$

Or by:-

$$a \ln(q_1 - s_1) + (1-a) \ln(q_2 - s_2) = constant$$

Hence a utility function which represents these preferences is:-

$$U(q_1, q_2) = (q_1 - s_1)^a (q_2 - s_2)^{(1-a)}$$

or

$$U(q_1, q_2) = a \ln(q_1 - s_1) + (1-a) \ln(q_2)$$

or

$$U(q_1, q_2) = f[(q_1 - s_1)^a (q_2 - s_2)^{(1-a)}] \text{ for any } f(.)$$

- **Exercise**

About production, demand and supply of goods and utility maximization using GAMS

DD and SS equilibrium using GAMS

- A simple supply and demand model of a single market, a partial equilibrium model.
- There are two equations, supply and demand, and two variables, price and quantity.
- Economic equilibrium problems are thus represented as a system of n equations/inequalities in n unknowns.
- Formulate the equations and variables as a complementary problem
- **This involves:-** (a) associating each equation with a particular variable, called the complementary variable.
- (b) if the variables are restricted to be non-negative (prices and quantities), then the equations are written as **weak inequalities**.
- If the equation holds as an **equality** in equilibrium, then the **complementary variable** is **generally strictly positive**.
- If the equation holds as a **strict inequality** in equilibrium, the **complementary variable** is **zero**.

DD and SS equilibrium using GAMS

- Consider first supply of good X with price P.
- The supply curve exploits the **firm's optimization decision**, equating price with marginal cost: **P = MC**
- MC $\geq$ P with the complementarily condition that X $\geq$ 0
- Note that the *price* equation is complementary with a *quantity* variable.
- Suppose that COST = aX + (b/2) X^2.
- Marginal cost is then given by MC = a + bX.
  a + bX $\geq$ P complementary with X$\geq$0.
- Optimizing consumer utility for a given income and prices will yield a demand function of the form X = D(P, I) where I is income.
- X$\geq$D(P, I) with the complementary condition that P $\geq$ 0.
  Note that the *quantity* equation is complementary with a *price* variable.
- We will suppress income and assume a simple function: X = c + dP where c > 0, d <0
- X $\geq$ c + dP complementary with P $\geq$0.

DD and SS equilibrium using GAMS

- Coding an economic equilibrium problem in GAMS.
- Solve through **Mixed Complementarity Problem (MCP).**

DD and SS equilibrium using GAMS

- **Reading the output**

- Here are just the relevant parts of our model runs.
- The *LEVEL* is the solution value of the variables.
- *MARGINAL* indicates the degree to which the equation corresponding to the variable is out of equality.
- For *P* (price), the equation is *DEMAND* and the value of the marginal is supply minus demand.
- For *X* (quantity), the equation is *SUPPLY* and the value of the marginal is the excess of marginal cost over price.
- Variables that have *positive values* in the solution should have *zero marginals*.
- Variables that have *zero values* in the solution should have *positive marginals*.
- This is the zero-output case. The price equation holds, but the quantity equation is slack. The marginal of 1.0 indicates that, at the solution, marginal cost exceed price by 1.0.
- This is the free-good case. Now the price equation is slack, and the marginal of 1.0 indicates that, at the solution, supply exceeds demand by 1.0.

Open Market case and Current Account

Chapter Three

A Simple General Equilibrium Model

- Specification of Products and Factor Market Equations
- Identification of Endogenous and Exogenous Variables and Application of Walras' Law
- Calibration of Numerical values of Parameters
- Numerical Experiments

A Simple General Equilibrium Model

- **General Equilibrium Models of the Economy**
- Under the assumptions of:
  - (1) concave technologies;
  - (2) quasi-concave preferences;
  - (3) price-taking behavior
  - (4) profit maximization by producers;
  - (5) utility maximization by households.
- Characterization of a competitive general equilibrium (Excess demand is less than or equal to zero in every market):
  - Existence
  - Uniqueness
  - Optimality

Why Applied (Computable) General Equilibrium (CGE) Models?

- Analytical indeterminacy of effects
- Need to know magnitude as well as direction
- Analytical intractability--substitution of numerical simulation for analysis
- Sensitivity analysis….

Specification of Products and Factor Market Equations

- Use Simple Static Applied General Equilibrium Model.
- Economic agents:- Households (utility functions)
  - Firms (production functions)
- Goods and factors
- Initial Endowments:- Leisure, Inventory and Capital
- Behavior: Utility maximization and Profit maximization
- Markets:-Simultaneous clearing with zero excess demand of all goods
- Choice of a numeraire good (zero degree homogeneity)
- Choice of assumptions on the utility and production functions
- Choice of functional forms for utility and production functions
- Households (Preferences):- Demander of goods for consumption, Supplier of labor, Supplier of saving & Owner of capital
- Firms (Technologies):- Demander of capital, Demander of labor & Supplier of goods for consumption and investment.
- No government, no external sector, no money and no financial sector

