Industrial Management and Engineering Economics

CHAPTER 4

INVENTORY MANAGEMENT & CONTROL

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What is Inventory?

- **Inventory** is stock of items kept to meet future demand.
- Is one of the most expensive assets of many companies, representing as much as **50%** of the total invested capital.
- Operations managers around the globe have long recognized that inventory management is crucial.
- **Inventory management** is a core operations management activity. Effective inventory management is important for the successful operation of most businesses and their supply chains.
- Poor inventory management hampers operations, diminishes customer satisfaction, and increases operating costs.

An example of simple inventory

Types of Inventory

- Raw materials
- Purchased parts and supplies
- Work-in-process (partially completed) products (WIP)
- Items being transported
- Spare parts, Tools, and equipment

Functions of Inventory

Inventories serve a number of functions. Among the most important are the following:

1. To maintain independency of operation.
2. To meet variation in product demand.
3. To allow flexibility in production scheduling.
4. To provide a safeguard for variation in raw material delivery time.
5. To take advantage of economic purchase order size.

The overall objective of inventory management is to achieve satisfactory levels of customer service while keeping inventory costs within reasonable bounds.

REQUIREMENTS FOR EFFECTIVE INVENTORY MANAGEMENT

The requirements for effective inventory management are the following:

i. A system to keep track of the inventory on hand and on order.

ii. A reliable forecast of demand that includes an indication of possible forecast error.

iii. Knowledge of lead times and lead time variability.

iv. Reasonable estimates of inventory holding costs, ordering costs, and shortage costs.

v. A classification system for inventory items.

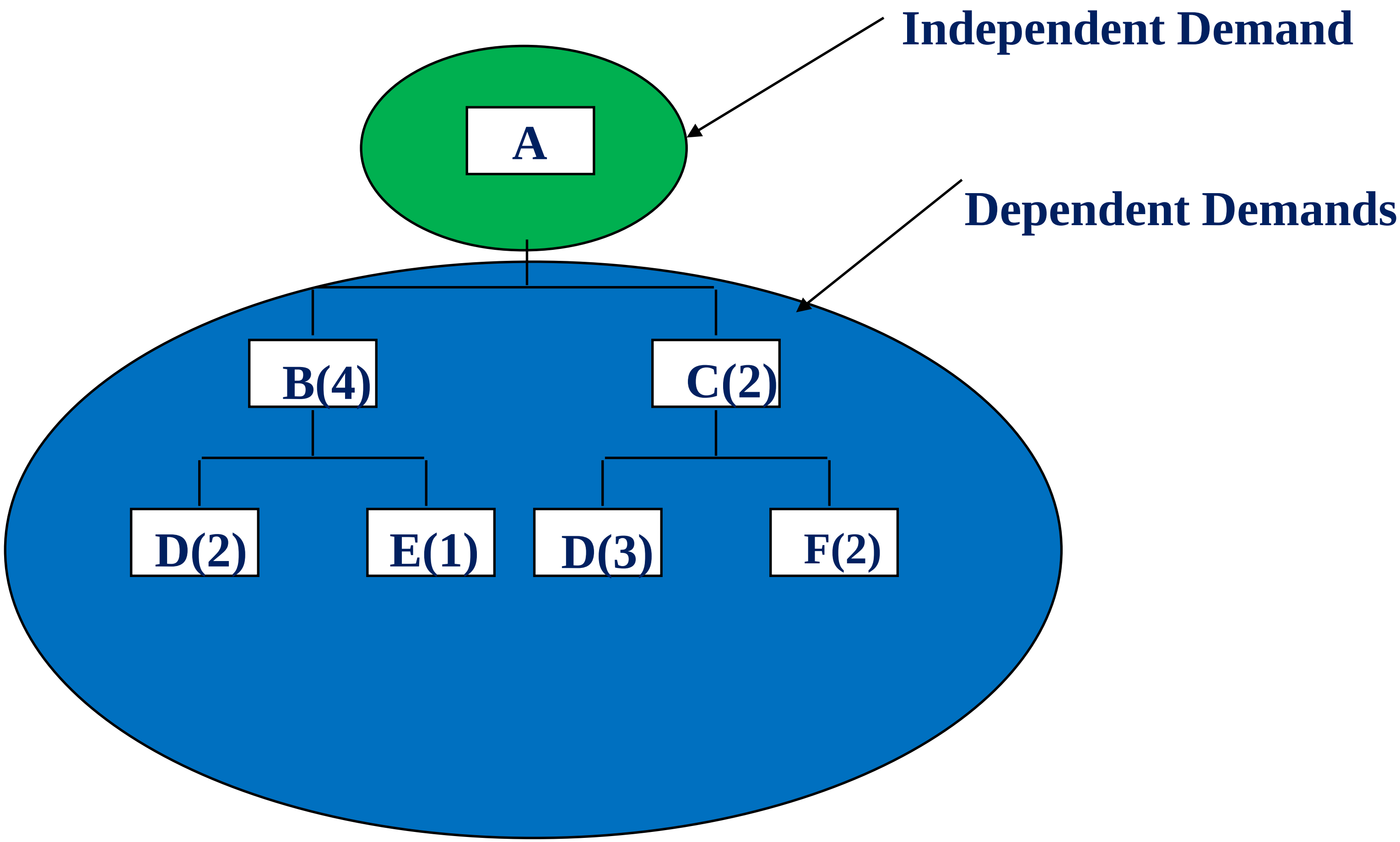
Demand Forecasts and Lead Time Information

- ✓ Inventories are used to satisfy demand requirements, so it is essential to have reliable estimates of the amount and timing of demand.
- ✓ Similarly, it is essential to know how long it will take for orders to be delivered.
- ✓ In addition, managers need to know the extent to which demand and lead time (the time between submitting an order and receiving it) might vary; the greater the potential variability, the greater the need for additional stock to reduce the risk of a shortage between deliveries. Thus, there is a crucial link between forecasting and inventory management.

NB:- Lead time is the time interval between ordering and receiving the order.

Two forms of demand inventory

- **Independent**
  - Demand for items used by external customers
  - Cars, appliances, computers, and houses are examples of independent demand inventory
- **Dependent**
  - Demand for items used to produce final products
  - Tires stored at a plant are an example of a dependent demand item



- Independent demand is uncertain.
- Dependent demand is certain.

Techniques of Inventory Control

- Items held in inventory are not of equal importance in terms of dollars invested, profit potential, sales or usage volume. Therefore, a more reasonable technique is required to control items according to their relative importance.

1. A-B-C analysis

- The most widely used method of inventory control is known as ABC analysis.
- The A-B-C approach classifies inventory items according to their annual dollar value.
- Typically, three classes of items are used: A (very important), B (moderately important), and C (least important).

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(a) **A-Item**: Very tight control, the items being of high value. The control need be exercised at higher level of authority.

