Chapter Three

Characteristics of Feedback control system.

3.1 Introduction.

What is a purpose of feedback in control systems?

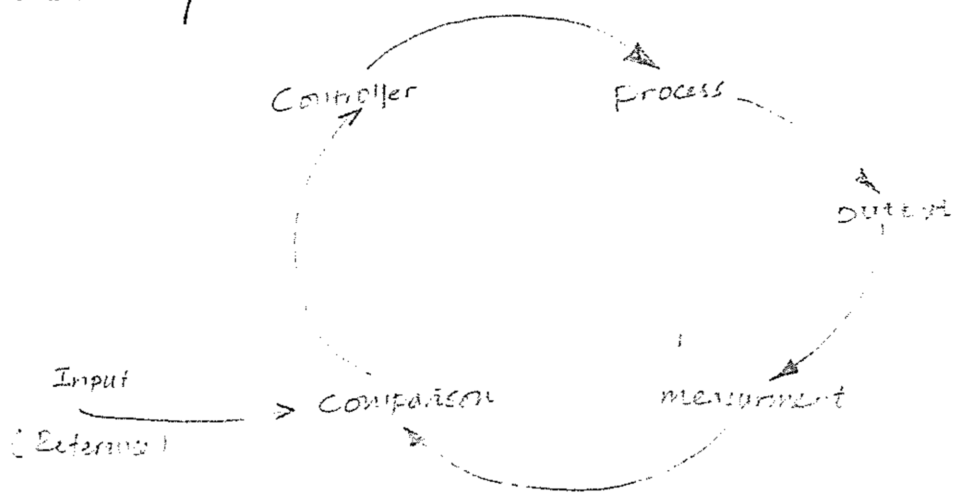
In simple terms

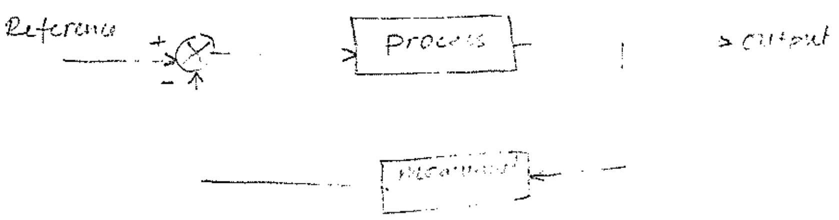
The use of feedback is for the purpose of reducing the error between the reference input and the system output.

What are its effects of Feedback in control systems?

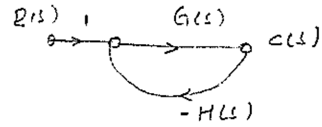
- The reduction of system error is merely one of the many important effects that the feedback may have upon a system.
- Feedback also has effects on a system Performance Characteristics such as stability, Bandwidth, Overall gain, Disturbance and sensitivity.

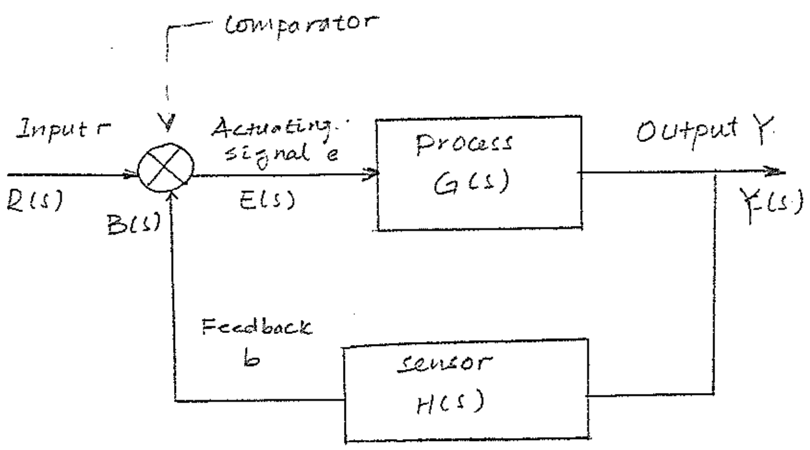
Feedback exists whenever there is a closed sequence of cause-and-effect relationship.





