Debre Markos University

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Department of Mathematics

Linear Optimization worksheet I

1. If C1 and C2 are convex, then C1 + C2 := {x + y : x ∈ C1, y ∈ C2} is convex. If C1 and C2 are convex, then C1 − C2 := {x − y : x ∈ C1, y ∈ C2} is convex.   **(2 points)**

2. $X = \{x \mid x = \alpha x_1 + \beta x_2, \text{where} x_2 \in S_1 \text{ and } x_2 \in S_2\}$ is a convex set, if S_1 and S_2 are convex sets.

3. Your city's cultural center is sponsoring a concert to raise at least $30,000 for the city's Youth Services. Tickets are $20 for balcony seats and $30 for orchestra seats. If the center has 500 orchestra seats, how many of each type of seat must they sell?

4. The sum of three numbers is 14. The largest is 4 times the smallest, while the sum of the smallest and twice the largest is 18. Find the numbers.

5. Sarah is selling bracelets and earrings to make money for summer vacation. The bracelets cost $2 and earrings cost $3. She needs to make at least $500. Sarah knows that she will see more than 50 bracelets. How many bracelets and earrings can Sarah sell?

6. Debre Markos University is going construct a swimming pool with the following restrictions in mind: the perimeter is to be at least 20 meters; the difference between the length (*y* meters) and the width (*x* meters) must be 5 meters or more and neither length nor width can exceed 15 meters.
   a. Write the restrictions in terms of algebraic inequalities;
   b. Graph the solution to the system of linear inequalities.
   c. Show that a pool of length of 10 m and a width of 10 m is not possible

7. A paint manufacturer produces two types of paint, one type of standard quality (S) and the other of top quality (T). To make these paints, he needs two ingredients, the pigment and the resin. Standard quality paint requires 2 units of pigment and 3 units of resin for each unit made and is sold at a profit of R1 per unit. Top quality paint requires 4 units of pigment and 2 units of resin for each unit made and is sold at a profit of R1.50 per unit. He has stocks of 12 units of pigment, and 10 units of resin. Formulate the above problem as a linear programming problem to maximize his profit?

8. A small brewery produces Ale and Beer. Suppose that production is limited by scarce resources of corn, hops and barley malt. To make Ale 5kg of Corn, 4kg of hops and 35kg of malt are required. To make Beer 15kg of corn, 4 kg of hops and 20kg of malt are required. Suppose that only 480 kg of corn, 160kg of hops and 1190 kg of malt are available. If the brewery makes a profit of E13 for each kg of Ale and E23 for each kg of Beer, how much Ale and Beer should the brewer produce in order to maximize profit?

9. A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 8 units of drug B, and

each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C, 3 units per gram in M and 1 unit per gram in N. How many grams of substances M and N should be mixed to meet the minimum daily requirements at the same time minimize the intake of drug C? How many units of the undesirable drug C will be in this mixture? **(2points)**

10. A laboratory technician in a medical research Centre is asked to formulate a diet from two commercially packaged foods, food A and food B, for a group of animals. Each kg of food A contains 8 units of fat, 16 units of carbohydrates, and 2 units of protein. Each Kg of food B contains 4 units of fat, 32 units of carbohydrate and 8 units of protein. The minimum daily requirements are 176 units of fat, 1024 units of carbohydrate, and 384 units of protein. If food A costs 5c per Kg and food B costs 5c per Kg, how many kilograms of each food should be used to meet the minimum daily requirements at the least cost? What is the cost of this amount?

11. A manufacturing plant makes two types of inflatable boats, a two-person boat and a four-person boat. Each two-person boat requires 0.9 labor hours from the cutting department and 0.8 labor hours from the assembly department. Each four-person boat requires 1.8 labor hours from the cutting department and 1.2 labor hours from the assembly department. The maximum labor hours available per month in the cutting department and the assembly department are 864 and 672 respectively. The company makes a profit of E25 on each two-person boat and E40 on each four-person boat. Use the graphical method to find the maximum profit.

12. A company manufactures two electrical products: air conditioners and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of E25. Each fan assembled may be sold for a profit of E15. Formulate and solve this linear programming mix situation to find the best combination of air conditioners and fans that yields the highest profit.

13. A wheat and barley farmer has 168 hectare of ploughed land, and a capital of E2000. It costs E14 to sow one-hectare wheat and E10 to sow one hectare of barley. Suppose that his profit isE80 per hectare of wheat and E55 per hectare of barley. Find the optimal number of hectares of wheat and barley that must be ploughed in order to maximize profit? What is the maximum profit?

14. Namboard produces two gift packages of fruit. Package A contains 20 peaches, 15 apples and 10 pears. Package B contains 10 peaches, 30 apples and 12 pears. Namboard has 40 000 peaches, 60 000 apples and 27 000 pears available for packaging. The profit on package A is E2.00 and the profit on B is E2.50. Assuming that all fruit packaged can be sold, what number of packages of types A and B should be prepared to maximize the profit?

15. A farmer can buy two types of plant food, mix A and mix B. Each cubic metre of mix A contains 20 kg of phosphoric acid, 30 kg of nitrogen, and 5 kg of potash. Each cubic metre of

mix B contains 10 kg of phosphoric acid, 30 kg of nitrogen and 10 kg of potash. The minimum monthly requirements are 460 kg of phosphoric acid, 960 kg of nitrogen, and 220 kg of potash. If mix A costs E30 per cubic meter and mix B costs E35 per cubic meter, how many cubic meters of each mix should the farmer blend to meet the minimum monthly requirements at a minimal cost? What is the cost?

16. Cohen Chemicals, Inc., produces two types of photo-developing fluids. The first, a black-and-white picture chemical, costs Cohen $2,500 per ton to produce. The second, a color photo chemical, costs $3,000 per ton. Based on an analysis of current inventory levels and outstanding orders, Cohen's production manager has specified that at least 30 tons of the black-and-white chemical and at least 20 tons of the color chemical must be produced during the next month. In addition, the manager notes that an existing inventory of a highly perishable raw material needed in both chemicals must be used within 30 days. To avoid wasting the expensive raw material, Cohen must produce a total of at least 60 tons of the photo chemicals in the next month.

17. A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators much be shipped each day. If each scientific calculator sold results in a $2 loss, but each graphing calculator produces a $5profit, how many of each type should be made daily to maximize net profits? **(2points)**

18. You need to buy some filing cabinets. You know that Cabinet X costs $10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Cabinet Y costs $20 per unit, requires eight square feet of floor space, and holds twelve cubic feet of files. You have been given $140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinets. How many of which model should you buy, in order to maximize storage volume?

19. In order to ensure optimal health (and thus accurate test results), a lab technician needs to feed the rabbits a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protein. But the rabbits should be fed no more than five ounces of food a day. Rather than order rabbit food that is custom-blended, it is cheaper to order Food X and Food Y, and blend them for an optimal mix. Food X contains 8 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs $0.20 per ounce. Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of $0.30 per ounce. What is the optimal blend? **(2points)**