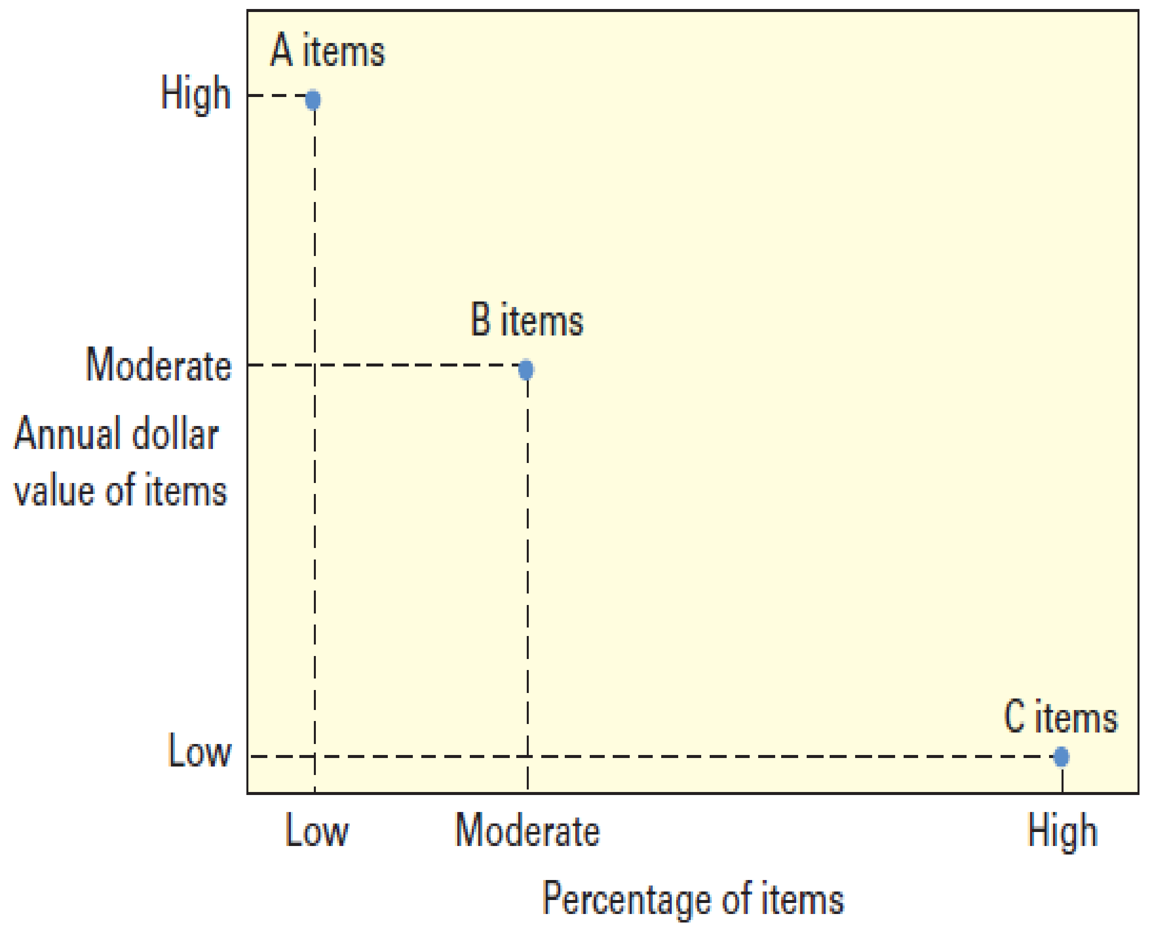
(b) **B-Item:** Moderate control, the items being of moderate value. The control need be exercised at middle level of authority.

(c) **C-Item**: The items being of low value, the control can be exercised at gross root level of authority, i.e., by respective user department managers.

A - very important

B - mod. important

C – least important



To solve an A-B-C problem, follow these steps:

a) For each item, multiply annual volume by unit price to get the annual dollar value.

b) Arrange annual dollar values in descending order.

c) The few (10 to 15 percent) with the highest annual dollar value are A items. The most (about 50 percent) with the lowest annual dollar value are C items. Those in between (about 35 percent) are B items.

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- **E.g.** A manager has obtained a list of unit costs and estimated annual demands for 10 inventory items. Categorize the items on an A-B-C basis.

Item Number	Annual Demand (00)	×	Unit Cost	=	Annual Dollar Value (00)
1	25		$ 360		$ 9,000
2	10		70		700
3	24		500		12,000
4	15		100		1,500
5	7		70		490
6	10		1,000		10,000
7	2		210		420
8	10		4,000		40,000
9	80		10		800
10	5		200		1,000
					75,910

Step 2. Arranging the annual dollars values in descending order can facilitate assigning items to categories:

Item Number	Annual Dollar Value	Classification	Percentage of Items	Percentage of Annual Dollar Value
8	$40,000	A	10	52.7
3	12,000	B		
6	10,000	B	30	40.8
1	9,000	B		
4	1,500	C		
10	1,000	C		
9	800	C	60	6.5
2	700	C		
5	490	C		
7	420	C		
	75,910		100	100

Note that category A has the fewest number of items but the highest percentage of annual dollar value, while category C has the more items but only a small percentage of the annual dollar value.

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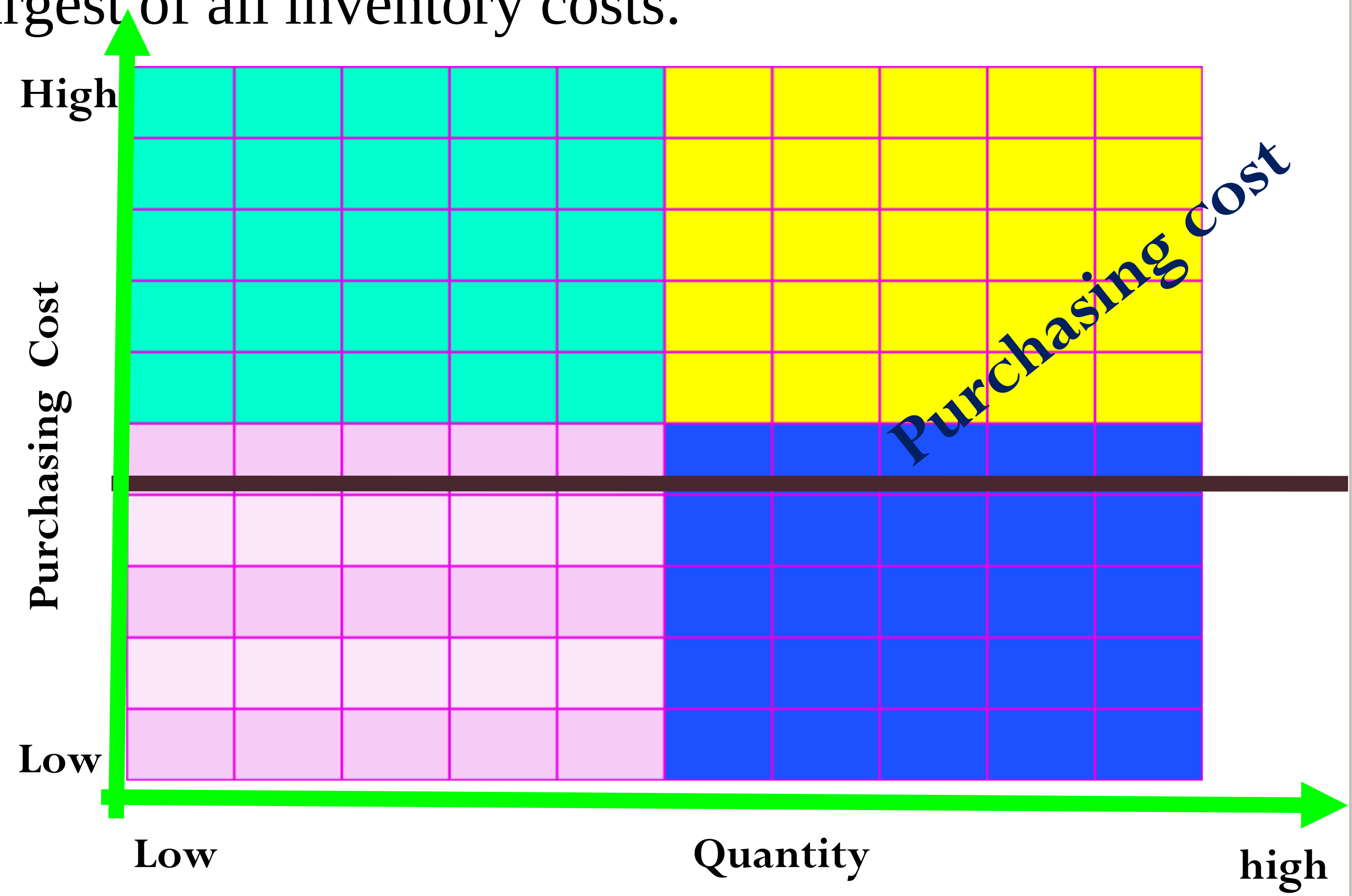
2. **HML analysis:** In this analysis, the classification of existing inventory is based on unit price of the items. They are classified as **high price, medium price and low cost items.**
3. **VED analysis:** In this analysis, the classification of existing inventory is based on criticality of the items. They are classified as **vital, essential and desirable items.** It is mainly used in spare parts inventory.
4. **FSN analysis:** In this analysis, the classification of existing inventory is based consumption of the items. They are classified as **fast moving, slow moving and non-moving items.**
5. **SOS analysis:** In this analysis, the classification of existing inventory is based nature of supply of items. They are classified as seasonal and off-seasonal items.