3-2 Feedback Control System.





A closed-loop system uses a measurment of the output signal and a comparison with the desired output to generate an error signal that is applied to the actuator.

The output signal Y is measured by a sensor H(s) which produces a feedback signal b. The comparator compares the ~~feedback~~ Error produces (r-b) signal e, which is a measure of discrepancy between r and b. The actuating signal is applied to the process G(s) so as to influence the output Y in a manner which tends to reduce the error.

From the block diagram

1. $Y(s) = E(s)\,G(s)$
2. $B(s) = H(s)\cdot Y(s)$
3. $E(s) = R(s) - B(s)$
   $= R(s) - H(s)Y(s)$
4. substituting eq 3 in 1
   $$Y(s) = E(s)\,G(s)$$
   $$= \left[R(s) - H(s)Y(s)\right]G(s)$$

The output becomes

$$Y(s) = \frac{G(s)}{1+H(s)G(s)}\,R(s) \Rightarrow$$

Transfer function T(s)

$$T(s) = \frac{Y}{R} = \frac{G(s)}{1+G(s)H(s)}$$

5. The actuating Error signal from eq 1.

$$E(s) = \frac{Y(s)}{G(s)} = \frac{\dfrac{G(s)}{1+H(s)G(s)}\,R(s)}{G(s)}$$

$$E(s) = \frac{R(s)}{1+H(s)G(s)}$$

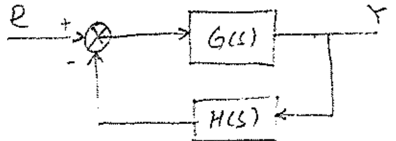
E(s) provides a measure of the error signal.

It is clear, to reduce the error, the magnitude of $[1+H(s)G(s)]$ must be greater than 1 over the range of 'S' under consideration.

3.3 Effect of Feedback On Over all gain

Consider the overall transfer function of negative feedback system

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)}$$



As seen from the above transfer function eqn feedback affects the gain G of a nonfeedback system ($T = \frac{Y}{R} = G$) by a factor of $1 + HG$

In practical control systems, G and H are functions of frequency, so the magnitude of $1+GH$ may be greater than 1 in one frequency range but less than 1 in another. Therefore;

"Feedback could increase the system gain in one frequency range but decreases it in another."

3.4 Effect of Feedback on Stability.

Stability is a notion that describes whether the system will be able to follow the input command, or be useful in general.

"A system is said to be unstable if its output is out of control."

To investigate the stability of a system

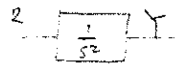
$$T = \frac{G}{1+GH} \quad , \quad Y = \frac{G}{1+GH} R$$

if $GH = -1$, the output of a system is infinite for any finite input, the system is said to be unstable.

We may state

feedback can cause a system that is orginally stable to become unstable.

One of the advantage of incorporating feedback is that it can stablize an unstable system by proper selection of feedback gain.



$G = \frac{1}{s^2}$ unstable
- poles at the origin

$\Longrightarrow$

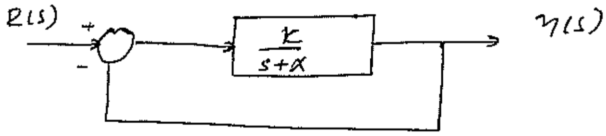
$$\frac{Y}{R} = T = \frac{1/s^2}{1 + \frac{1}{s^2}} = \frac{1}{s^2+1}$$

pair complex poles at $j\omega = \pm j$
→ results in a sinusoidal impulse response

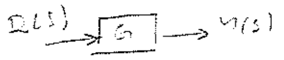
inherently unstable → marginal stable.

