Industrial Management and Engineering Economics

Chapter 2
forecasting

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What comes in your mind when one talks about forecasting?

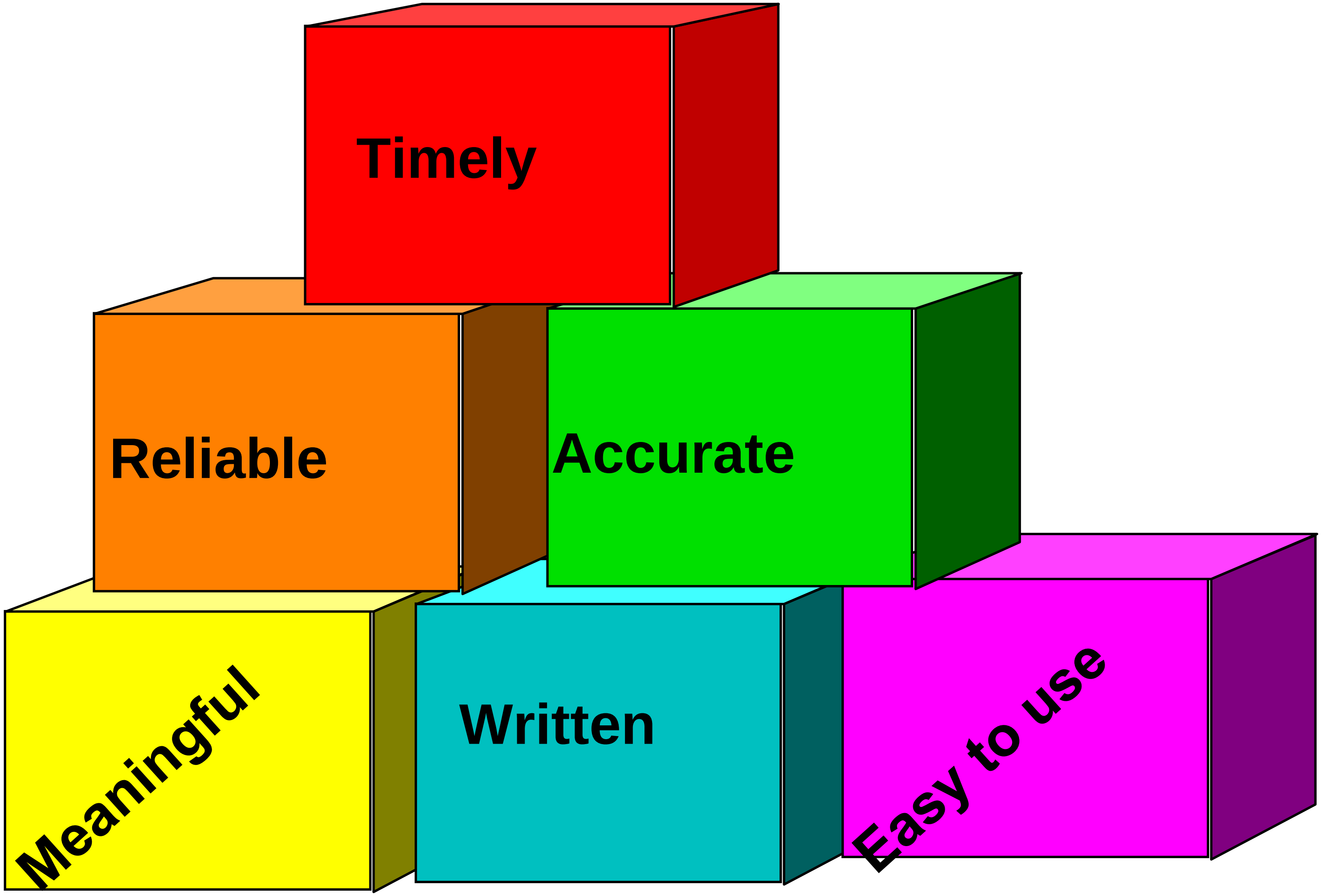
- Forecasting is the art and science of predicting **future events.**
- Focus on the forecasting of demand for output from the operations function (Demand may differ from sales, forecasting may serve for developing operation planning)
- Demand management is coordinating and controlling all source of demand so the production system can be used efficiently and product be delivered on time.

What is the difference between forecasting and planning?

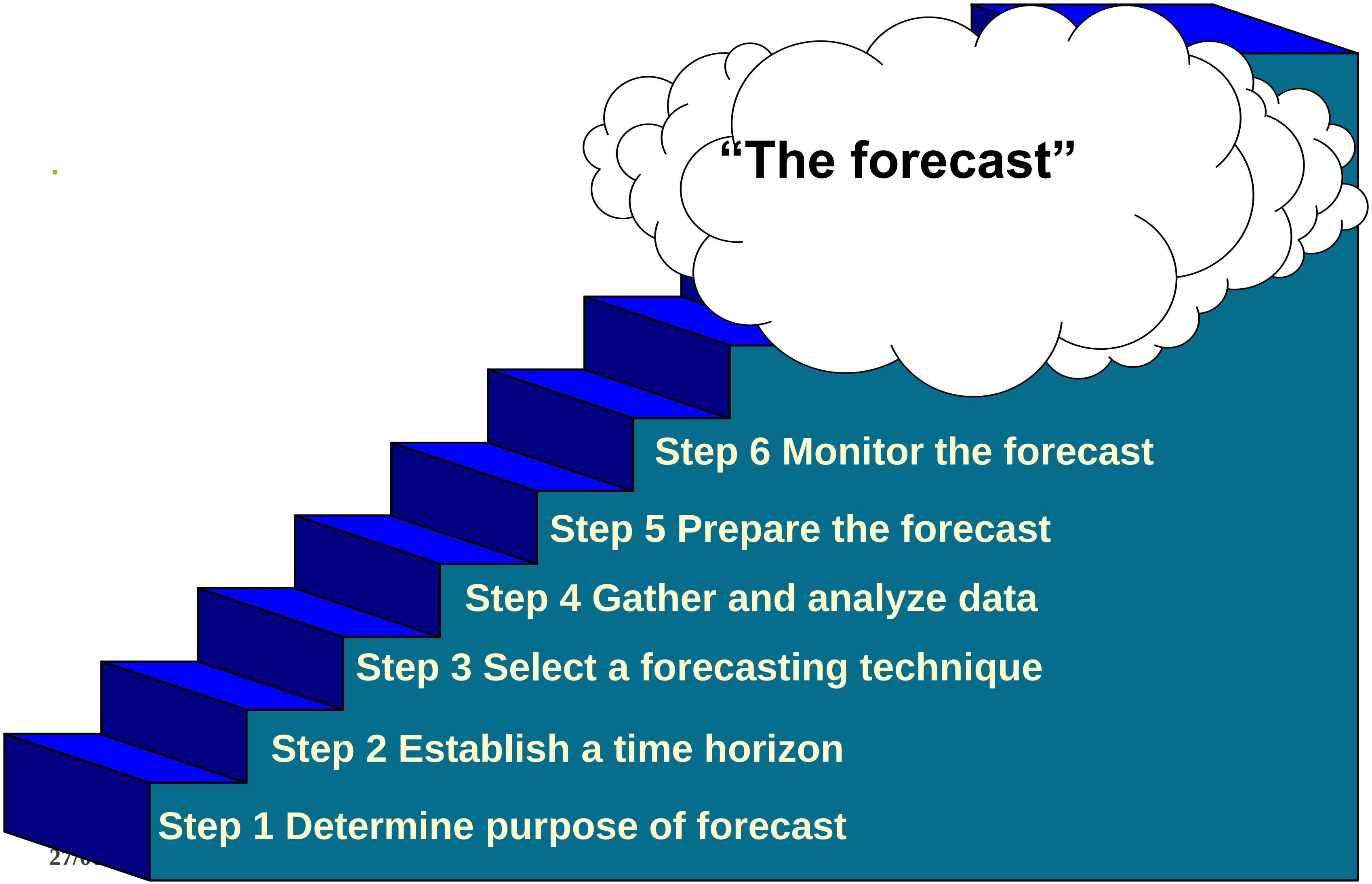
- Difference between **forecasting** and **planning**.
  - Forecasting: what we think will happen
  - Planning: what we think should happen
- Forecasting is an input to all business planning and control.
  - Marketing uses for planning product, pricing, promotion and placement.