Inventory Costs

1. Purchase cost is the actual price paid for the item. It is typically the largest of all inventory costs.



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2. Carrying Cost

- Cost of holding an item in inventory such as storage facility, handling, insurance, breakage, obsolescence, depreciation, taxes etc.
- It is generally estimated that carrying costs range from **10 to 40%** of the value of a manufactured item.

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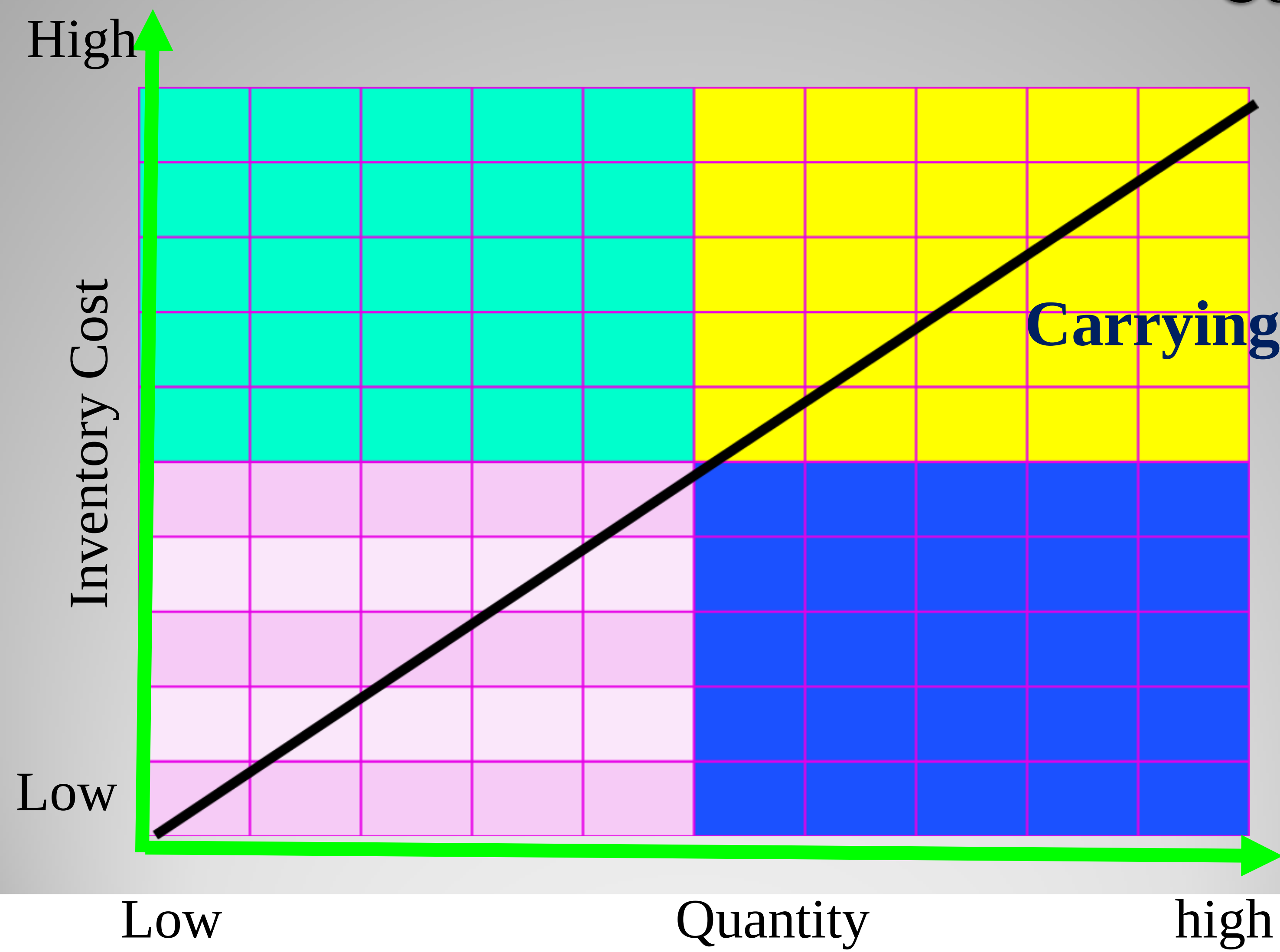

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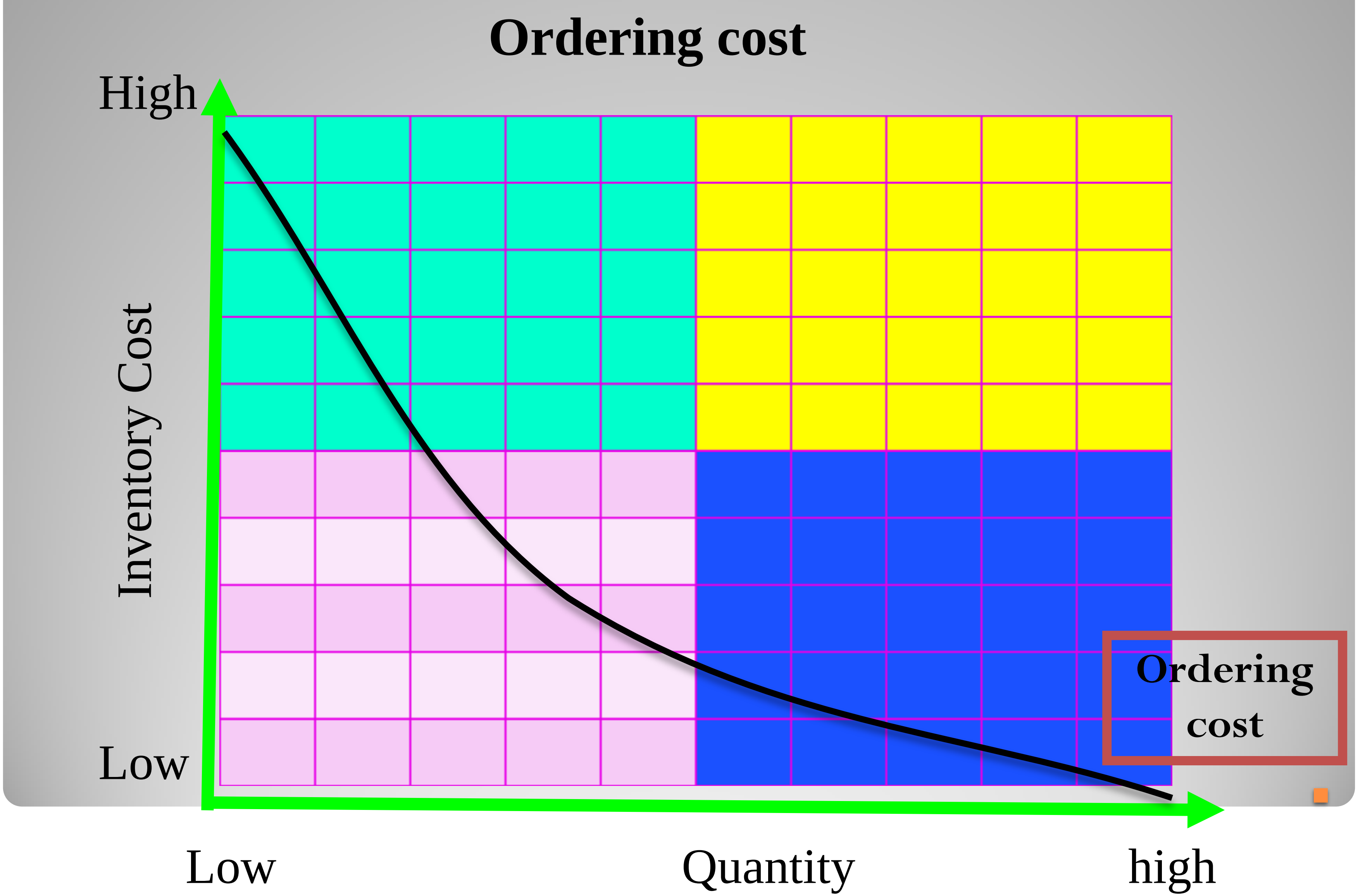
3. Ordering cost