The Simplest System of Equations

- **Households**

- Demand for consumption = D_C (r*, w*, K_{-1}, S_S)
- Supply of labor = S_L (r*, w*, K_{-1}, S_S)
- Supply of savings = S_S (exogenously given)
-
- **Firms**
-
- Demand for capital = D_K (r*, w*)
- Demand for labor = D_L (r*, w*)
- Supply of output = S_O (r*, w*)

General Equilibrium

- **General Equilibrium**

-

- Demand for capital = Supply of capital

$$D_K (r^*, w^*) = K_{-1}$$

- Demand for labor = Supply of labor

$$D_L (r^*, w^*) = S_L (r^*, w^*, K_{-1}, S_S)$$

- Supply of output = Demand for consumption + Savings

$$S_O (r^*, w^*) = D_C (r^*, w^*, K_{-1}, S_S) + S_S$$

-

Determination of the Parameters: Calibration Vs Econometric Estimation

- The derivation of the numerical values of the parameters
- The calibration approach
  - matching quantities and prices in the base period
  - overly dependent on assumptions on the functional forms
- The econometric approach
  - estimating parameters on the basis of a time-series of observations
  - permits validation of estimated values of parameters with actual empirical experience
  - functional form and other assumptions can be empirically tested

– Solution of the Model:
The Choice of Algorithms

Fixed point algorithms (Scarf)

Chapter Four
Input-Output Table, SAM and Multipliers Analysis

- Input-Output Table and Multiplier Analysis
- Social Accounting Matrix and Multiplier Analysis
- Python Software Application and SimSIP SAM Application

Basic Components of Input-Output Models

- **It has 3 basic Components**
- **Transactions Table**
- **Direct Requirements Table**
- **Total Requirements Table**

Input-Output (IO)Table and Multiplier Analysis

- IO analysis creates a picture of a regional economy describing flows to and from industries and institutions.
- IO Analysis is an accounting framework.
- IO analysis can be used to predict changes in overall economic activity as a result of some change in the local economy.
- IO analysis tries to estimate inter-industry transactions and use those figures to estimate the economic impacts of any changes to the economy.
- It provides a description of a local economy.
- It is predictive model to estimate impacts.
- The IO model is centered on the idea of inter-industry transactions: –Industries use the products of others to produce their own products.

1.Transactions Table

- Contains basic data on the flows of goods and services among suppliers and purchasers in a study period.
- A transactions table shows the monetary flows of goods and services in a local economy.
- Represents monetary flows for a given time period, usually one year.
- Total outlays = Total output
- Intermediate purchases are goods and services purchased and used in the local production process.
- **Final demands** are purchases for final consumption
- **Final payments** are payments for factors or inputs outside intermediate production process.

Example Transactions Table

Purchasing Sectors ($ million)

Selling Sectors ($ million)

	Agriculture	Health	Services	Final Demands	Total Output
Agriculture	10	6	2	18	36
Health	4	4	3	26	37
Services	6	2	1	35	44
Final Payments	16	25	38	0	79
Total Input	36	37	44	79	196

Predictive Use of Input-Output Analysis

- Impacts are tracked throughout the economy
- The multipliers are derived from regional economic accounts
- Only local transactions are used to create the multiplier effect.

2. Direct Requirements Table

- Direct requirements are the purchases of resources (inputs) by a sector from all sectors to produce one dollar of output
- Creates a production recipe/formula .
- Derived from the transactions table, this shows the inputs required directly from different suppliers by each intermediate purchaser for each unit of output that purchaser produces.

Direct Requirements Table

Purchasing Sectors

Selling Sectors		Agriculture	Health	Services
	Agriculture	0.278	0.162	0.045
	Health	0.111	0.108	0.068
	Services	0.167	0.054	0.023
	Final Payments	0.444	0.676	0.864
	Total	1.000	1.000	1.000

What are Multipliers?

- Multipliers measure total change throughout the economy from one unit change for a given sector.

Three Types of Multipliers from Model

- **1. Output**
- **2. Employment**
- **3. Income**

Three levels of Multipliers

- **Type I Multipliers**
- **Type II Multipliers**
- **Type III Multipliers**

Type I Multipliers

- Include **direct** or **initial spending**.
- Include indirect **spending** or **businesses** buying and selling to each other.
- The **multiplier** is **direct** plus **indirect effect** divided by **direct effect** .

Type II Multipliers

- Includes Type I Multiplier effects.
- Plus **household** spending based on the income earned from the direct and indirect effects – the **induced effects**.