Control over system dynamics by use of Feedback

Consider an elementary system



a) The open loop transfer function



$$G(s) = \frac{K}{s+\alpha}$$ real pole $s = -\alpha$

The response of a unit impulse $R(s) = 1$

$$Y(s) = G(s)\,R(s) = \frac{K}{s+\alpha} \Rightarrow \mathcal{L}^{-1}[Y(s)] = y(t) = K e^{-\alpha t}$$

b) For a feedback system $T = \frac{G}{1+G} = \frac{\frac{K}{s+\alpha}}{1+\frac{K}{s+\alpha}} = \frac{K}{s+\alpha+K}$

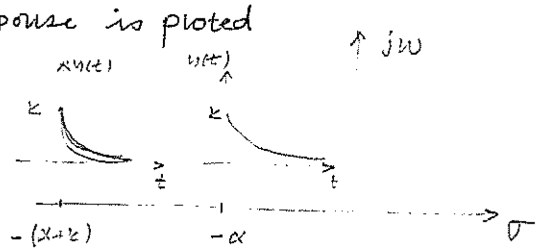
$$Y(s) = T(s)\,R(s) = \frac{K}{s+(\alpha+K)}$$

The laplace transform

$$y(t) = K e^{-\alpha t}$$ – open loop (Time constant $\tau = \frac{1}{\alpha}$

$$= K e^{-(\alpha+K)t}$$ · feedback system. (Time constant $\tau = \frac{1}{\alpha+K}$

These response is ploted



For +ve value of K, the effect of feedback is to shift the pole to $-(\alpha+K)$ so the time constant reduces to $\tau = \frac{1}{K+\alpha}$. This implies as K increases the system dynamics ~~increases~~ continously become faster.

↳ Feedback controls the dynamics of the system by adjusting the location of its poles.

However it is important to note here that a feedback introduces the possibility of unstability i.e a closed-loop system may be unstable even though the open-loop is stable.

3.5 Sensitivity of Control systems to Parameter Variation. 3

A process, represented by the transfer function $G(s)$, whatever its nature, is subjecte to envicroment, aging, ignoranu of the exact values of the process parameters, and other natural factors that affects a control process.

- In open-loop system, all these errors and changes result on a changing and inaccurate output.

But
- A closed-loop systems senses the change in the output due to the process change and attempts to correct the output.

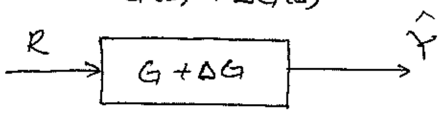
A primary advantage of a feedback control system is its ability to reduce the system senseitivity to variation in the parameters of the forward path.

A) Parameter Variation of open loop system.

i) process $G(s)$

$$Y = RG$$

ii) New process with parameter Variation of ΔG: $G(s) + \Delta G(s)$



$$\hat{Y} = R[G + \Delta G]$$
$$= RG + R\Delta G$$
$$= Y + \Delta Y$$

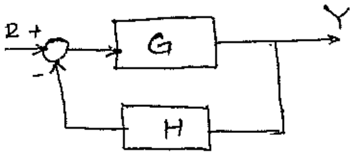
where $Y = RG$ - origional output

$\Delta Y = \Delta G R$ - change in the output b/s of variation of the transforms

B) Parameter Variation of closed loop system.

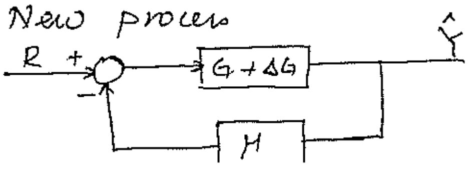
A good control system should be very insensetive to parameter variation but sensentive to the input command.

process i)



$$Y(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

ii) New process



$$\hat{Y}(s) = \frac{G + \Delta G}{1 + [(G + \Delta G)H]} R(s)$$
$$= Y(s) + \Delta Y(s)$$

The output of the closed-loop parameter variation becomes

$$\hat{Y}(s) = Y(s) + \Delta Y(s) = \frac{G(s) + \Delta G(s)}{1 + (G+\Delta G)H} R(s)$$

Substituting the value of $Y(s) = \frac{G}{1+GH} R(s)$

$$\frac{G}{1+GH} R + \Delta Y = \frac{G+\Delta G}{1+(G+\Delta G)H} R(s)$$

The change in output becomes

$$\Delta Y = \hat{Y} - Y$$

$$= \frac{G+\Delta G}{1+(G+\Delta G)H} R - \frac{G}{1+GH} R$$

$$= \frac{\Delta G}{[1+GH+\Delta GH][1+GH]} R(s)$$

When $GH(s) \gg \Delta GH(s)$, the change in output becomes

$$\Delta Y(s) = \frac{\Delta G(s)}{[1+GH(s)]^2} R(s)$$

The change in the output of the closed-loop system is reduced by the factor $[1+GH(s)]^2$, which is usually much greater than one over the range of complex frequencies of interest.