NB:- Forecasting is a tool used for predicting future demand (event) based on past demand information.

Elements of a Good Forecast



Steps in the Forecasting Process



A Decision Making Process

- The basic aim of management is to transform a company's strategic objectives into decision and action.
- The constantly increasing volatility of business dynamics emphasizes the critical importance forecasting in decision making process.

Forecasting Horizon

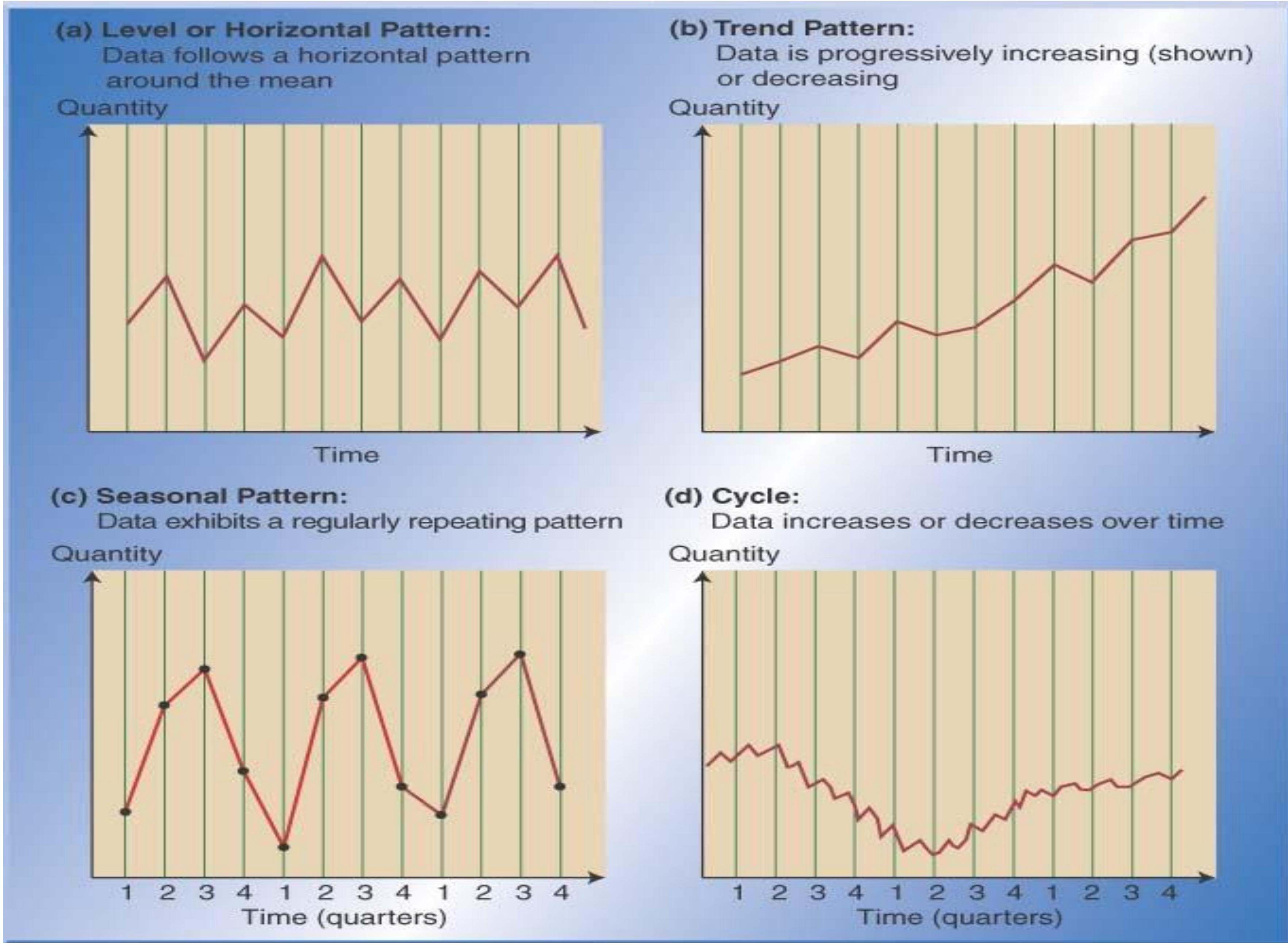
- ✓ **Short term (0 to 3 months):** for inventory management and scheduling.
- ✓ **Medium term (3 month to 2 years):** for production planning, purchasing and distribution.
- ✓ **Long term (2 years and more):** for capacity planning, facility location and strategic planning.

Characteristics of demand over time

- **Time series analysis**

Plot the demand data on a time scale, study the plot and look for consistent shape or pattern. The time series of demand might have the following pattern

- Constant
- Trends
- Seasonal and cyclical pattern
- Random variation (cause by chance of event)
- Some combinations of these patterns



(a) Level or Horizontal Pattern:
Data follows a horizontal pattern around the mean
Quantity
Time
(b) Trend Pattern:
Data is progressively increasing (shown) or decreasing
Quantity
Time
(c) Seasonal Pattern:
Data exhibits a regularly repeating pattern
Quantity
1 2 3 4 1 2 3 4 1 2 3 4
Time (quarters)
(d) Cycle:
Data increases or decreases over time
Quantity
1 2 3 4 1 2 3 4 1 2 3 4
Time (quarters)

Useful Forecasting Models

1. Qualitative and judgment method: based on estimates and opinions. E.g. Delphi method

- **Naive:** Assumes demand in next period is the same as demand in most recent period (e.g., If January sales were 68, then February sales will be 68). Sometimes cost effective and efficient . Can be good starting point

2. Time series (quantitative/Extrapolative model): based on data related to past demand can be used to predict future demand

3. Causal relationship (quantitative) or Explanatory model: - using linear regression techniques

Time Series Forecasting

Decomposition" of time-series

- Data are broken into three components

I. Simple Moving Average

II. Weighted Moving Averages

III. Exponential Smoothing

a. Simple Moving Average

- Assumes no trend, seasonal or cyclical components.
- Simple moving average combines demand data from several of the most recent periods; their average being the forecast for next period.
- As general rule: the longer the averaging period, the slower response to demand change.

$$A_t = \frac{D_t + D_{t-1} + \ ... + D_{t-N+1}}{N}$$

- Forecast Ft is average of n previous observations or actuals Dt :

$$F_{t+1} = \frac{1}{n}(D_t + D_{t-1} + \cdots + D_{t+1-n})$$

$$F_{t+1} = \frac{1}{n} \sum_{i=t+1-n}^{t} D_i$$

Note that the n past observations are equally weighted.