TYPE III MULTIPLIERS

- Type III Multipliers are **modified** Type II multipliers.
- Therefore, Type III Multipliers also include the direct, indirect, and induced effects.
- Type III Multipliers adjust Type II Multipliers based on spending patterns **amongst different income groups.**

Type I Multipliers include:

- Direct
- Indirect (Business Spending)
- Type I Multipliers are derived from the
- **Total**

Total Requirements Table

Purchasing Sectors ($ million)

Selling Sectors ($ million)

	Agriculture	Health	Services
Agriculture	1.446	0.268	0.085
Health	0.199	1.163	0.090
Services	0.258	0.110	1.043
Total	1.903	1.541	1.218

Type I Multiplier

- For a $1.00 change in final demand sales in the local economy, the total direct and indirect impacts are $1.541

Type II Multipliers include:-

- **Direct**
- **Indirect (Businesses)**
- **Induced (Households)**

Type II Multipliers

- **They are derived from the Total Requirements Table with**
- **Households**

Transactions Table with Households

Purchasing Sectors ($ million)

Selling Sectors ($ million)

	Ag	Health	Services	House-holds	Final Demands	Total Output
Ag	10	6	2	2	16	36
Health	4	4	3	10	16	37
Services	6	2	1	7	28	44
Households	3	6	10	0	0	19
Final Payments	13	19	28	0	0	60
Total Input	36	37	44	19	60	196

Total Requirements Table with Households

Selling Sectors	Purchasing Sectors: Agriculture	Health	Services	Households
Agriculture	1.536	0.369	0.197	0.429
Health	0.386	1.370	0.318	0.879
Services	0.388	0.256	1.203	0.619
Households	0.279	0.311	0.341	1.319
Total	2.589	2.307	2.059	3.245

Explaining Type II Multiplier

For a $1.00 change in final demand sales in the local economy, the total direct, indirect and induced impacts are $2.307.

Multipliers

- Direct requirements represent direct or initial spending

- Direct and indirect effects include the direct spending plus the indirect spending or businesses buying and selling to each other.

- Direct, indirect and induced effects include direct and indirect plus household spending earned from direct and indirect effects

Other Multipliers

- Employment Multipliers
  - Type I
  - Type II
  - Type III
- Income Multipliers
  - Type I
  - Type II
  - Type III

Example -Type I Employment Multiplier

- Agricultural Sector Type I Employment Multiplier = 1.43

When the Agricultural Sector realizes a 1 employee change, total employment in the study area changes by 1.43 jobs from direct and indirect linkages

Example –
Type II Employment Multiplier

- Agricultural Sector Type II Employment Multiplier = 2.25

When the Agricultural Sector realizes a 1 employee change, total employment in the study area changes by 2.25 jobs from direct, indirect and induced linkages

Breakdown of Type II Employment Multiplier - Agricultural Sector

Direct Effects	=	1.00
Indirect Effects	=	0.43
Induced Effects	=	<u>0.82</u>
Total	=	2.25

Example –Type I Income Multiplier

- Agricultural Sector Type I Income Multiplier = 1.96

When the Agricultural Sector realizes a $1.00 change in income, total income in the study area changes by $1.96 from direct and indirect linkages.

Example - Type II Income Multiplier

- Agricultural Sector Type II Income Multiplier = 2.50

When the Agricultural Sector realizes a $1.00 change in income, total income in the study area changes by $2.50 from direct, indirect and induced linkages

Breakdown of Type II Income Multiplier -Agricultural Sector

Direct Effects	=	$1.00	
Indirect Effects		=	$0.96
Induced Effects		=	<u>$0.54</u>
Total		=	$2.50

Caution When Using Multipliers

- Multiplier values include direct effects.
- Do not aggregate sector multipliers to derive an aggregate multiplier.
- Be cautious of large multipliers.
- Be cautious in using a multiplier from another study area.

The Economic Base Model

E_T = Total Employment
E_X = Export Employment
E_L = Local Employment

$$E_T = E_X + E_L \qquad (1)$$

Define $a = E_L/E_T$
Multiply by E_T and substitute into (1):
$$E_T = E_X + aE_T$$
Solve for E_T:

$$E_T = (1/1\text{-}a)E_X$$

- **SIMPLE EXAMPLE**

The Transaction Table

Sales (Supplies)	Purchase (Demands)			
	Agriculture	**Manufacturing**	Households	**Total**
Agriculture	10	30	60	100
Manufacturing	5	10	35	50
Households	85	10	15	110
Total	100	50	110	260

Direct Requirements Table		
Agriculture	**0.1**	**0.6**
Manufacturing	**0.05**	**0.2**
Households	**0.85**	**0.2**
Total purchase (Input)	**1.00**	**1.00**

First Round of Economic Impacts

- Assume sales of 300 to the two primary sectors (Agriculture and Manufacturing).

	Sales to Agri. & Manu.	Sales as direct input		
		Agri.	Manu.	Total
Agriculture	200	20	60	80
Manufacturing	100	10	20	30
Households	0	170	20	190
Total indirect	300	200	100	300

Second Round of Economic Impacts

- The total produce of the two primary sectors will go to second round production as an input.

	Sales to Agri. & Manu.	Sales as direct input		
		Agri.	Manu.	Total
Agriculture	**80**	**8**	**18**	**26**
Manufacturing	**30**	**4**	**6**	**10**
Households	**0**	**68**	**6**	**74**
Total indirect	**110**	**80**	**30**	**110**

Third Round of Economic Impacts

- The total produce of the two primary sectors in the second round production will go to the third round as an input.