Definition.

System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function $G(s)$ (or parameter) for a small incremental change

system transfer function $T = \frac{Y}{R}$

$$\text{system sensitivity} = \frac{\text{Percentage change in } T}{\text{Percentage change in } G}$$

$$= \frac{\frac{\Delta T(s)}{T(s)}}{\frac{\Delta G(s)}{G(s)}} = \frac{\partial T / T}{\partial G / G}$$

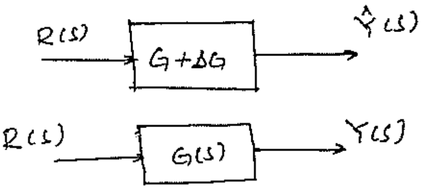
$$S_G^T = \frac{G}{T}\frac{\partial T}{\partial G}$$

Example.

1) Determine the sensetivity of open-loop system in the effect of parameter variation

$e^{(\alpha+j\omega)t}$

$\frac{1}{s-(\alpha+j\omega)}$



$T(s) = \frac{Y(s)}{R(s)} = G(s)$

$$\hat{Y}(s) = Y(s) + \Delta Y = [G + \Delta G] R(s)$$

$$= GR + \Delta G R$$

$$\Delta Y = \Delta G R$$

$$\Delta T = \frac{\Delta Y}{R} = \Delta G$$

$$= \frac{\Delta G R}{R} = \Delta G$$

The sensetivity of open-loop system becomes

$$S_G^T = \frac{G}{T} \frac{\Delta T}{\Delta G}$$

$$= \frac{G}{G} \cdot \frac{\Delta G}{\Delta G}$$

$$= 1$$

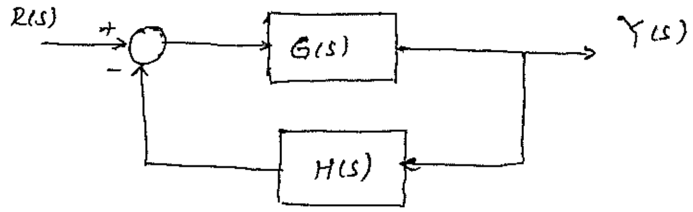
$T = G$ open loop

The sensetivity of open-loop system becomes 1

2) Determine the sensetivity of feedback control system.

a) The sensetivity of the system changes with gain G(s)

b) The sensetivity of the system changes with feedback gain H(s)



a) soln

1. The transfer function of a closed-loop Feedback system is given by

$$T(s) = \frac{Y}{R} = \frac{G}{1+GH}$$

2. The sensetivity of the system change with gain G(s)

$$S_G^T = \frac{G}{T} \frac{\partial T}{\partial G}$$

$$= \left(\frac{G}{\frac{G}{1+GH}} \right) \frac{\partial}{\partial G} \left(\frac{G}{1+GH} \right)$$

$$= \frac{G(1+GH)}{G}\left[\frac{1(1+GH) - G(0+H)}{(1+GH)^2}\right]$$

$$= 1+GH\left[\frac{1+GH-GH}{(1+GH)^2}\right]$$

$$S_G^T = \frac{1}{1+GH}$$

6 The price for improvement of in senseitivity by use of feedback is paid in terms of loss of system gain. The open loop system has a gain $G(s)$, while the gain of the closed loop system is $\frac{G}{1+GH}$. Hence by use of feedback, the system gain is reduced by the same factor as by which the senseitivity of the system to parameter variations is reduced.