- Issues with moving average forecasts:
- All n past observations treated equally;
- Observations older than n are not included at all;
- Requires that n past observations be retained;

Compute three period moving average (number of periods is the decision of the forecaster)

Period	Actual Demand	Forecast
1	10	
2	18	
3	29	
4		19

(10+18+29)/3 = 19

Period 5 will be (18+29+actual for period 4)/3

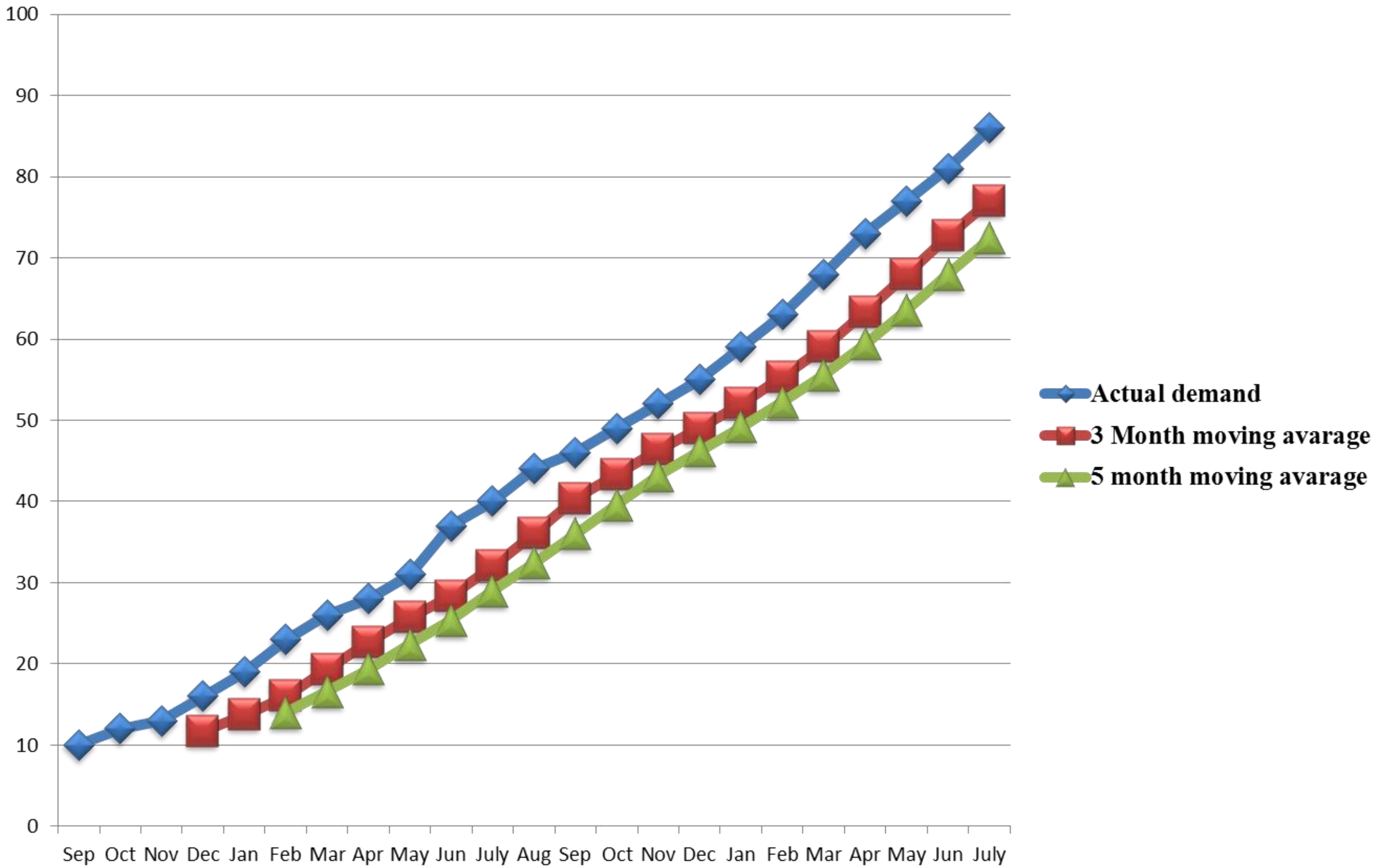
Simple Moving Average Example

Month	Actual Shed Sales	3-Month Moving Average
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\,^{2}/_{3}$
May	19	$(12 + 13 + 16)/3 = 13\,^{2}/_{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\,^{1}/_{3}$

Compute three and five month moving average and find August forecast (number of periods is the decision of the forecaster)

Month	Actual demand	3 Month moving average	5 month moving avarage
Sep	10		
Oct	12		
Nov	13		
Dec	16	12	
Jan	19	14	
Feb	23	16	14
Mar	26	19	17
Apr	28	23	19
May	31	26	22
Jun	37	28	25
July	40	32	29
Aug	44	36	32
Sep	46	40	36
Oct	49	43	40
Nov	52	46	43
Dec	55	49	46
Jan	59	52	49
Feb	63	55	52
Mar	68	59	56
Apr	73	63	59
May	77	68	64
Jun	81	73	68
July	86	77	72
		81	**77**

Graph of actual demand, three & five month moving average



b. Weighted Moving Average

Weighted moving average :-

- ✓ during this more recent values in a series are given more weight in computing the forecast.
- ✓ wants to use the moving average but does not want to have all n periods equally weighted. This makes responsive:

- $F_{t+1} = A_t = W_1 D_t + W_2 D_{t-1} + \cdots + W_N D_{t-N+1}$

Weighted moving average example

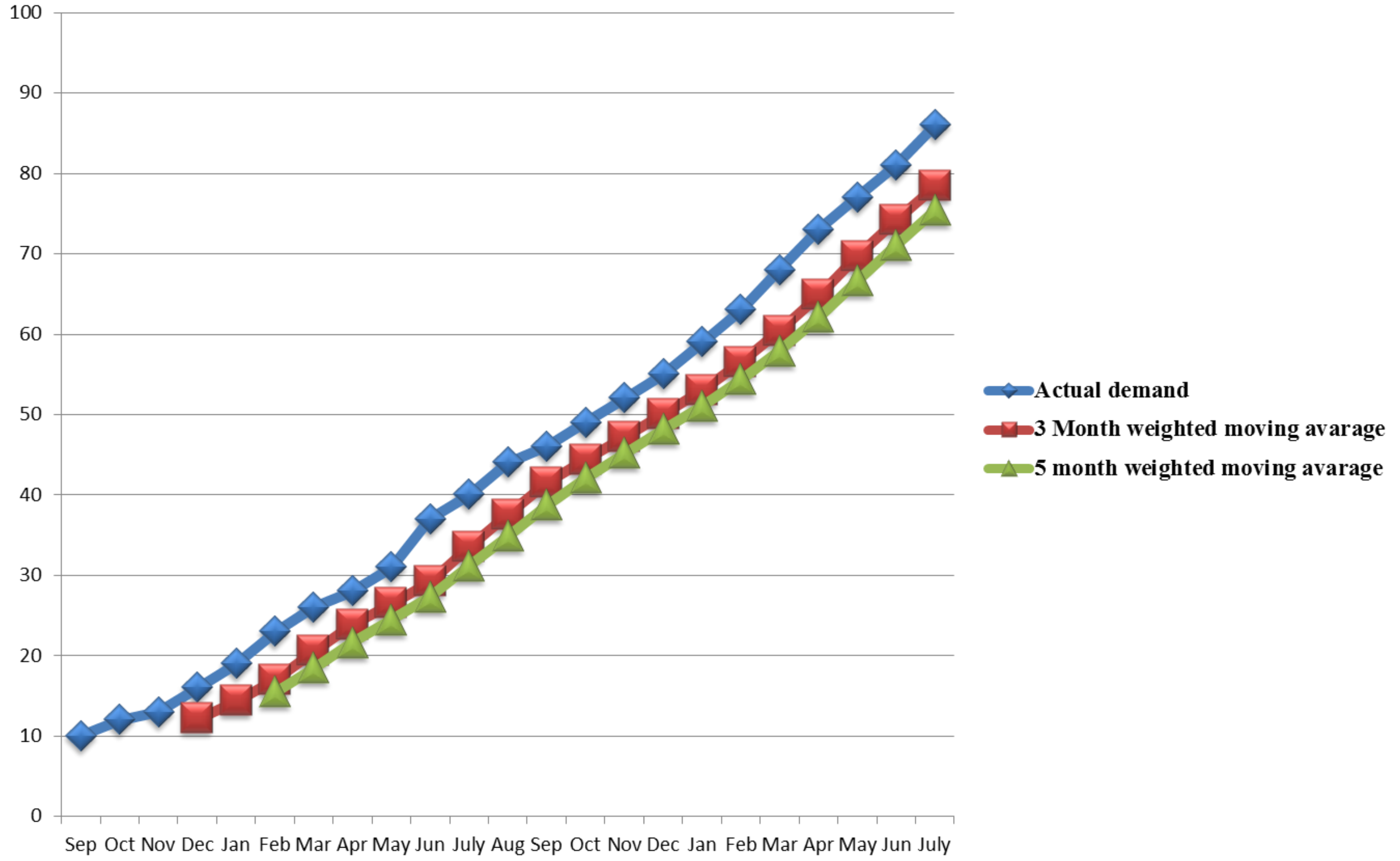
Weights Applied	Period
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Month	Actual Shed Sales	3-Month Weighted Moving Average
January	10	
February	12	
March	13	
April	16	[(3 x 13) + (2 x 12) + (10)]/6 = $12^{1}/_{6}$
May	19	[(3 x 16) + (2 x 13) + (12)]/6 = $14^{1}/_{3}$
June	23	[(3 x 19) + (2 x 16) + (13)]/6 = 17
July	26	[(3 x 23) + (2 x 19) + (16)]/6 = $20^{1}/_{2}$

Compute three and five month weighted moving average to forecast August demand (number of periods is the decision of the forecaster)

Month	Actual demand	3 Month weighted moving average	5 month weighted moving avarage
Sep	10		
Oct	12		
Nov	13		
Dec	16	12	
Jan	19	14	
Feb	23	17	15
Mar	26	21	18
Apr	28	24	22
May	31	27	24
Jun	37	29	27
July	40	34	31
Aug	44	38	35
Sep	46	42	39
Oct	49	44	42
Nov	52	47	45
Dec	55	50	48
Jan	59	53	51
Feb	63	57	54
Mar	68	60	58
Apr	73	65	62
May	77	70	67
Jun	81	74	71
July	86	78	75
		83	**80**

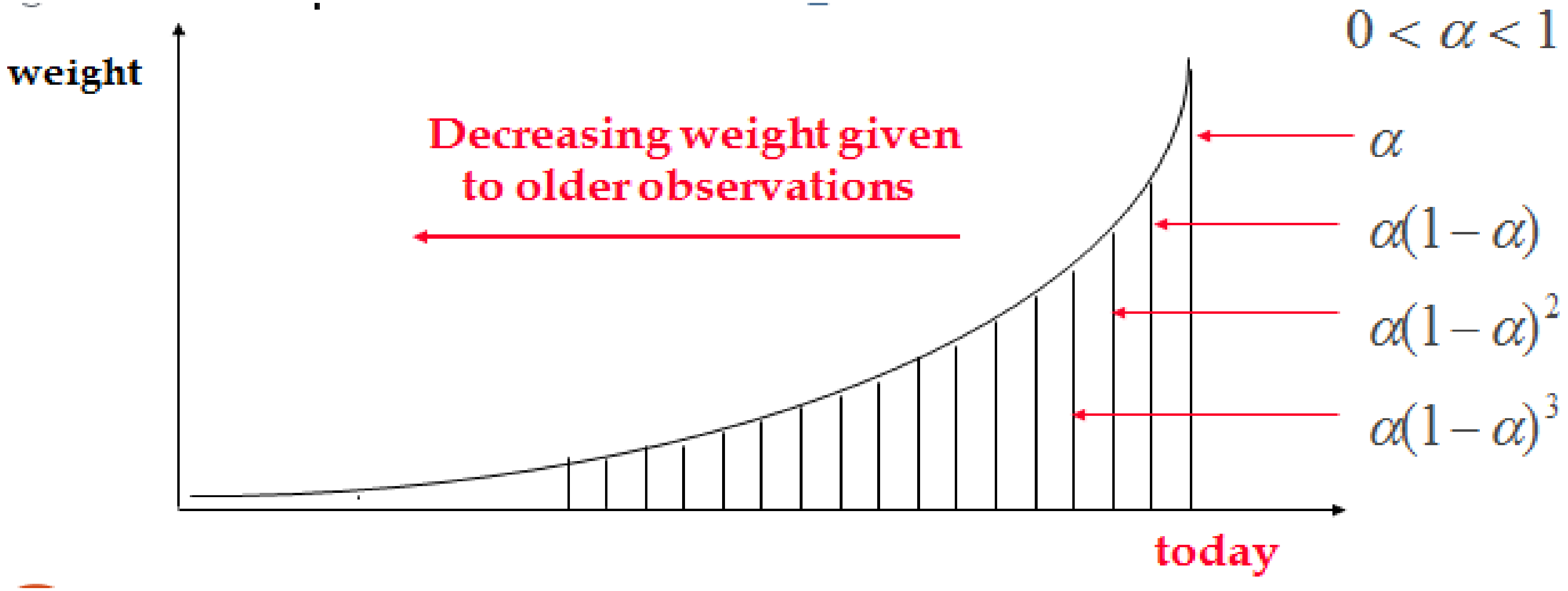
Graph of actual demand, three & five month weighted moving average



NB. The advantage of a WMA over a SMA is that the WMA is more reflective of the most recent occurrences.

c. Exponential Smoothing: Concept

- Include all past observations
- Form of weighted moving average
- Weight recent observations much more heavily than very old observations:



$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

- If F_{t-1} is to be very responsive to recent demand, choose the large value of α
- No trend, cyclical or seasonal components.
- the most recent occurrences are more indicative of the future than in the more distant past

Exponential Smoothing Example

Predicted demand (t-1)= 142 Ford Mustangs

Actual demand = (t-1)153

Smoothing constant α = .20

New forecast (t) = 142 + .2(153 – 142)

Cont.…

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant $\alpha = 0.20$

New forecast = 142 + 0.2(153 – 142)