- Cost of replenishing inventory such as managerial and clerical costs
- Include requisition and purchase orders, transportation and shipping, receiving, inspection, handling, and accounting and auditing costs.


Ordering cost
High
Inventory Cost
Low
Ordering cost
Low
Quantity
high

Cont.…

4. Shortage cost

- Temporary or permanent loss of sales when demand cannot be met.
- It is resulted when demand exceeds the supply of inventory on hand. It includes backorder costs, loss of customer goodwill and similar costs.

5. Setup costs :- When a firm produces its own inventory instead of ordering it from a supplier, machine **setup costs** (e.g., preparing equipment for the job by adjusting the machine, changing cutting tools) are analogous to ordering costs.

- Setup costs are expressed as a fixed charge per production run, regardless of the size of the run.

Inventory Control Systems

1. **Continuous system (fixed-order-quantity)**
   - Constant amount ordered when inventory declines to predetermined level.

2. **Periodic system (fixed time period)**
   - Order placed for variable amount after fixed passage of time.

HOW MUCH TO ORDER: ECONOMIC ORDER QUANTITY MODELS

- The question of how much to order can be determined by using an **economic order quantity (EOQ)** model.
- EOQ models identify the optimal order quantity by minimizing the sum of certain annual costs that vary with order size and order frequency. Three order size models are described here:

    1. The basic economic order quantity model.
    2. The economic production quantity model.
    3. The quantity discount model.

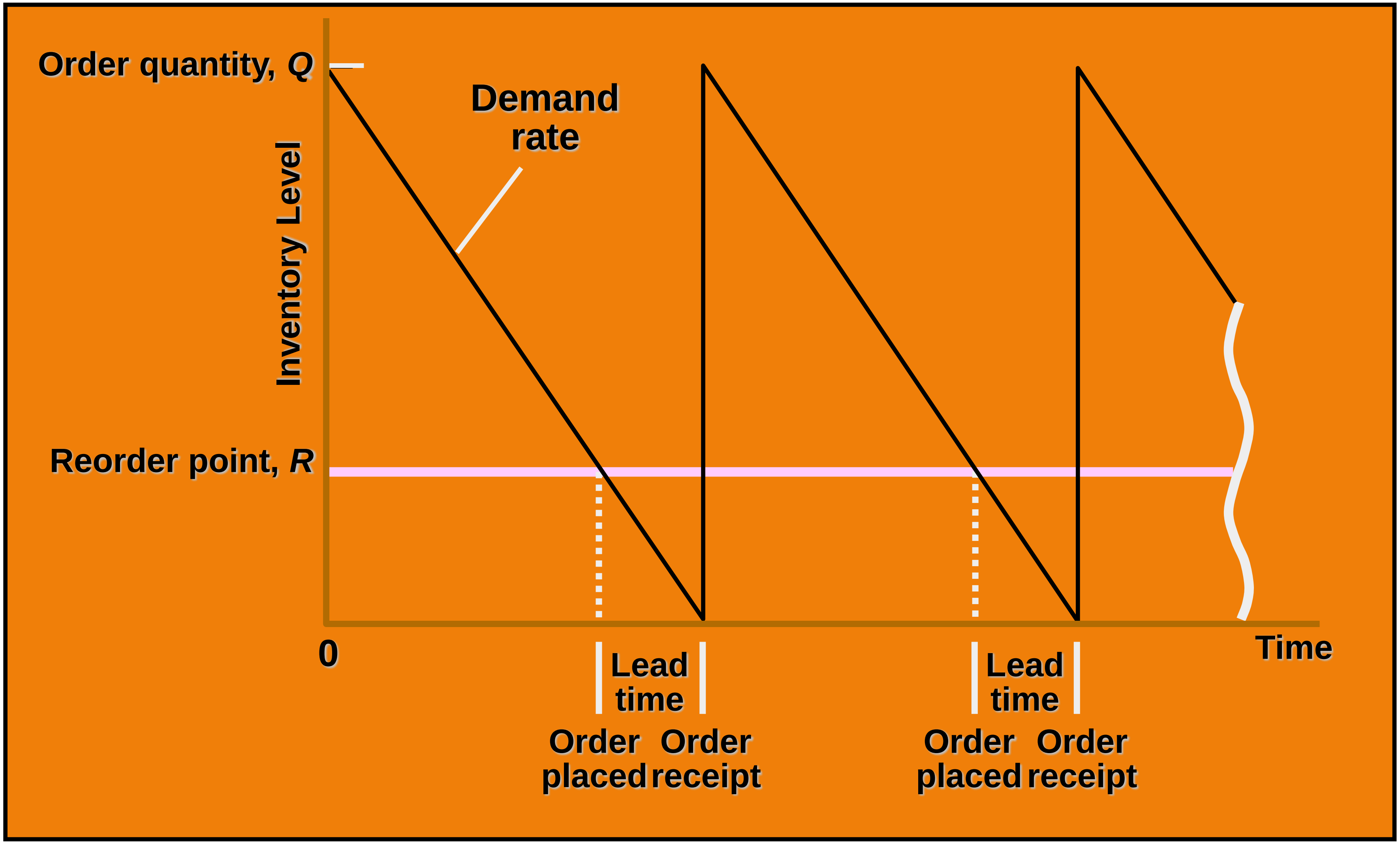
1. The basic economic order quantity model.

- ✓ The basic EOQ model is the simplest of the three models.
- ✓ It is used to identify a fixed order size that will minimize the sum of the annual costs of **holding inventory** and **ordering inventory.**
- ✓ The unit purchase price of items in inventory is not generally included in the total cost because the unit cost is unaffected by the order size unless quantity discounts are a factor. If holding costs are specified as a percentage of unit cost, then unit cost is indirectly included in the total cost as a part of holding costs.

Assumptions of Basic EOQ Model

- ✓ Only one product is involved.
- ✓ Annual demand requirements are known.
- ✓ Demand is spread evenly throughout the year so that the demand rate is reasonably constant.
- ✓ Lead time is known and constant.
- ✓ Each order is received in a single delivery.
- ✓ There are no quantity discounts.