		Sales as direct input		
	Sales to Agri. & Manu.	**Agri.**	**Manu.**	**Total**
Agriculture	**26**	**2.6**	**6**	**8.6**
Manufacturing	**10**	**1.3**	**2**	**3.3**
Households	**0**	**22.1**	**2**	**24.1**
Total indirect	**36**	**26.0**	**10**	**36**

Fourth Round of Economic Impacts

- The total produce of the two primary sectors in the third round production will go to the fourth round as an input.

		Sales as direct input		
	Sales to Agri. & Manu.	**Agri.**	**Manu.**	**Total**
Agriculture	**8.6**	**0.9**	**2.0**	**2.9**
Manufacturing	**3.3**	**0.45**	**0.7**	**1.15**
Households	**0**	**7.3**	**0.7**	**8.0**
Total indirect				

- NB: The process will continue until the total contributions become zero for the two primary sectors (**Agri.** & Manu.)

Chapter 5
A Standard and Advanced General Equilibrium Model

- Types of Models: IFPRI model, PEP model, STAGE model, GTAP model, Dual- Dual Model, MAMS model etc
- Social Accounting Matrix and an overview of the Standard CGE model
- Mathematical Model Statement
- The Standard Model in GAMS
- Exercises using GAMS

Types of Models

- IFPRI,
- PEP,
- GTAP,
- MAMS
- STAGE,
- Dual- Dual model etc

IFPRI impact model

- **International Food Policy Research Institute (IFPRI)**
- IFPRI's IMPACT model allows policy makers, analysts, and civil society to explore future scenarios for food security, impact of climate change on agricultural production and hunger…
- It integrated economic, climate, water, crop models.
- International Model for Policy Analysis of Agricultural Commodities and Trade (IMPACT) was developed in the early 1990s to consider the long term challenges facing policymakers in reducing hunger, and poverty in a sustainable fashion.
- IMPACT model has been expanded and improved repeatedly to respond to increasingly complex policy questions and the state-of-the-art of modeling.
- Currently, IMPACT is a network of linked economic, water, and crop models.
- Its core is a partial equilibrium multi-market economic model, which simulates national and international agricultural markets.
- The links to water and crop models support the integrated analysis of changing environmental, biophysical, and socio-economic trends.
- It allows varied and in depth analysis on a diversified critical issues of policymakers at national, regional, and global levels.

IFPRI impact model

- Projections may reported by region and country include aggregate food production, per capita food consumption, and numbers at risk of hunger, all in “with” and “without” climate change scenarios, as well as detailed breakdowns for major crops.
- The IMPACT model is designed to examine alternative futures for global food supply, demand, trade, prices, and food security.
- The IMPACT model allows IFPRI to provide both fundamental, global baseline projections of agricultural commodity supply, demand, trade, prices and malnutrition outcomes.
- It identified cutting-edge research results on bio-energy, climate change, changing diet/food preferences, and many other themes.
- E.G. the model projected that global food production will grow by 60% over 2010 levels by 2050 in the context of climate change-10% points less than would be the case without climate change.

IFPRI impact model

- IMPACT covers over 40 commodities, which account for virtually all of world food production and consumption, including all crop and livestock products in a partial equilibrium framework.
- It is specified as a set of 115 country-level supply and demand equations where each country's model is linked to the rest of the world through trade.
- The model was on GAMS language and it was a relatively straightforward partial equilibrium on global agricultural sector.
- Solution of the system of equations seeks a world market prices for all commodities that satisfies market-clearing conditions for the sum of global net trade to equal zero.
- The basic IMPACT model is combined with the IMPACT Water Simulation Model (IWSM) to estimate interactions between water supply and demand, and food supply, demand, and trade.
- *In order to explore food security effects, IMPACT projects the percentage and number of malnourished preschool children (0 to 5 years old) in developing countries as a function of average per capita calorie availability, the share of females with secondary schooling, the ratio of female to male life expectancy at birth, and the percentage of the population with access to safe water.*

African Growth and Development Policy (AGRODEP)

- AGRODEP provides a platform for sharing existing core economic models dealing with sector, national, and international policy issues as well as long-term projections on areas like agricultural growth and poverty, trade, nutrition, climate change, natural resources management, and science and technology.
- The shared modeling organized around two main pillars: **simulation** models and **econometric** models.
- **Simulation models** mainly perform ex-ante analysis of economic policies (e.g. investment decision in agricultural R&D in different sectors) or to disentangle different effects of existing policies (e.g. interaction between export taxes and import duties in international trade relations).
- **Simulation models** include CGE and Partial Equilibrium, both in deterministic and stochastic versions.
- The main soft-wares needed to use these models are **GAMS** or **MS Excel**.
- **Econometric models** mainly aimed to assess effects of policies (e.g. consequences of exchange rate on agricultural exports) or to identify and quantify the drivers of an economic phenomenon (e.g. determinants of trade in a gravity equation).
- They are also used to estimate key parameters used in the simulation models (e.g. price transmission magnitude between international and domestic markets).