= 142 + 2.2

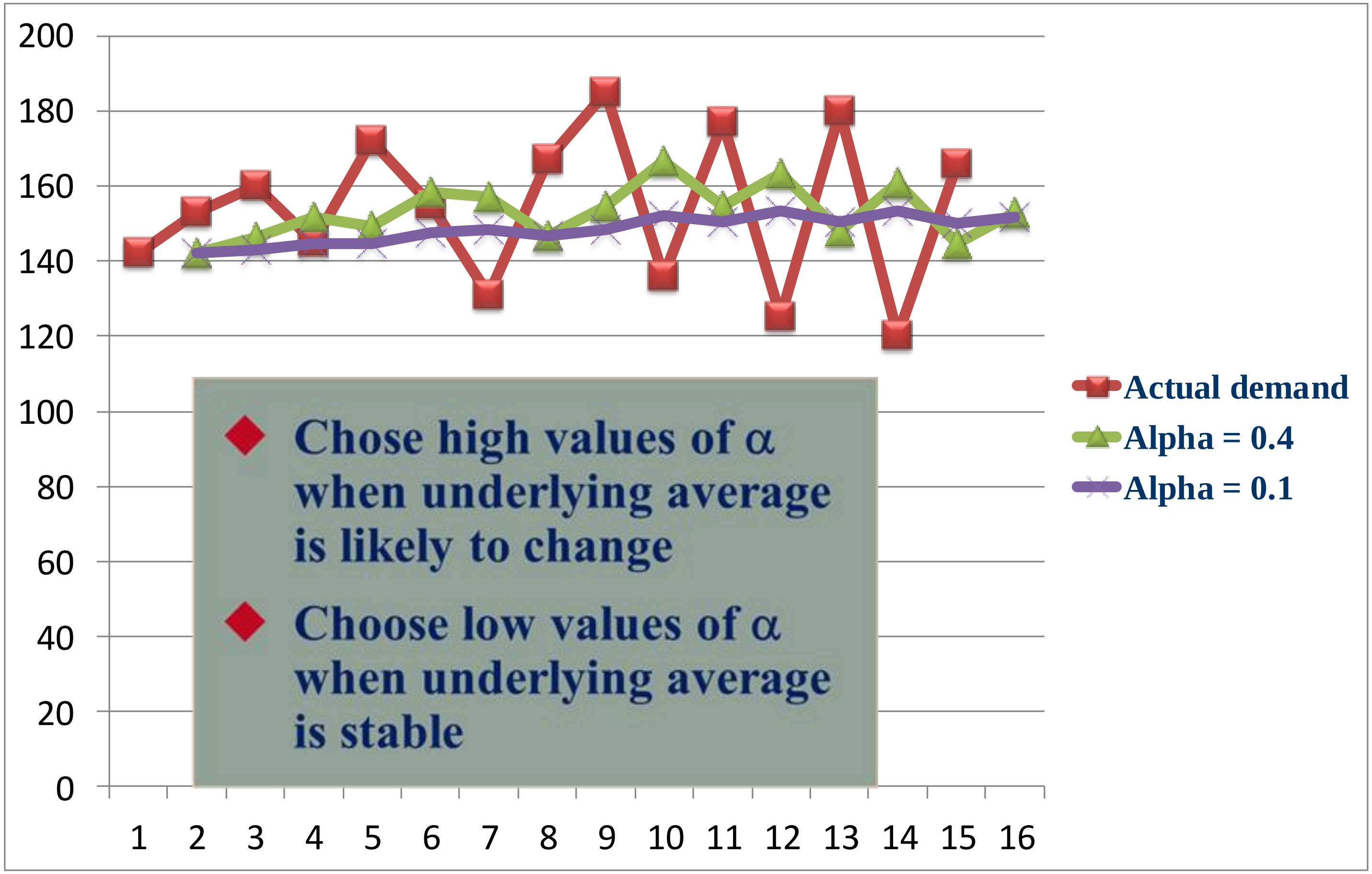
= 144.2 ≈ 144 cars

By using this method calculate the 16^{th} period forecasted demand using $\alpha = 0.1$ & $\alpha = 0.40$ for the following table.

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Period	Actual demand (A_{t-1})	(F_t), α= 0.4	(F_t), α = 0.1
1	142		
2	153	142	142
3	160	146	143
4	145	152	145
5	172	149	145
6	155	158	148
7	131	157	148
8	167	147	147
9	185	155	149
10	136	167	152
11	177	155	151
12	125	164	153
13	180	148	150
14	120	161	153
15	166	145	150
		153	152

Cont….



NB:- MA requires all N past data points to compute new forecast estimate while ES only requires last forecast and last observation of ‘demand’ to continue.

– ES is more accurate than MA

Choosing α

- The objective is to obtain the most accurate forecast no matter the technique.
- We generally do this by selecting the model that gives us the lowest forecast error
- Forecast error = Actual demand - Forecast value

$$= A_t - F_t$$

forecast error = A_t - F_t

Period	Actual demand (A_t)	(F_t) , α = 0.4	(F_t), α = 0.1	Error-1	Error-2
1	142				
2	153	142	142	11	11
3	160	146	143	14	17
4	145	152	145	-7	0
5	172	149	145	23	27
6	155	158	148	-3	7
7	131	157	148	-26	-17
8	167	147	147	20	20
9	185	155	149	30	36
10	136	167	152	-31	-16
11	177	155	151	22	26
12	125	164	153	-39	-28
13	180	148	150	32	30
14	120	161	153	-41	-33
15	166	145	150	21	16
		153	152	Sum = 28	Sum = 96

Hence the forecasted demand using high value of alpha will have smaller error value.

Forecasting Performance

How good is the forecast?

- **Mean Forecast Error (MFE or Bias)**:

  Measures average deviation of forecast from actuals.

- **Mean Absolute Deviation (MAD)**:

  Measures average absolute deviation of forecast from actuals.

- **Mean Absolute Percentage Error (MAPE)**: Measures absolute error as a percentage of the forecast.

- **Standard Squared Error (MSE):**

  Measures variance of forecast error

Mean Absolute Deviation (MAD)

$$MAD = \frac{1}{n}\sum_{t=1}^{n}\left|D_t - F_t\right|$$

✓Measures absolute error

✓Positive and negative errors thus do not cancel out (as with MFE).

✓Want MAD to be as small as possible.

✓No way to know if MAD error is large or small in relation to the actual data.

Formulas of forecast errors

Cumulative sum of Forecast Errors

$$CFE = \sum_{i=1}^{n} e_t$$

Mean Absolute Percentage Error

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{e_t}{D_t} \right| 100}{n}$$

Mean Square Error

$$MSE = \frac{\sum_{i=1}^{n} e_t^2}{n}$$

Tracking Signal

$$TS = \frac{\sum_{i=1}^{n} e_t}{MAD}$$

Mean Absolute Deviation

$$MAD = \frac{\sum_{i=1}^{n} |e_t|}{n}$$

Mean Error

$$ME = \frac{\sum_{i=1}^{n} e_t}{n}$$

Example for Accuracy of forecasts

The manager of a big hydroelectric power station is interested to choose between the two alternative forecasting techniques. Both techniques have been used to prepare forecasts for a six month period. Using MAD as a criterion, which technique has the better performance record?

Month	Demand	Technique 1	Technique 2
1	492	488	495
2	470	484	482
3	485	480	478
4	493	490	488
5	498	497	492
6	492	493	493

Solution

Check that each forecast has an average error of approximately zero. (See computations that follow.)