Inventory Order Cycle



EOQ Cost Model

A - cost of placing order

D - annual demand

C - annual per-unit carrying cost

Q - order quantity

$$\text{Annual ordering cost} = \frac{AD}{Q}$$

$$\text{Annual carrying cost} = \frac{CQ}{2}$$

$$\text{Total cost} = \frac{AD}{Q} + \frac{CQ}{2}$$

EOQ Cost Model

Deriving Q_{opt}

$$TC = \frac{AD}{Q} + \frac{CQ}{2}$$

$$\frac{\partial TC}{\partial Q} = -\frac{AD}{Q^2} + \frac{C}{2}$$

$$0 = -\frac{AD}{Q^2} + \frac{C}{2}$$

$$Q_{opt} = \sqrt{\frac{2AD}{C}}$$

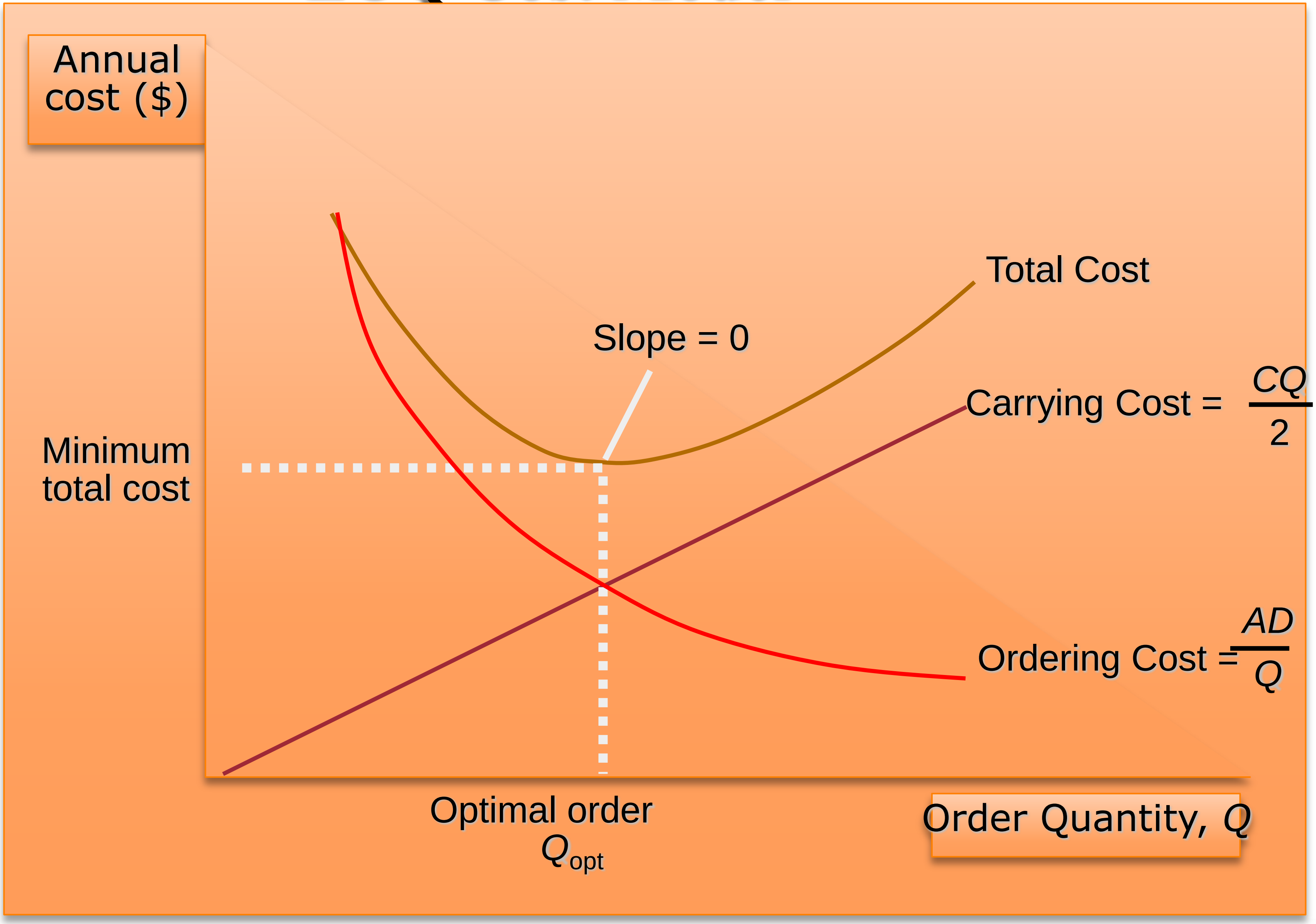
Proving equality of costs at optimal point

$$\frac{AD}{Q} = \frac{CQ}{2}$$

$$Q^2 = \frac{2AD}{C}$$

$$Q_{opt} = \sqrt{\frac{2AD}{C}}$$

EOQ Cost Model



Example 1

A local distributor for a national tire company expects to sell approximately 9,600 steel-belted radial tires of a certain size and tread design next year. Annual carrying cost is $16 per tire, and ordering cost is $75. The distributor operates 288 days a year.

a. What is the EOQ?

b. How many times per year does the store reorder?

c. What is the length of an order cycle?

d. What is the total annual cost if the EOQ quantity is ordered?

Given

D = 9,600 tires per year

H = $16 per unit per year

S = $75

Solution

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}600)75}{16}} = 300$ tires.

b. Number of orders per year: $D/Q = \frac{9{,}600 \text{ tires/year}}{300 \text{ tires/order}} = 32$ orders.

c. Length of order cycle: $Q/D = \frac{300 \text{ tires}}{9{,}600 \text{ tires/year}} = \frac{1}{32}$ of a year, which is $\frac{1}{32} \times 288$, or nine workdays.

d. $$\begin{aligned} \text{TC} &= \text{Carrying cost} + \text{Ordering co} \\ &= (Q/2)H + (D/Q)S \\ &= (300/2)16 + (9{,}600/300)75 \\ &= \$2{,}400 + \$2{,}400 \\ &= \$4{,}800 \end{aligned}$$

Note that the ordering and carrying costs are equal at the EOQ.

Example 2

- The Paint Store stocks paint in its warehouse and sells it online on its internet web site. The store stocks several brands of paint; however, its biggest seller is Sharman-Wilson Iron coat paint. The company wants to determine the optimal order size and total inventory cost for Iron coat paint given an estimated annual demand of 10,000 gallons of paint, an annual carrying cost of $0.75 per gallon, and an ordering cost of $150 per order. They would also like to know the number of orders that will be made annually and the time between orders (i.e., the order cycle).

EOQ Example

C = \$0.75 per galloons A = \$150 D = 10,000 galloons

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} \qquad TC_{min} = \frac{AD}{Q} + \frac{CQ}{2}$$

$$Q_{opt} = \sqrt{\frac{2(150)(10{,}000)}{(0.75)}} \qquad TC_{min} = \frac{(150)(10{,}000)}{2{,}000} + \frac{(0.75)(2{,}000)}{2}$$

$$Q_{opt} = 2{,}000 \text{ galloons} \qquad TC_{min} = \$750 + \$750 = \$1{,}500$$

Orders per year = D/Q_{opt}
= 10,000/2,000
= 5 orders/year

Order cycle time = 311 days/(D/Q_{opt})
= 311/5
= 62.2 store days

Reorder Point

Level of inventory at which a new order is placed

$$R = d*L$$

where

d = demand rate per period

L = lead time

Reorder Point: Example

Demand = 10,000 galloons/year

Store open 311 days/year

Daily demand =10,000/311 = 32.154 galloons/day

Lead time = L = 10 days

R = d*L = (32.154)(10) = 321.54 galloons

2. Production Quantity Model

- An inventory system in which an order is received gradually, as inventory is simultaneously being depleted.
- Non-instantaneous receipt model

✓ assumption that **Q** is received all at once is relaxed

- **p -** daily rate at which an order is received over time, production rate and **d -** daily rate at which inventory is demanded.