If GH is a +ve constant, the magnitude of the senseitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable.

b) Senseitivity of the system changes with Feedback gain H(s)

$$S_H^T = \frac{\text{Percentage Change in } T}{\text{percentage change in } H} = \frac{\Delta T/T}{\Delta H/H}$$

$$= \frac{H}{T}\frac{\partial T}{\partial H}$$

$$= \left(\frac{H}{\frac{G}{1+GH}}\right) \cdot \frac{\partial}{\partial H}\left[\frac{G}{1+GH}\right]$$

$$= \frac{H(1+GH)}{G} \cdot \left(\frac{0(1+GH) - G(0+G)}{(1+GH)^2}\right)$$

$$= \frac{H(1+GH)(-G^2)}{G(1+GH)^2}$$

$$S_H^T = \frac{-GH}{1+GH}$$

When GH is large, the senseitivity approaches unity, and the changes in H(s) directly affects the output response.
Therefore it is important to use feedback components that will not vary with envoromental changes or that can be maintained constant.

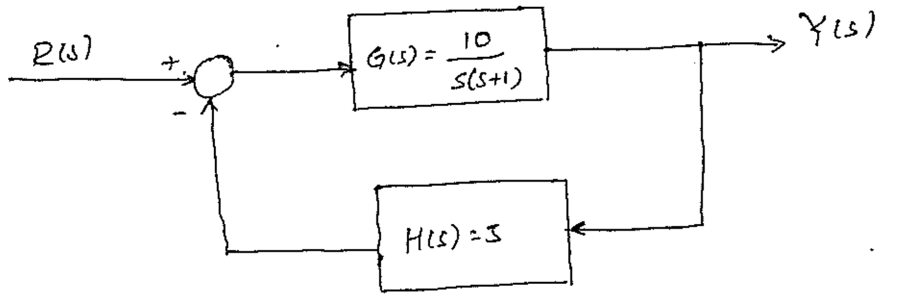
Often we seek to determine S_α^T, where α is a parameter within the transfer function of a block G. Using chain rule, we find

$$S_\alpha^T = S_G^T S_\alpha^G \quad \text{where } S_G^T = \frac{G}{T}\frac{\partial T}{\partial G}$$

$$S_\alpha^G = \frac{\alpha}{G}\frac{\partial G}{\partial \alpha}$$

Example.

The block diagram of a position control system is shown below. Determine the senestivity of the closed loop system transfer function with respect to G and H for $\omega = 1\ rad/sec$



sol/

1- The Closed loop transfer function

$$T = \frac{G}{1+GH} = \frac{10}{s^2+s+50}$$

a) The senestivity of T with respect to G is given by

$$S_G^T = \frac{G}{T}\frac{\partial T}{\partial G} = \frac{1}{1+GH} = \frac{1}{1+\frac{10}{s(s+1)} * s}$$

$$= \frac{s^2+s}{s^2+s+50}$$

S_G^T at $\omega = 1\ rad/sec$

$\Rightarrow s = \cancel{\omega + \sigma j}\ \sigma + j\omega$

$= j1$

$$S_G^T \Big|_{s=j1} = \frac{(j1)^2 + j1}{(j1)^2 + j1 + 50} = \frac{-1+j1}{-1+j1+50}$$

$$= \frac{-1+j1}{49+j1}$$

The magnitude of the senestivity

$$\left| S_G^T \right| = \frac{\sqrt{1^2+1^2}}{\sqrt{49^2+1^2}} = 0.029$$

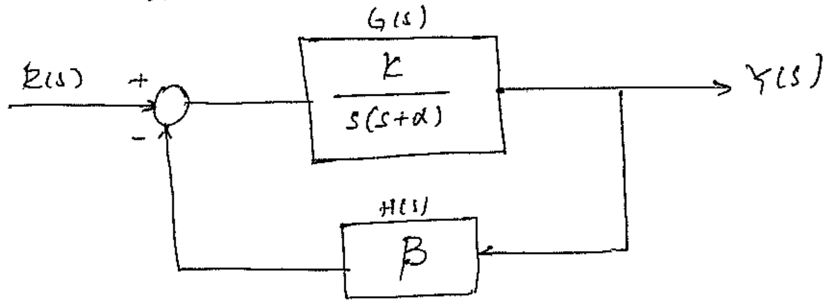
b) senestivity of transfer function with respect to H is given by

$$S_H^T = \frac{H}{T}\frac{\partial T}{\partial H} = \frac{-GH}{1+GH} = \frac{-50}{s^2+s+50}$$

$$S_H^T \Big|_{s=j1} = \frac{-50}{49+j1} \Rightarrow \left| S_H^T \right|_{s=j1} = \frac{50}{\sqrt{49^2+1^2}} = 1.02.$$