African Growth and Development Policy (AGRODEP)

- Econometric methods used **parametric** and **non-parametric** approaches.
- Most of the **econometric programs** will require the use of **STATA**.
- The library includes models developed and fully supported by AGRODEP team, as well as links to models developed and managed by other partners.
- For instance, CGE, IFPRI Standard Model or the PEP 1-1 models, while the later include the GTAP (Global Trade Analysis Project) model.
- Modeling tools will be expanded to include a set of specialized modules devoted to specific technical areas and economic sectors.

AGRODEP Disclaimer

- Information on the African Growth and Development Policy Modeling.
- It makes every effort to ensure, but does not guarantee, the accuracy or completeness of, and declines responsibility for any loss, damage, liability or expense suffered which is claimed to result from its use.
- It also provides many support on African economic development modeling.

PEP (Partnership for Economic Policy)

- The Modeling and Policy Impact Analysis (MPIA) program assists developing country researchers in constructing models of their national economy to simulate impact of macroeconomic shocks/policies on various dimensions of poverty and welfare.
- To do so, it applies a combination of macro-micro modeling and simulation techniques including, CGE framework.
- CGE models account for the structural aspects of a country's economy, *i.e*. the interactions among sectors and institutions, and their links with the global economy.
- In recent years, MPIA leaders identified that there is a need for reference models that are more elaborate and closer to the real-life conditions than the ones that had been used so far.
- Moreover, policymakers face new challenges that call for impact assessments that look beyond national boundaries, global economy.
- Researchers of the MPIA program have devoted time and energy to create a series of new standard CGE models that can respond to these needs.

PEP (Partnership for Economic Policy)

The growing family of PEP standard CGE models

- Since the beginning of 2011, three new PEP standard CGE models have been put online.
- The first one, PEP 1-1, designed for country-level studies, was developed as an operational tool for researchers to easily adapt a relatively standard model to their national economy.
- It was 1 period – 1 country economic policy model.
- It is a static general equilibrium model.
- The model is designed for the study of an archetypal national economy.
- Based on the above basic model, PEP 1-t, was created to include evolution in time.
- Technically, PEP 1-t is a «recursive dynamic» version of PEP 1-1, which is a static or single-period model.
- The analysis extended to multiple periods linking each one to the past through variables inherited from the previous period.

PEP (Partnership for Economic Policy)

- PEP-w-1 is a single-period **world** model and PEP-w-t is its recursive dynamic version. These new models are based on the GTAP, world-level data base.
- The latest innovation of the **PEP-MPIA** (**M**odeling and **P**olicy **I**mpact **A**nalysis) team is, PEP-w-t-fin, which is the only **recursive dynamic world model** that includes **international financial assets**.
- They propose a static CGE model designed for the study of a national economy and intended to be an operational tool for PEP researchers and other users.
- Through it, researchers will be able to develop a relatively standard model, and easily apply it to their country, whatever the particular structure of the SAM is.
- Quoting the co-authors: ***PEP-1-1 is to be our basis, from which to further deepen our understanding of CGE analysis and develop modeling techniques that will tackle new problems. But, both in sharing our experience and exploring new paths, we want to remain in the realm of operational model building. So our intentions are all at once pedagogical, experimental, and practical.***

PEP (Partnership for Economic Policy)

- **PEP-1-1** differs significantly from Decaluwé, Martens and Savard's EXTER model, which has been used extensively in the past by PEP researchers.
- In many respects, the PEP-1-1 model is richer than the more pedagogical EXTER.
- First, the PEP-1-1 model distinguishes several categories of workers and of capital.
- PEP-1-1 is capable of considering broader set of tax instruments, and models all possible transfers between institutions (agents).
- the GAMS code, will facilitate the application of PEP-1-1 to variously aggregated SAMs, by means of a few simple steps to make the SAM directly readable into the GAMS program.

PEP (Partnership for Economic Policy)

- The PEP 1-1 model is different from Decaluwé, Martens, and Savard's EXTER model, which has been used extensively in the past by PEP researchers.
- The model distinguishes several categories of factors (labor and capital).
- It also takes into account all possible transfers between agents and a large set of tax instruments.
- Furthermore, the aggregate output of each industry consists of several products, consistent with non-square input-output tables.
- The authors have written the GAMS code in a general form in order to facilitate the application of the model to a broader set of aggregated SAMs.

SAM and the Standard CGE model

- SAM is a snapshot of an economy, for a given year.
- It must be square since each account appears both as a row and column of the table: the account's income is registered in the corresponding row, and its expenditures in the corresponding column.
- The value in every cell of the matrix, is an expenditure for the corresponding column, and income for the corresponding row.
- The SAM to be used as the basis of a CGE model must be balanced, which means for each account, the sum of income from all sources must be equal to the sum of expenditures.
- In modeling SAM the order in which the accounts are listed does not have to be followed; any order will do.
- To adopt SAM and use in PEP-1-1, the user must however stick to the following guidelines:- The user must build SAM using Excel.
  - Accounts in the SAM should be grouped into five main categories: factors of production, institutions (agents), commodities, industries and accumulation.