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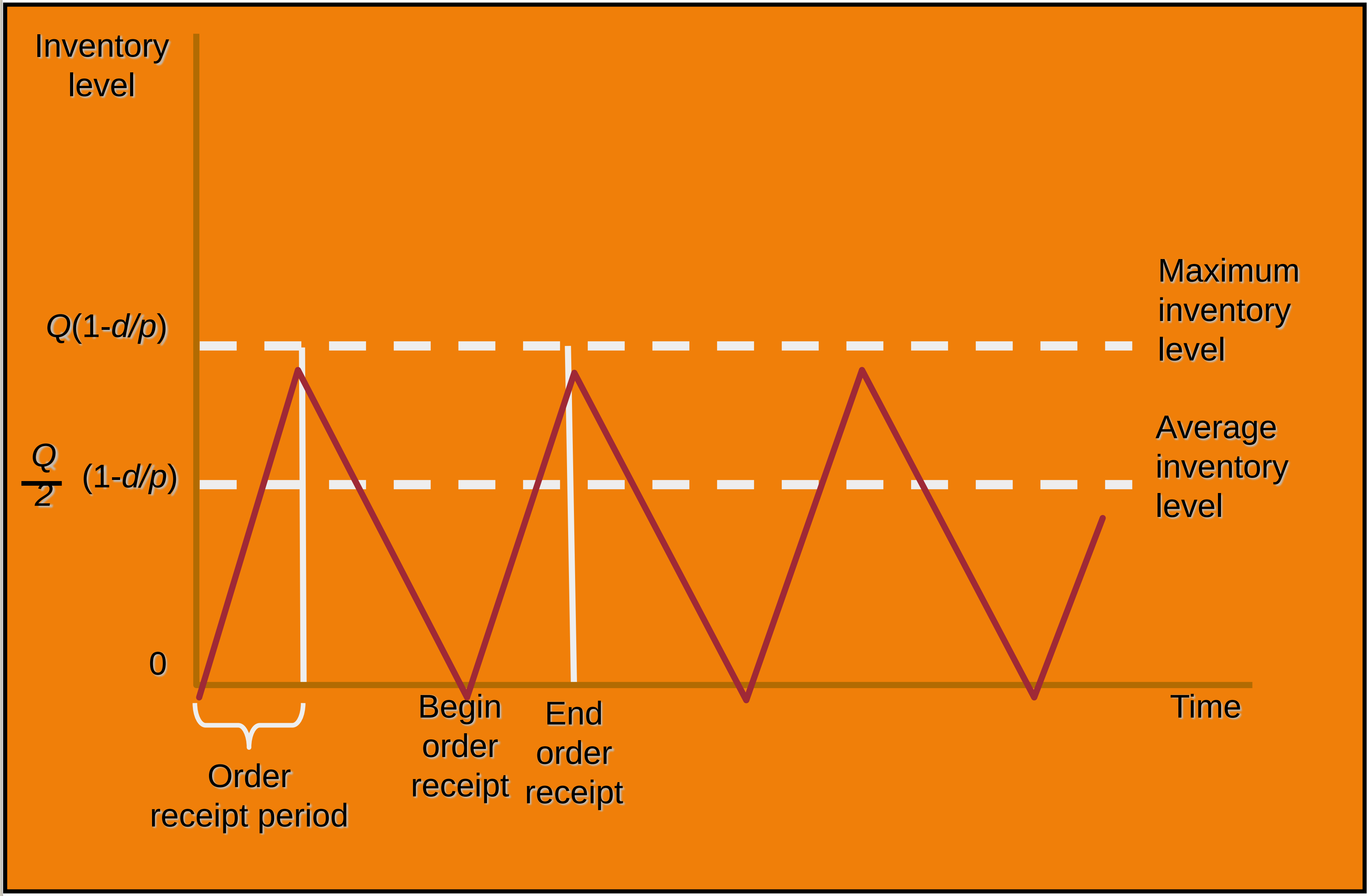


Inventory level
Q(1-d/p)
Q/2 (1-d/p)
0
Maximum inventory level
Average inventory level
Time
Order receipt period
Begin order receipt
End order receipt

Cont'd

p = production rate d = demand rate

Maximum inventory level $= Q - \frac{Qd}{p}$

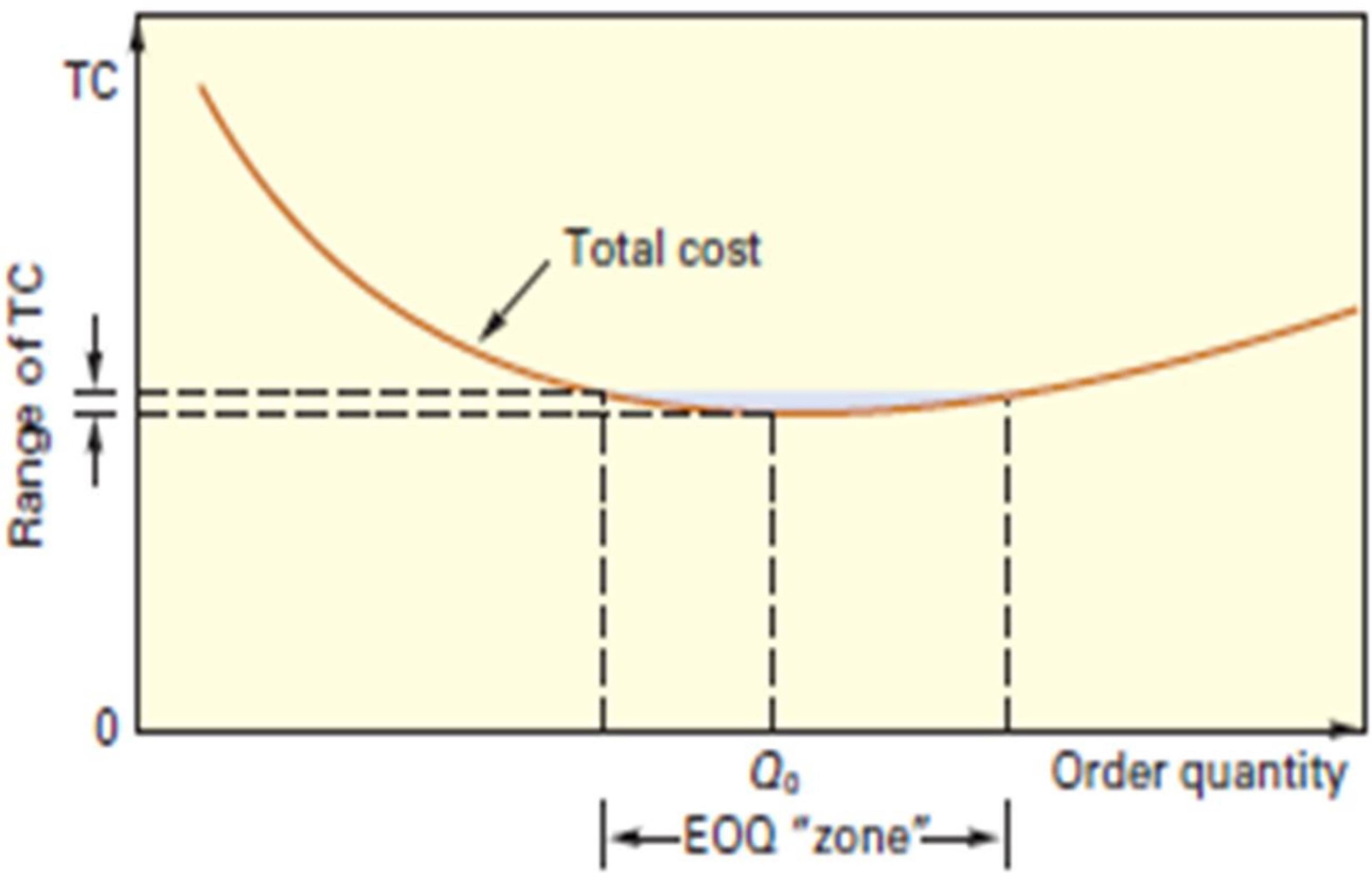
$= Q\left(1 - \frac{d}{p}\right)$

Average inventory level $= \frac{Q}{2}\left(1 - \frac{d}{p}\right)$

$$TC = \frac{AD}{Q} + \frac{CQ}{2}\left(1 - \frac{d}{p}\right)$$

$$Q_{opt} = \sqrt{\frac{2AD}{C\left(1 - \frac{d}{p}\right)}}$$

The total cost curve is relatively flat near the EOQ



Production Quantity Model: Example

C = \$0.75 per yard	A = \$150	D = 10,000 yards
d = 10,000/311 = 32.2 yards per day		p = 150 yards per day

$$Q_{opt} = \sqrt{\frac{2AD}{C\left(1 - \frac{d}{p}\right)}} = \sqrt{\frac{2(150)(10{,}000)}{0.75\left(1 - \frac{32.2}{150}\right)}} = 2{,}256.8 \text{ yards}$$

$$TC = \frac{AD}{Q} + \frac{CQ}{2}\left(1 - \frac{d}{p}\right) = \$1{,}329$$

$$\text{Production run} = \frac{Q}{p} = \frac{2{,}256.8}{150} = 15.05 \text{ days per order}$$