Example.

consider the simple closed-loop system shown below, which represents a position control system using a DC servo motor in the armature control mode. The nominal value of the gain constant is 10, that of $\alpha = 2$ and the feedback parameter β is equal to 1.
Determine the sensitivity of the closed loop transfer function with respect to α and the change in Transfer function for a 5 % change in the parameter α.



sol/n

1) Closed loop transfer function

$$T(s) = \frac{G(s)}{1+GH(s)} = \frac{K}{s^2+\alpha s+\beta K}$$

$$S_\alpha^T = \frac{\alpha}{T}\frac{\partial T}{\partial \alpha} = \frac{\alpha(s^2+\alpha s+\beta K)}{K} \cdot \frac{\partial}{\partial \alpha}\left[\frac{K}{s^2+\alpha s+\beta K}\right]$$

$$= \frac{\partial T}{\partial G}\cdot\frac{\alpha}{G}\frac{\partial G}{\partial \alpha} = \frac{\alpha(s^2+\alpha s+\beta K)}{K} \cdot \frac{(-Ks)}{(s^2+\alpha s+\beta K)^2}$$

$$= \frac{\alpha}{T}\frac{\partial T}{\partial \alpha} = \frac{-\alpha s}{s^2+\alpha s+\beta K}$$

Given $K = 10$
$\alpha = 2$
$\beta = 1$

$$S_\alpha^T = \frac{-2s}{s^2+2s+10}$$

Now consider the effect a 5% change in parameter α,

we know $\frac{\Delta\alpha}{\alpha} = 5\%$

$$S_\alpha^T = \frac{\Delta T/T}{\Delta\alpha/\alpha} \Rightarrow \frac{\Delta T}{T} = S_\alpha^T * \frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta T}{T} = \frac{-2s}{s^2+2s+10} * 0.05$$

$$\Delta T = \frac{-0.1s}{s^2+2s+10}\, T(s) = \frac{-0.1s}{s^2+2s+10} * \frac{10}{s^2+2s+10}$$

$$= \frac{-s}{(s^2+2s+10)^2}$$

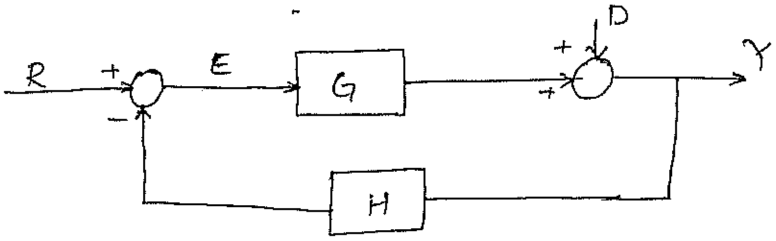
3.6 Effect of feedback on External Disturbance.

All physical systems are subjected to some type of extraneous signals, or noise during operation.

Eg. • thermal noise in electronic ckt
- brush or commutator noise in Electric motors
- wind gust on an antenna

In the design of a control system, consideration should be given so that the system is insensitive to noise and disturbance and sensitive to input command.

Consider a feedback system with input $R(s)$ and disturbance $D(s)$



open loop

We may write the eqn

1) output eqn $\quad Y = GE + D$

2) Error signal $\quad E = \cancel{Y+}\, R - YH$

3) From the above two eqns

$$Y = G(R - YH) + D$$

$$Y + GYH = GR + D$$

$$Y[1 + GH] = GR + D$$

$$Y = \frac{G}{1+GH} R + \frac{1}{1+GH} D$$

Insensitive to Noise $\frac{1}{1+GH}$

Sensitive to input command $\frac{G}{1+GH}$

Feedback can reduce the effect of noise and disturbance on a system performance by the factor $1+GH$.