PEP (Partnership for Economic Policy)

- Each account must have two titles (both in rows and in columns): one representing the set of accounts to which it belongs, the other indicating which element it is in that set.
- CGE modeling and SAM based research requires the use of the most recent economic data available in a coherent framework.
- These data may be from diverse sources like input-output tables, national accounting, household, firm and/or labor market surveys, government accounts, international trade accounts, etc., and correspond to different periods of time.
- Input-output data are generally prepared every five years or more, whereas national data on income, production, trade, etc. are generated annually.
- In some cases, it is possible to start with an existing SAM, and update using new data. SAM must be constructed in its entirety.
- The problem in estimating a disaggregated SAM for a recent year is to find an efficient (cost-effective) way to incorporate and reconcile information from a variety of sources, including data from the prior years.

A GAMS Program for Balancing a SAM

- Steps to balance an unbalanced SAM on GAMS.
- **Step 1: Define Matrix Accounts**
- The user must first declare the SAM accounts.
- The GAMS command “SET” is used. The set of accounts set is named I and contains all accounts including the account “TOT” for total.
- Instead of actual account names, it is possible to assign numbers to the accounts when the dimensions are large, e.g. A1, A2, A3… A4000.
- If the set of accounts forms a sequence as above, it may be defined by the first and last element, separated by an asterisk, as follows:-

SETS I MATRIX ACCOUNTS / A1 * A4000 /;

Input names of matrix accounts

A GAMS Program for Balancing a SAM

- **Step 2: Input Data**
- In this section, the user must input data from the unbalanced matrix "SAM0".
- This can be done in many ways. First, the data can be entered manually as a GAMS "TABLE".
- Alternatively, the TABLE statement can be saved as a space-delimited ASCII file ("Save as" *.prn format in Excel) and then directly imported into the GAMS code with the "$include" command.
- Finally, data can be imported directly from an Excel spreadsheet using the GAMS GDX facility11 to first transform the Excel matrix into a GDX file, and then to read the data from the GDX file.
- In this case, matrix position, name and range should be given and must **exclude** the row and column totals.

A GAMS Program for Balancing a SAM

- **Step 3: Treatment of Negative Values**
- The standard entropy method does not allow negative values.
- To solve this problem, simply transpose these values to their counterpart cell before balancing the SAM.
- Indeed, as the SAM represents flows from one account to another, a negative flow from account A to B is equivalent to an equal positive flow from account B to A.
- E.G, if SAM0(6,5) is negative, an equal amount is **added** to cell SAM0(5,6) and the cell SAM0(6,5) is set equal to zero.
- To recall where these negative values belong, the program first defines a new matrix, NEG.
- The positions of all negative values are indicated in the new matrix by a "1".
- This step does not require any modifications by the user.

A GAMS Program for Balancing a SAM

- **Step 4: Normalize Initial Matrix Cell Values**
- The program defines a total of cell values for the unbalanced matrix, “TOTO”, and calculates its value.
- All matrix transaction flows are then normalized by dividing by “TOTO”. This step does not require any modifications by the user.
- **Step 5: Treatment of Zero Values**
- To avoid having to take the log of zero in the entropy method, the program adds a small amount (delta) to each cell value.
- This parameter must be as small as possible to avoid influencing the results.
- By default, it is equal to .000000000001.
- This step does not require any modifications by the user.

A GAMS Program for Balancing a SAM

- **Step 6: Definition of Variables used in Optimization Process**
- A new matrix “NSAM” and variable “OPT” are declared. OPT is the objective function variable which has to be minimized.
- The number of variables is I × I matrix cells, plus the optimum variable. This step does not require any modifications by the user.

Step 7: Declaration and Statement of Model Equations

- Three sets of equations are used to balance the matrix: an optimization equation “OPTIMIZE”, defined differently for the OLS and CE approaches, and two sets of constraint equations, the first equalizing each row total to its corresponding column total, the second setting the sum of all proportions to one.

A GAMS Program for Balancing a SAM

Step 8: Initialization of Variables

- All variables are initialized in this section. Also, matrix cell values are limited between 0 and infinity and empty cells remain empty.
- No user modifications are necessary, but the user can fix any cell values (or combination of cell values), as desired.
- Be careful not to fix too many values, as the program may not find a solution (infeasible problem).

Step 9: Model Solving

- All equations are used in the model solving statement. The solver is chosen in the OPTION statement.
- The SOLVE command minimizes the variable OPT using a non-linear solver algorithm.
- This step does not require any modifications by the user.

Step 10: Results Copied to A New Matrix

- Results are copied into a new matrix NSAM defined previously.
- Negative values are re-transposed to their original position in the new matrix .
- The new SAM is transformed into transaction flows

A GAMS Program for Balancing a SAM

- The new SAM is then stored in a GDX file, which is subsequently transformed into an Excel file. The “GDX2XLS” routine12 creates an Excel spreadsheet in the form of a database, with one cell value per line.
- It can be returned to matrix form using Excel’s Pivot Table facility, but the accounts may not be in the desired order. A better option is to use the GDXXRW.EXE program13, which enables to create an Excel table.
- Here, the NSAM parameter of the GDX file Results.gdx is written to output file NewSAM.xls; the data range in the Excel sheet, including row and column headings, is A1:W23, with one row dimension (rdim) and one column dimension (cdim).
- **Step 11: Problem Detection**
  A parameter matrix named PROBS is created. It takes the value 1 if the corresponding value in the new balanced matrix is zero, while the corresponding value in the *a priori* matrix is non-zero.
- All values of PROBS should be zero.