Cont'd….

$$\text{Number of production runs} = \frac{D}{Q} = \frac{10{,}000}{2{,}256.8} = 4.43 \text{ runs/year}$$

$$\text{Maximum inventory level} = Q\left(1 - \frac{d}{p}\right) = 2{,}256.8\left(1 - \frac{32.2}{150}\right)$$

$$= 1{,}772 \text{ yards}$$

3. Quantity Discounts Model

Price per unit decreases as order quantity increases

$$TC = \frac{AD}{Q} + \frac{CQ}{2} + PD$$

where

P = per unit price of the item
D = annual demand

- The goal : is to reduce price (P) for an item when it is purchased in larger quantities.

Discount Number	Discount Quantity	Discount (%)	Discount Price (P)
1	0 to 999	no discount	$5.00
2	1,000 to 1,999	4	$4.80
3	2,000 and over	5	$4.75

Total Cost = Setup cost(order cost) + Holding cost + Product cost

$$TC = DS/Q + QH/2 + PD$$

Q:- Quantity ordered,

D:- Annual demand in units

S:- Ordering or setup cost per order per setup

P:- Price per unit, **H-:** Holding cost per unit per year

- The Steps to determine Optimum Quantity order :

a) Calculate value of optimum order for each discount

b) $Q^* = \sqrt{2DS/IP}$, Holding cost (I) as percentage of unit price (P)

c) If order quantity is too low, adjust the order qty. upward to the lowest qty that will qualify for the discount

d) Compute total cost for each

e) Select Q* with the lowest total cost

- Example : The store stocks toy race cars. Recently, the store has been given a quantity discount schedule for these cars. The normal cost for the race car is $5.00. For orders between 1,000 and 1,999 units, the unit cost drops to $4.80; for orders 2,000 or more units, the unit cost is only$4.75. Furthermore, ordering cost is $49.00 per order, annual demand is 5,000 race cars and inventory carrying charge as a percent of cost is 20%. What order quantity will minimize the total inventory cost?

Q1* $= \sqrt{2DS/IP} = \sqrt{2(5{,}000)(49)/(20\%)(5.00)}$ = 700 cars order

Q2* $= \sqrt{2DS/IP} = \sqrt{2(5{,}000)(49)/(20\%)(4.80)}$ = 714 cars order

Q3* $= \sqrt{2DS/IP} = \sqrt{2(5{,}000)(49)/(20\%)(4.75)}$ = 718 cars order step 1 & 2.

- Q1* = 700
- Q2* = 1,000 adjusted
- Q3* = 2,000 adjusted

Step 3. Calculate Total Cost each discount

TC = DS/Q + QH/2 + PD

Discount Number	Unit rice (P)	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total Cost
1	$5.00	700	$25,000.00	$350	$350	$25,700
2	$4.80	1,000	$24,000.00	$245	$480	$24,725
3	$4.75	2,000	$23,750.00	$122,5	$950	$24,823

- Annual Product Cost = PD
- Annual ordering cost = DS/Q
- Annual Holding cost = QH/2 = QIP/2

- 4. Select Order Quantity at the lowest cost 1,000 units

Quantity Discount: Example

Avtek, a distributor of audio and video equipment, wants to reduce a large stock of televisions. It has offered a local chain of stores a quantity discount pricing schedule, as follows:

Quantity	Price
1–49	$1400
50–89	1100
90+	900

The annual carrying cost for the stores for a TV is $190, the ordering cost is $2,500, and annual demand for this particular model TV is estimated to be 200 units. The chain wants to determine if it should take advantage of this discount or order the basic EOQ order size.

Solution

QUANTITY	PRICE
1 - 49	$1,400
50 - 89	1,100
90+	900

A = $2,500

C = $190 per computer

D = 200

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_c}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ PCs}$$

For Q = 72.5

$$TC = \frac{AD}{Q_{opt}} + \frac{CQ_{opt}}{2} + PD = \$233,784$$

For Q = 90

$$TC = \frac{AD}{Q} + \frac{CQ}{2} + PD = \$194,105$$

Safety Stocks

- **Safety stock**

  Buffer added to on hand inventory during lead time

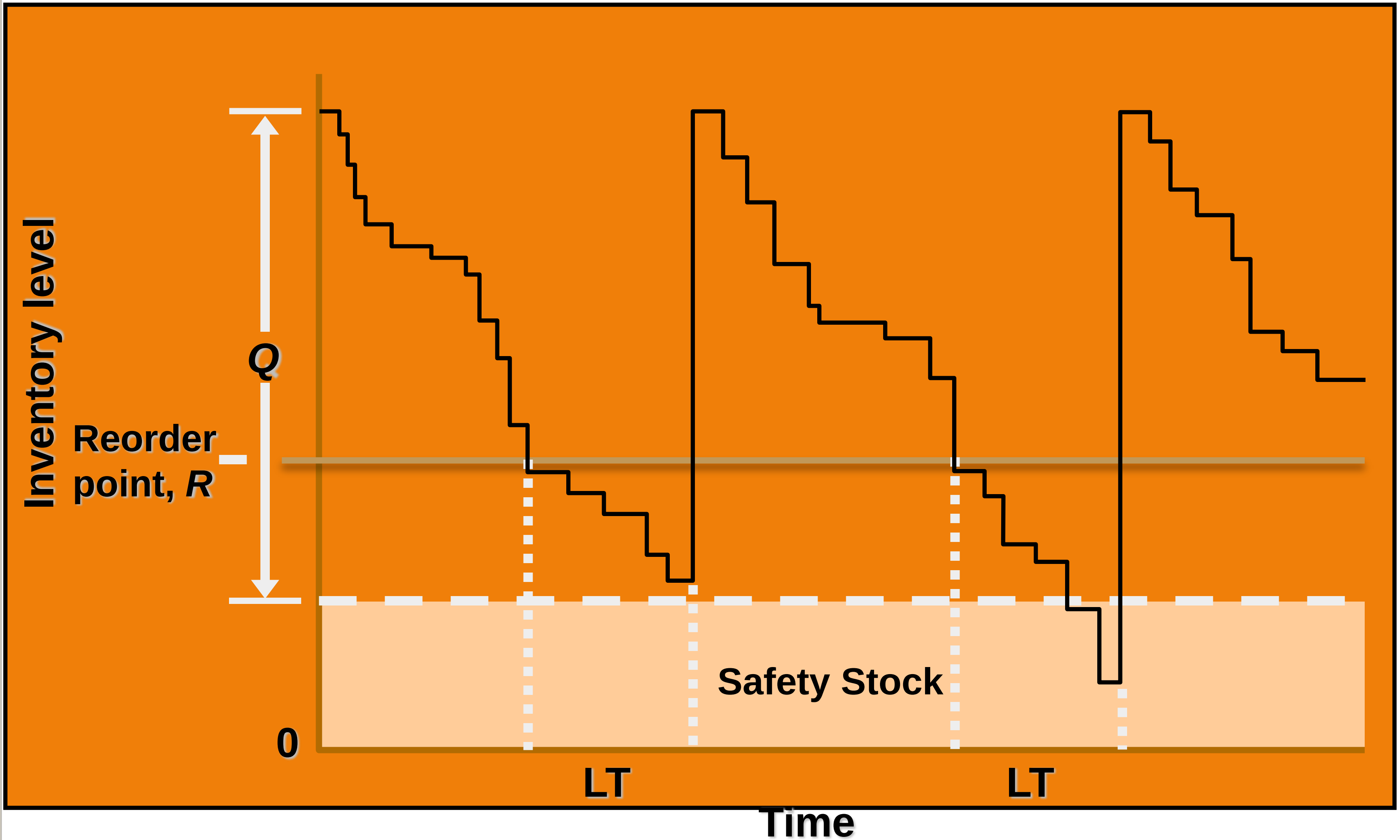
- **Stock out**

  An inventory shortage

- **Service level**

  Probability that the inventory available during lead time will meet demand.