Month	Demand	Technique 1	e	| e |	Technique 2	e	| e |
1	492	488	4	4	495	-3	3
2	470	484	-14	14	482	-12	12
3	485	480	5	5	478	7	7
4	493	490	3	3	488	5	5
5	498	497	1	1	492	6	6
6	492	493	1	1	493	1	1
			-2	28		+2	34

-

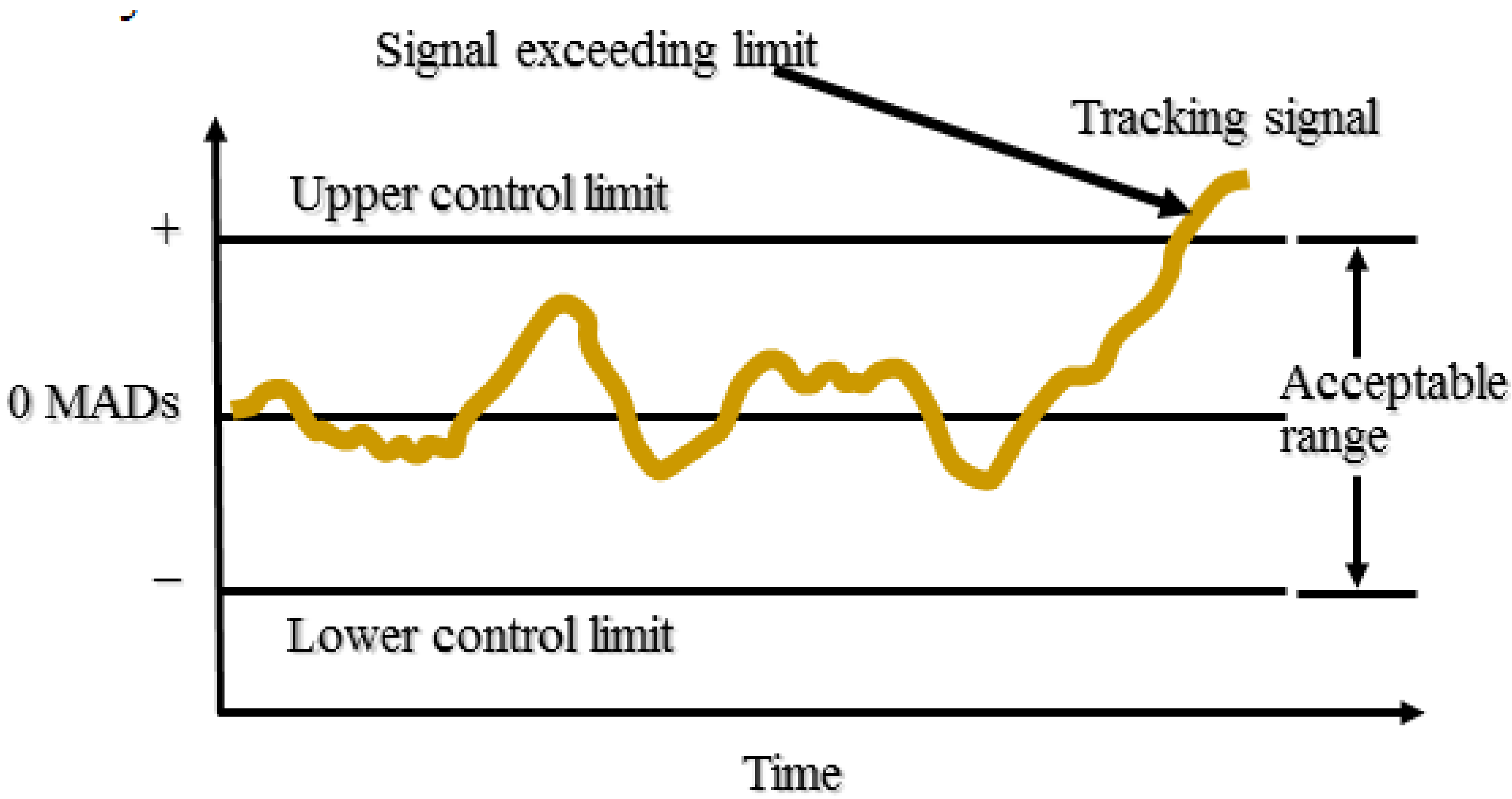
$$MAD_1 = \frac{\sum|e|}{n} = \frac{28}{6} = 4.67$$

$$MAD_2 = \frac{\sum|e|}{n} = \frac{34}{6} = 5.67$$

- ✓ Technique 1 is superior in this comparison because its MAD is smaller, although six observations would generally be too few on which to base a realistic comparison.

Tracking Signal

- Analogous to control charts in quality control, *viz*. if there is no bias, its values should fluctuate around zero.
- Is a relative measure, i.e. the numbers mean the same for any forecast.



Tracking Signal Example

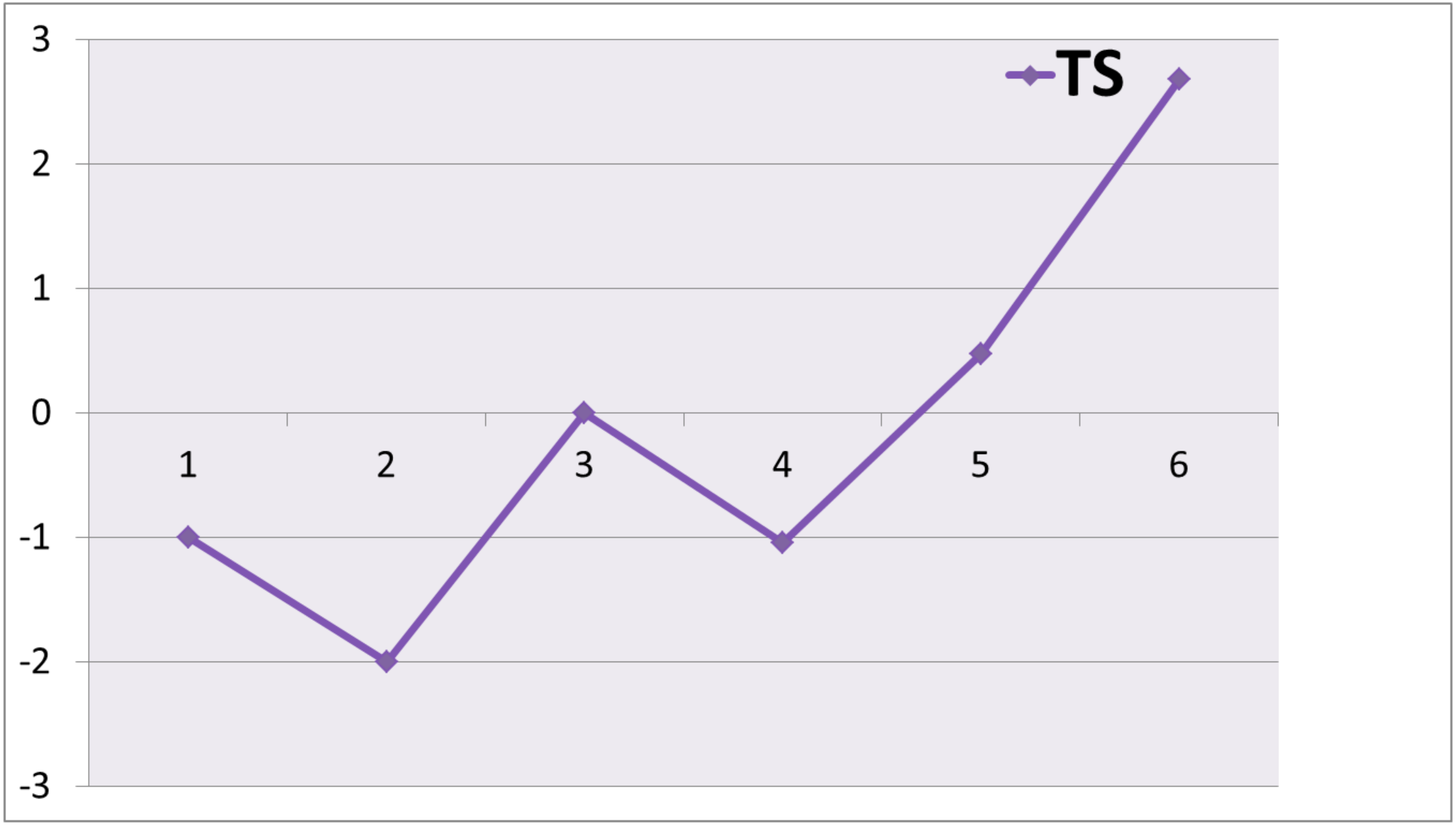
Period	Actual demand (Dt)	Forecasted demand (Ft)	Error	CSFE	AFE	CAFÉ	MAD	**TS**
1	90	100	-10	-10	10	10	10	**-1**
2	95	100	-5	-15	5	15	7.5	**-2**
3	115	100	15	0	15	0	11	**0**
4	100	110	-10	-10	10	10	10	**-1**
5	125	110	15	5	15	5	11	**0**
6	140	110	30	35	30	35	13	**3**