Global Trade Analysis Project (GTAP) model

- The standard GTAP model is a multi-region, multi-sector, computable general equilibrium model, with perfect competition and constant returns to scale.
- GTAP-dynamics is a worldwide recursively dynamic CGE model, which extends the standard comparative static GTAP model (Hertel 1997) to include international capital mobility, capital accumulation, and an adaptive expectations theory of investment.
- GTAP-Dynamics differs from most recursive dynamic models in that it is formulated in continuous time, which is made possible by the differential equations approach of GEMPACK (General Equilibrium Modelling Package).
- The core GTAP model is a static, multi-regional model which tracks the production and distribution of goods in the global economy.

Global Trade Analysis Project (GTAP) model

- In GTAP the world is divided into regions (typically representing individual countries), and each region's final demand structure is composed of public and private expenditure across goods.
- The model is based on optimizing behavior.
- Consumers maximize welfare subject to budget constraint with fixed levels of investment and public output.
- Producers combine intermediate inputs, and primary factors (skilled and unskilled labor, land, resources and physical capital) at least cost subject for given technology.
- The dataset includes a full set of bilateral trade flows with associated transport costs, export taxes and tariffs.
- The Global Trade Analysis Project (GTAP) is a research program initiated in 1992 to provide the economic research community with a global economic dataset for use in the quantitative analysis of international economic issues.

Global Trade Analysis Project (GTAP) model

- GTAP has lead to the establishment of a global network of researchers who share a common interest of multi-region trade analysis and related issues, notably climate and energy policy.
- The GTAP research is coordinated by Thomas Hertel, Director of the Center for Global Trade Analysis at Purdue University (see notably Hertel, 1997).
- As Deputy Director of this Center, Robert McDougall oversees the data base work (e.g. McDougall, 2005).
- Software development within the GTAP project has been assisted greatly by the efforts of Ken Pearson, Mark Horridge and other researchers from Centre of Policy Studies, Monash University (see http://www.gtap.org for a list of applications based on the GTAP framework).
- Input-output matrices provide data on value flows between economic sectors and regions for primary production factors, intermediate goods and final consumption products.
- In their micro-consistent versions, such as those provided in the GTAP data, these matrices provide a complete representation of the economy, such that no value is lost in transaction. The ensuing dataset represents value flows in a closed economic system.

Global Trade Analysis Project (GTAP) model

- The project's objectives include the provision of a documented, publicly available, global, general equilibrium data base, and to conduct seminars on a regular basis to inform the research community about how to use the data in applied economic analysis.
- GTAP has lead to the establishment of a global network of researchers who share a common interest of multi-region trade analysis and related issues.
- The principal programming language for GTAP data and modeling work is GEMPACK [Harrison and Pearson, 1996].
- In the GEMPACK framework the model is solved as a system of nonlinear equations. The present paper describes a version of the GTAP model which has been implemented in GAMS. The GAMS model is essentially implemented as a nonlinear system of equations, although it can be posed either as a CNS or MCP.
- Along with the core model I have developed two ancillary programs for dataset management.
- The package is called "GTAP7inGAMS".
- These programs should be useful to economists who program in GAMS and wish to use GTAP in applied work.
- These programs include tools for translation of the GTAP files into GAMS readable form, GAMS programs for dataset aggregation and reconciliation.

Global Trade Analysis Project (GTAP) model

- The canonical GTAPINGAMS model is essentially a nonlinear system of equations, as the model does not include activity analysis nor does it rely on free disposal.
- Formulation as an MCP imposes some modeling discipline, as equations must be explicitly linked with variables.
- Extensions of the model to incorporate, e.g., tariff quotas (Hertel *et al*., 2009) or quantitative restrictions on carbon emissions which may or may not be binding (Bohringer *et al*. ¨ , 2016).
- In the core GTAP model "production" takes place under conditions of perfect competition with constant returns to scale, hence there are no excess profits, and the cost of inputs must equal the value of outputs.

Datasets in GTAP

- GTAP in GAMS datasets are stored in the GAMS Data exchange (GDX) format. Data
  stored in this format may be freely transferred to header array format using Mark Horridge's gdx2har.exe program.
- Data may also be transferred to Excel using any of several free utilities provided by GAMS Development Corporation (see, e.g., gdxrw.exe).
- Any GTAP in GAMS dataset may be aggregated into fewer regions, sectors and primary factors. This permits a modeller to do preliminary model development using a small dataset to ensure rapid response and a short debug cycle.
- After having implemented a small model, it is then a simple matter to expand the number of sectors and/or regions in order to obtain a more precise empirical estimate.
-

Datasets in GTAP

- All GTAP datasets are defined in terms of three primary sets: *i*, the set of sectors and produced commodities, *r* the set of countries and regions, and *f* the set of primary factors.
- Table 6.6 presents the identifiers for the 59 GTAP 7 sectors in their most disaggregate form.
- These sectors may be aggregated freely to produce more compact dataset.
- Regional identifiers in the full dataset correspond to standard UN three-character country codes.
- Users can define their own aggregations of the GTAP data and use any
  labels to describe regions.
- For technical reasons, if a GTAP dataset is to be used with MPSGE, then regional identifiers can have at most 4 characters.