Reorder Point with a Safety Stock



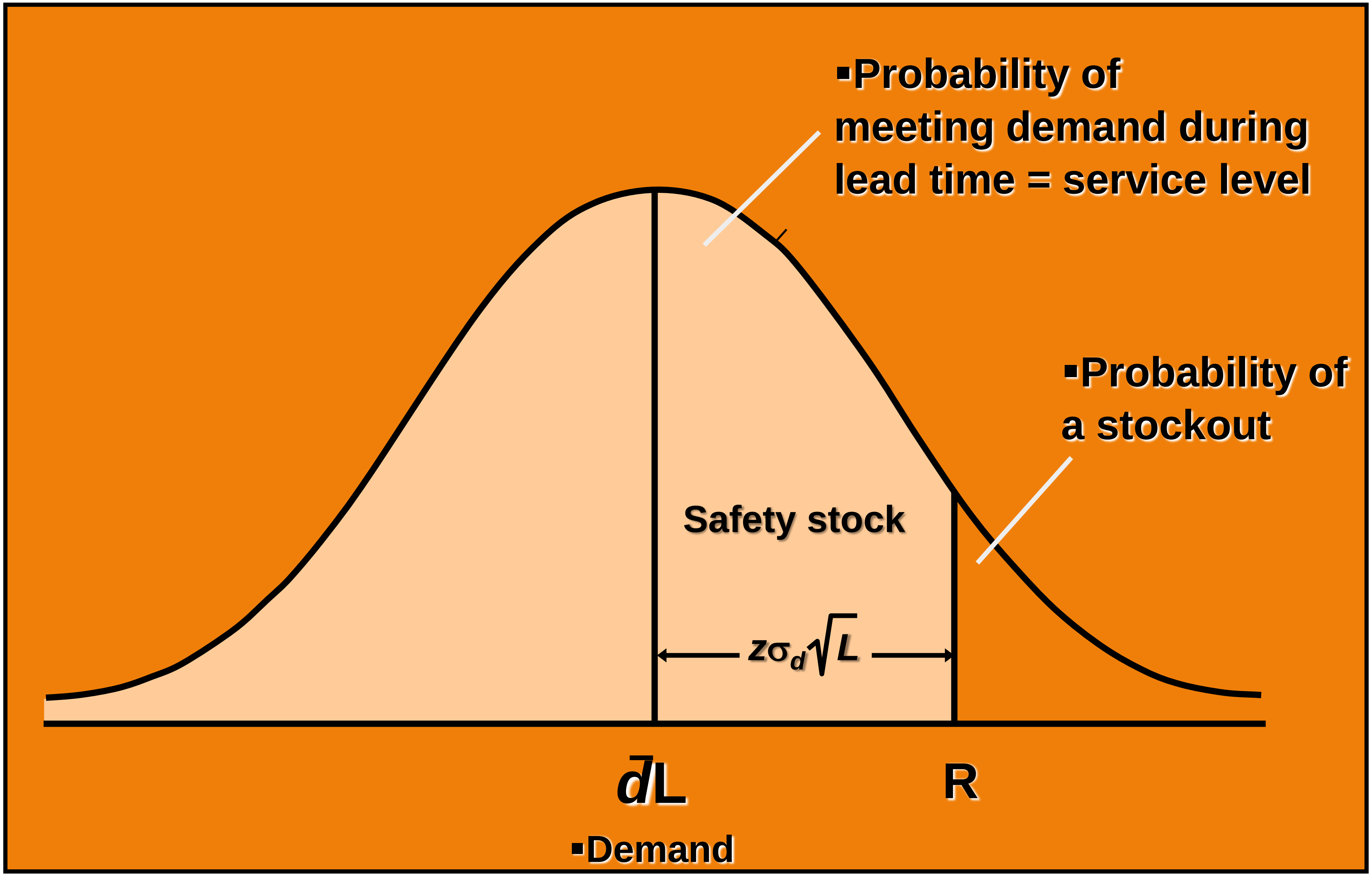
Reorder Point With Variable Demand

- $R = d^*L + z^*\sigma_d \sqrt{L}$

where

- ✓ d = average daily demand
- ✓ L = lead time
- ✓ σ_d = the standard deviation of daily demand
- ✓ z = number of standard deviations corresponding to the service level probability (service factor)
- ✓ $z^*\sigma_d \sqrt{L}$ = safety stock

Reorder Point for Service Level



Service factor values for CSL

Service Level	Service Factor	Service Level	Service Factor
50.00%	0.00	90.00%	1.28
55.00%	0.13	91.00%	1.34
60.00%	0.25	92.00%	1.41
65.00%	0.39	93.00%	1.48
70.00%	0.52	94.00%	1.55
75.00%	0.67	95.00%	1.64
80.00%	0.84	96.00%	1.75
81.00%	0.88	97.00%	1.88
82.00%	0.92	98.00%	2.05
83.00%	0.95	99.00%	2.33
84.00%	0.99	99.50%	2.58
85.00%	1.04	99.60%	2.65
86.00%	1.08	99.70%	2.75
87.00%	1.13	99.80%	2.88

Reorder Point for Variable Demand

- The carpet store wants a reorder point with a 95% service level and a 5% stock out probability

d= 30 m per day

$\bar{L}$= 10 days

σ_d= 5 m per day

- For a 95% service level, z = 1.64

$$R = d\bar{L} + z\,\sigma_d\sqrt{L}$$

$$= 30(10) + (1.64)(5)(\sqrt{10})$$

$$= 325.9 \text{ m}$$

$$\text{Safety stock} = z\,\sigma_d\sqrt{L}$$

$$= (1.64)(5)(\sqrt{10})$$

$$= 25.9 \text{ m}$$

Purchasing

- Purchasing is an important function of materials management.
- In any industry purchase means buying of equipments, materials, tools, parts etc. required for industry.

Objectives of Purchasing

- The basic objective of the purchasing function is to ensure continuity of supply of raw materials, sub-contracted items and spare parts and to reduce the ultimate cost of the finished goods.

Parameters of Purchasing

- The success of any manufacturing activity is largely dependent on the procurement of raw materials of right quality, in the right quantities, from right source, at the right time and at right price popularly known as **ten ‘R’s’** of the art of efficient purchasing.

Purchase parameters

- Right price
- Right quality
- Right time
- Right source
- Right quantity
- Right attitude
- Right transportation
- Right contract
- Right place of delivery
- Right material

Purchasing Procedure

- ✓Recognition of the need
- ✓The selection of the supplier
- ✓Placing the order
- ✓Follow-up of the order
- ✓Receiving and inspection of the materials
- ✓Payment of the invoice
- ✓Maintenance of the records
- ✓Maintenance of vendor relations

Thank you!!!