Tracking Signal

$$TS = \frac{\sum_{i=1}^{n} e_t}{MAD}$$

$$MAD = \frac{1}{n}\sum_{t=1}^{n} \left| D_t - F_t \right|$$

The variation of the tracking signal is between -2.0 and +3 as within acceptable limits.



3. Time Series vs. Causal Models

- Time series compares data being forecast over time, *i.e.* Time is the independent variable or x- axis or X-variable.
- Causal models compare data being forecast against some other data set which the forecaster may think is a cause of the forecasted data, *e.g.* population size causes newspaper sales.

Causal Forecasting Models

- The general regression model:

$$Y_t = a + b(x)$$

a – y-intercept and b - slope
t – time period (e.g. year)

- The Values of a and b

$$b = \frac{\sum XY - n\bar{X}\,\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

$$a = \bar{Y} - b\bar{X}$$

Least Squares Method

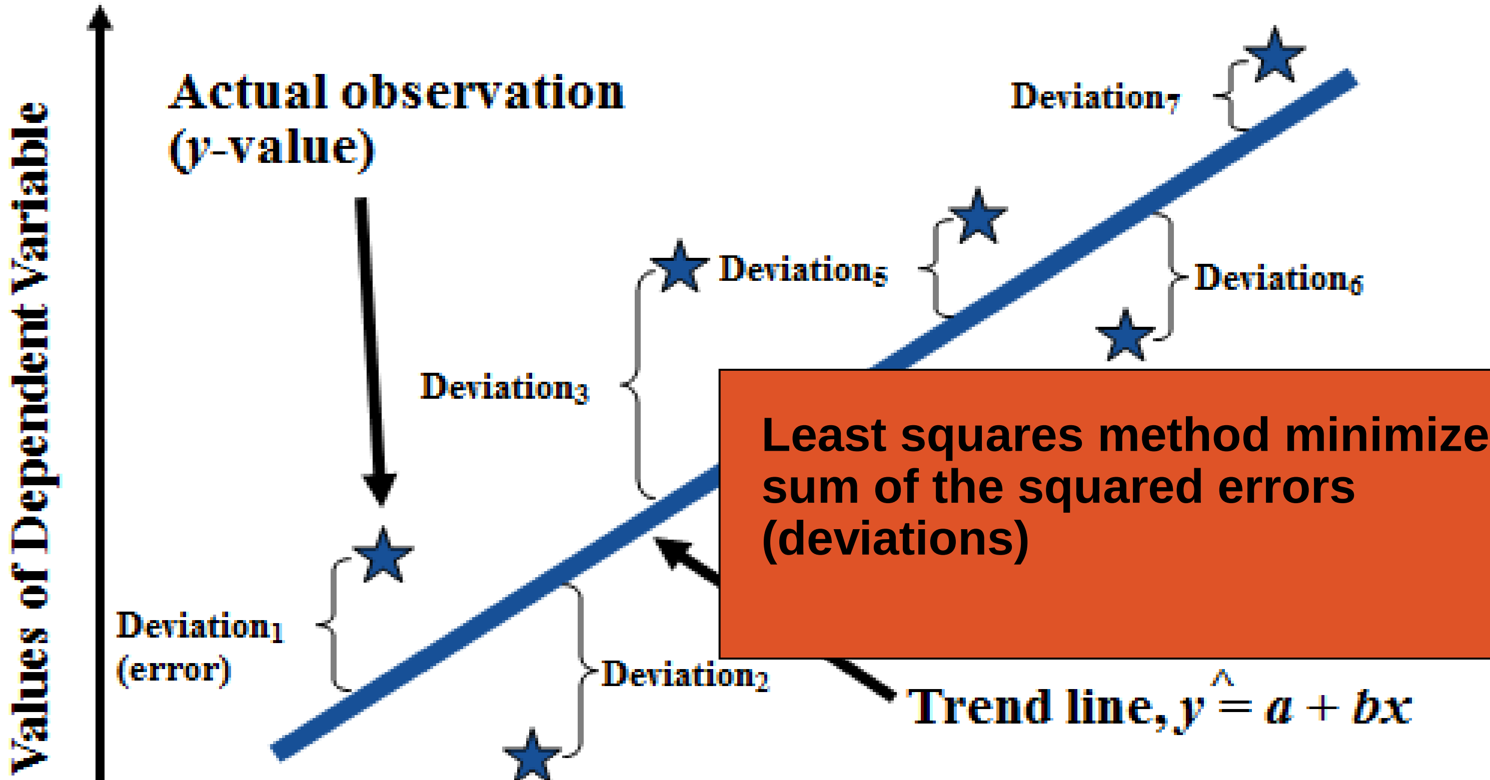


Figure 4.4

Least squares method minimizes the sum of the squared errors (deviations)