Maquette Microsimulation (MAMS)

- Maquette for MDG Simulations; a dynamic recursive CGE Model.
†
- Initially developed for country-level MDG strategies: How should government and aid policies be designed to achieve the MDGs?
- Evolved into a general framework for country-level, *ex-ante*, medium-to-long-run development policy analysis, with emphasis on fiscal issues and MDG indicators.
- Different versions ranging from aggregated macro version to disaggregated MDG version.
- In addition to major non-monetary MDGs, MAMS covers monetary poverty.
- Like other CGE models using two alternative approaches: representative household (RH) and microsimulation (MS).
- Most MAMS applications cover MDGs 1 (poverty), 2 (primary school completion) 4 (under fi**ve** mortality rate), 5 (maternal mortality rate), 7a (water access), and 7b (sanitation access)

Issues in MDG strategy analysis

- The main originality and extensions of MAMS compared to standard CGE models is the inclusion of (MDG- and/or education-related) *social services* and their impact on MDGs and other aspects of social and economic performance.
- Social services may be produced by the government and the private sector.
- MAMS is designed to consider the following aspects of MDG scenarios:
  1. Role of non-government service providers
  2. Demand-side conditions (incentives, infrastructure, incomes)
  3. Role of economic growth
  4. Macro consequences of increased government spending under different financing scenarios
  5. Diminishing marginal returns (in terms of MDG indicators) to services and other determinants
  6. Role of efficiency and input prices (e.g. wages) in

Model Structure

- MAMS ay be described as a may be described as an *extended*, dynamic recursive computable general equilibrium (CGE) model designed for MDG analysis.
  MAMS is coded in GAMS/Excel.
  MAMS is complementary to and synthesizes results from *sector and econometric research on MDGs*.
  Motivation behind the design of MAMS:
  „An economy wide, flexible-price model is required.
  „Standard CGE models provide a good starting point.
  „But standard CGE approach must be complemented by a satisfactory representation of 'social sectors'.

Features Common to Most CGE Models

- Computable solvable numerically †General economy-wide † Equilibrium agents have found optimal solutions subject to constraints„ quantities demanded = quantities supplied „macroeconomic account balance.
- Producers use factors and intermediates as inputs. †Imperfect transformability/substitutability in foreign trade.†
- Dynamic-recursive Î the solution in any time period depends on current and past periods, not the future. A “real” model: only relative prices matter; no modeling of inflation.

- **Flexible modeling framework**

- MAMS has a flexible disaggregation of production activities and commodities, factors and households (GAMS facilitates)
- Data readily available for virtually any country for the MAMS minimum version: simple two-sector (government – private) framework for dynamic macro analysis.
- MAMS may include:
  „Wide range of taxes
  „NGO + private MDG/HD services
  „Special-case sectors (resource-based export, regulated utility
- Special versions developed to deal with additional issues: demography, gender, and natural disasters.
- MAMS: a tool for analyzing the impact of alternative scenarios on economic development, including monetary poverty and other MDGs.
- DR simulation analysis illustrates the application of MAMS to the

MAMS vs. RMSM-X

Description	MAMS	RMSM-X
Time frame	**Medium- to long-run**	**Short - to medium-run**
Accounting consistency	**yes**	**yes**
Economic behavior	More emphasized	Less emphasized
Production function	**Labor, capital, land Intermediates.**	**Capital**
Monetary sector	**No**	**Yes**
Disaggregation	More	Less
Data requirements	More	Less
Software	**GAMS/Excel**	**Excel**

Poverty Analysis with MAMS

- Two basic approaches to poverty and inequality analysis using MAMS and other CGE models:

1. Representative household (RH)
2. Micro-simulation (MS)

- Both generate standard poverty and inequality indicators
- Representative Household Approach to Poverty Analysis
- MAMS includes one or more RHs.
  Each RH is characterized by:,,

1. Pattern of incomes (factors, transfers, interest),,
2. Pattern of outlays (taxes, saving, consumption, transfers)
3. Behavioral assumptions (given by elasticities)

- Changes in RH receipts and outlays are generated as part of model simulations.

Representative Household Approach

- Steps in the analysis:
- 1. MAMS provides changes in mean per-capita income for each RH (by scenario and year);
- 2. The survey observations (the distribution) for each RH are scaled on the basis of the changes under (1);
- 3. Simulated poverty (and inequality) statistics are computed for each RH and aggregated to the nation.
- MAMS is programmed to generate standard poverty indicators and the Gini coefficient for a household survey (provided in Excel) or assuming a log-normal distribution for each RH.

THANK

YOU

!!!