Least Squares Example 1

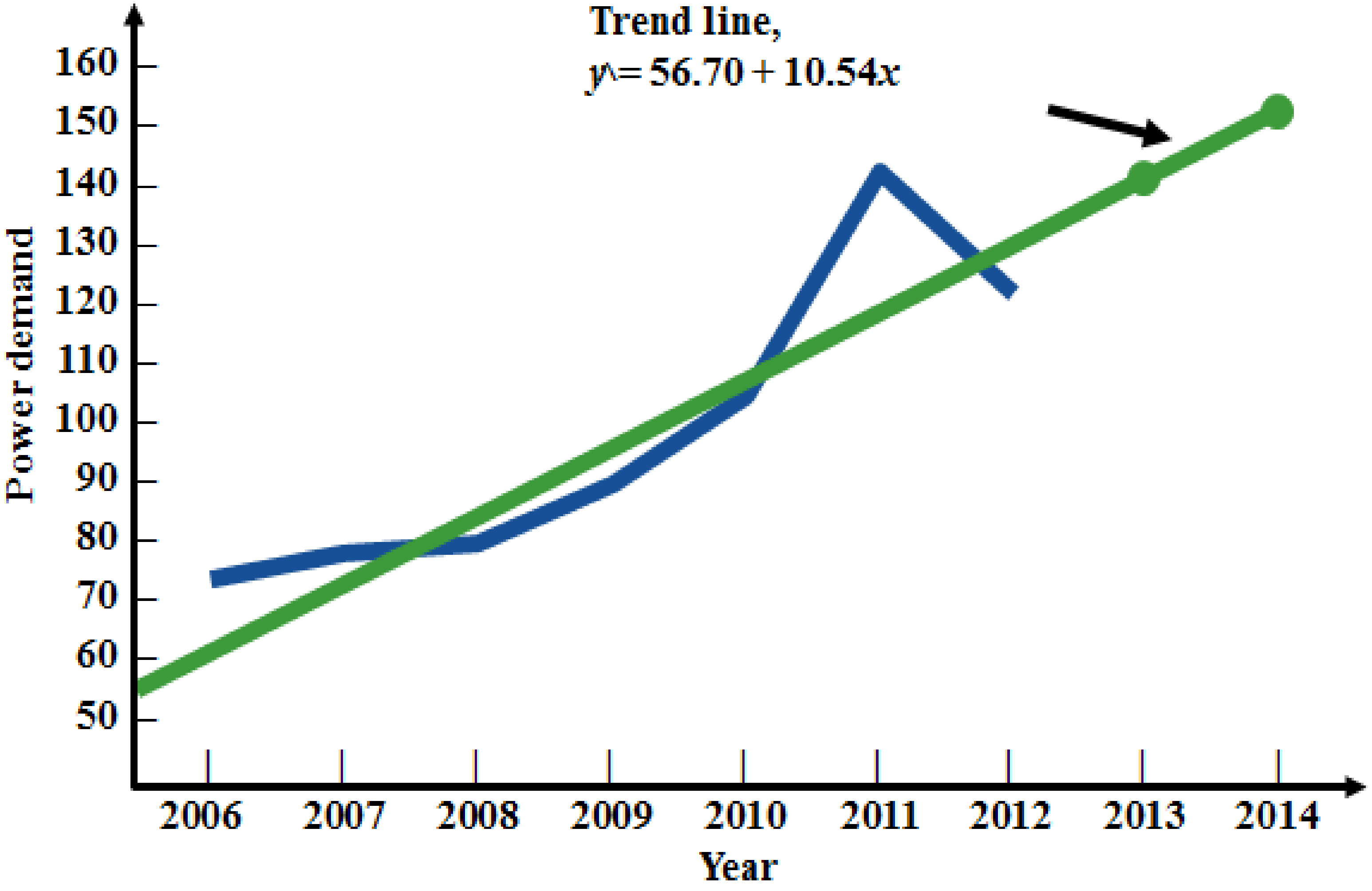
Year	Time Period (x)	Electrical Power Demand (megawatt)	x^2	xy
2006	1	74	1	74
2007	2	79	4	158
2008	3	80	9	240
2009	4	90	16	360
2010	5			25
2011	6			52
2012	7			54
	$\sum x = 28$			63
	$\bar{x} = 4$			

The trend line is

$$\hat{y} = 56.70 + 10.54x$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4^2)} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$


Trend line,
y^= 56.70 + 10.54x
Power demand
160
150
140
130
120
110
100
90
80
70
60
50
2006
2007
2008
2009
2010
2011
2012
2013
2014
Year

Least Squares Example 2

$$Y_t = a + b(x)$$

t (x)	D_t (y)	F_t
1	120	120
2	124	121
3	119	123
4	124	124
5	125	126
6	130	128
		132
Intercept (a)	118	
Slope (b)	2	

D_t = actual sales

F_t = forecasted sales

t = time period (e.g. year)

$$b = \frac{\sum XY - n\bar{X}\,\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

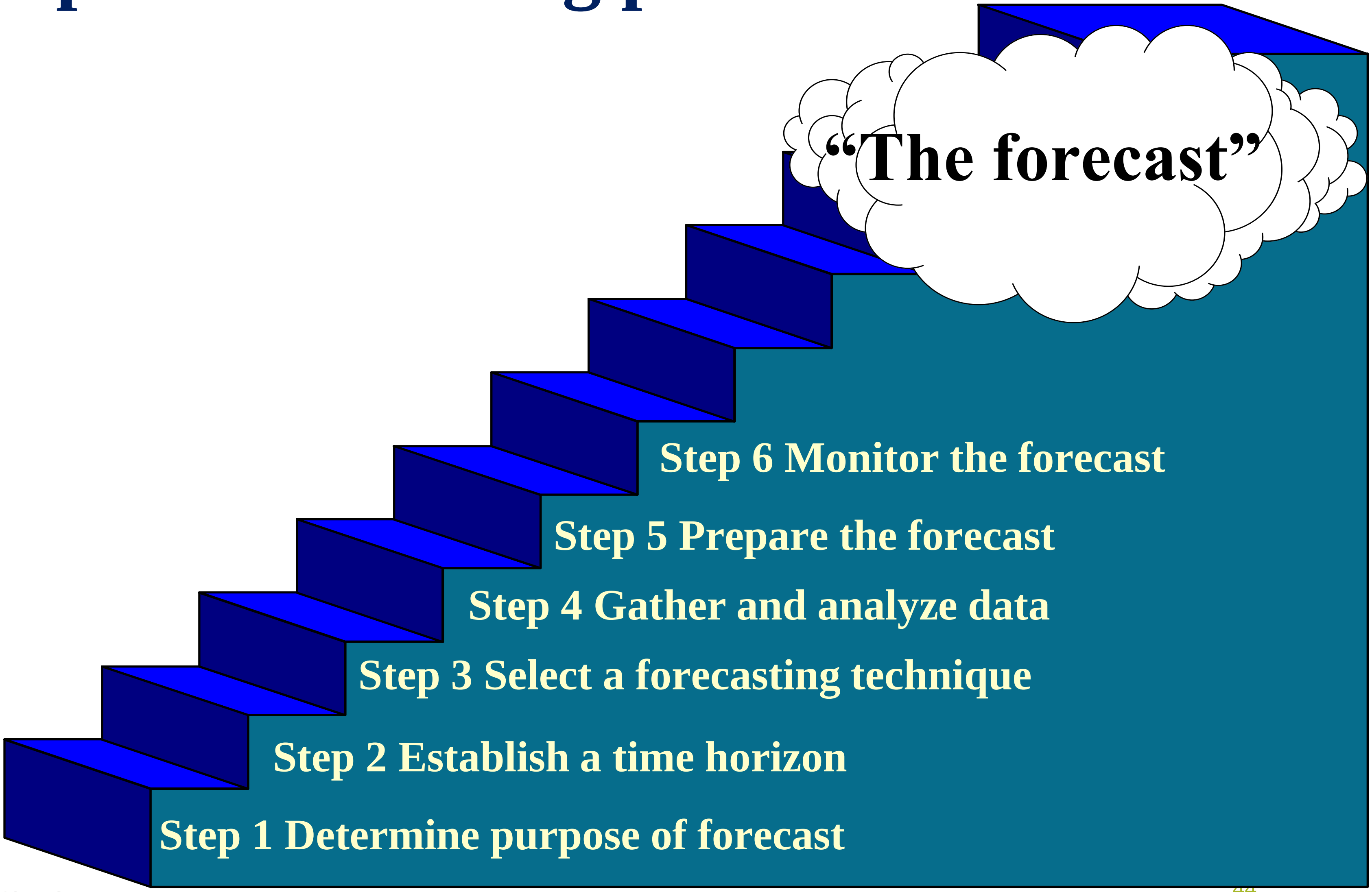
$$a = \bar{Y} - b\bar{X}$$

F_7 = 118 + 2(7) = 132 = sales forecast for next year

Selecting a forecasting methods

- Use and decision characteristics
  - Accuracy required, Time horizon
  - Pricing decision require highly accurate short ranged forecasts for large number of item
- Data availability and quality
- Data pattern affects the type of forecasting
  - If the time series is flat, A first order method can be used, where as if the data shows trend or seasonal pattern some advance method will be used
  - If the data is unstable over time, a qualitative method may be selected
- Don’t force the data to fit the model!

NB:- Don't forget remembering these Steps in forecasting process



Thank you!!!