Debre Markos University

Institute of Technology

Construction Technology & Management Academic program

Development and Construction Economics

(CoTM 5271)

April, 2020

CHAPTER 5

SENSITIVITY ANALYSIS

Introduction

- In all our preceding discussions we had arrived at a particular decision (in the form of acceptance or rejection of a proposal, and the selection of one alternative among different possible alternatives) in a given situation assuming that the estimates or the expected value for different variables such as initial cost, receipts, disbursements, interest rate, life of the assets, salvage value etc. were accurate and constant.

Unfortunately in real life situation it is not.

- Barring (except) few variable, such as initial cost of the asset, rest all the variables are all our estimates or forecasts which may prove to be wrong on most of the occasions.
- The life of the asset could be longer or shorter than our estimate;
- The interest rate could be higher or lower than the assumed value and so on.
- The salvage value may be more or less than the assumed value.

- We may like to know what will happen to the net present worth associated with a particular investment alternative when some variables like incomings (receipts) value or outgoings (Expense) value vary from its expected value.

Sensitivity analysis

It is aimed to study the impact of change in the value of variable(s) on the economic decision in a particular situation. In a sense it aims to answer “what if”.

- For example what will happen if the annual disbursement or cost value increases by 10% or 20% from the current value?

✓ Will it turn the positive present worth into negative?

✓ Will it change the earlier decision?

✓ The changes (increase or decrease in the assumed values) in the variable values may or may not lead to reversal of our earlier decision.

- If a slight change in one variable makes the reversal of decision from let's say acceptance of one alternative to the rejection we say that the variable is highly sensitive.
- Whereas, even if a large change in one variable does not change the decision we say that the variable is not sensitive or insensitive.
- The sensitivity analysis is also aimed to identifying the sensitivity of a particular variable.

- Once the identification has been made of variables in categories such as highly sensitive, less sensitive or insensitive the management can focus their attention to the highly sensitive variables.
- That is for such variables they can put more energy and effort in preparing their estimate.
- In some situations, external help in the form of engagement of consultants can also be thought of.

Different forms of sensitivity analysis

- In its simplest form, sensitivity analysis for a single alternative can be performed and the impact (on decision) of change in single variable can be studied.
- Then there could be simultaneous changes in two variables corresponding to a single alternative can be studied.
- Finally for a single alternative we can study the changes in more than two variables at a time.

- Sensitivity analysis can be performed for more than one alternative.
- Here also we can see the impact of variation of one variable on the decision.
- There can be cases in which we would like to change two variables at a time or more than two variables at a time and *see their impact on the decision* arrived earlier by assuming all the variables as fixed or constant.

- Sensitivity analysis can be performed with any method of evaluation of alternatives for example
  - ✓ Pw, Aw or Internal rate of return analysis.
- Also the analysis can be performed at different stages of project either
  - ✓ pre tax or post tax cash flows.
- However, it is preferable to perform sensitivity analysis with post tax cash flow.
  - ✓ It is customary to show the results of sensitivity analysis in the form of sensitivity graphs.

Examples

- Consider the given alternative to acquire a new piece of equipment. The acquisition cost, incomings, outgoings, salvage value, interest rate and service life associated with the alternative are provided in this table.

Description	Alternative 1
Acquisition cost	Birr. 500,000
Annual Incomings	Birr. 100,000
Annual Outgoings	Birr. 5,000
Salvage value	Birr. 50,000
Interest rate	12%
Service life	10 years

- Net present worth

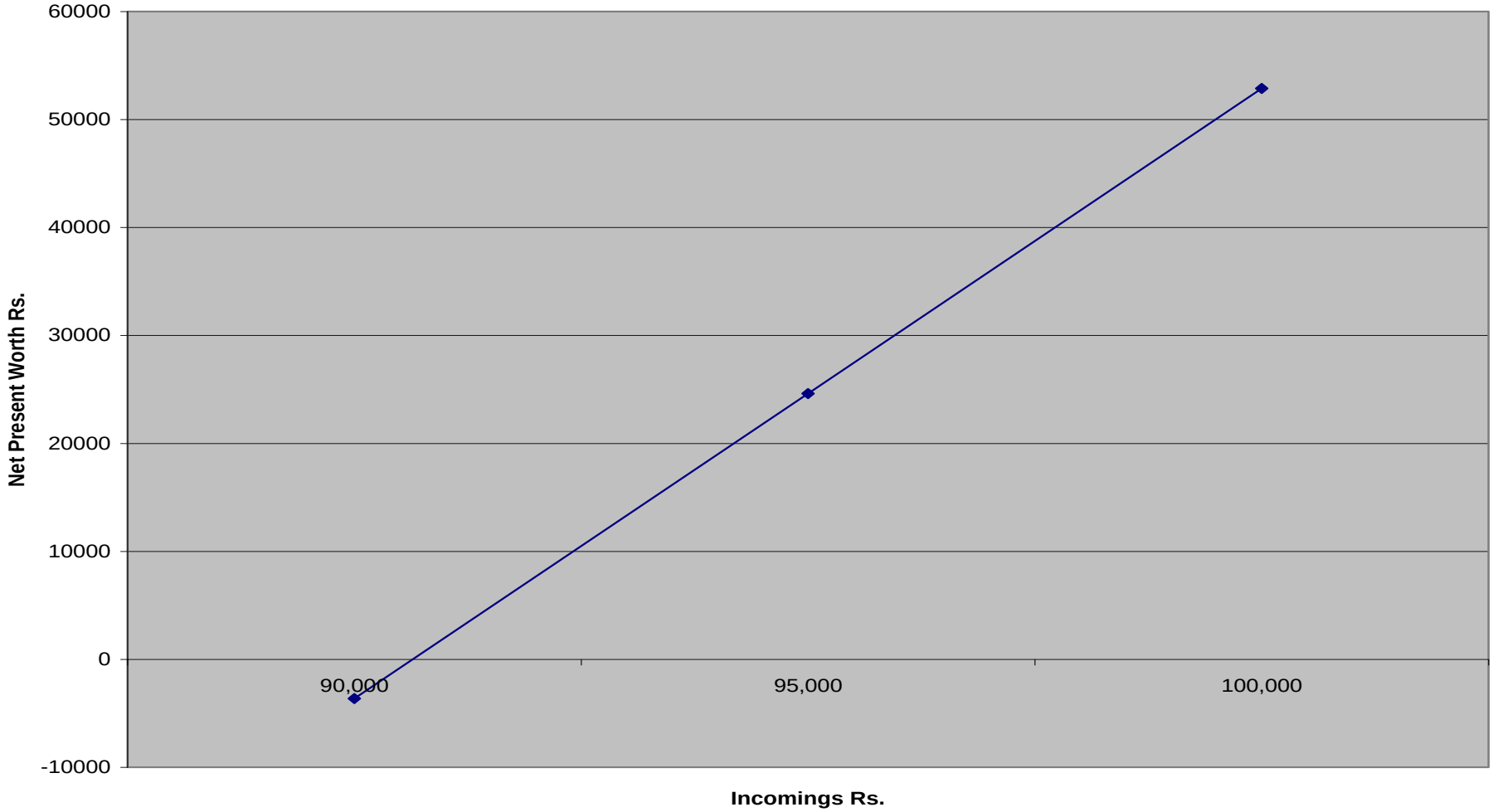
= -500,000 + 100,000(P/A, 12%, 10) – 5,000(P/A, 12%, 10) + 50,000(P/F, 12%, 10) **= 52869.4**

- Let's assume that the estimate of incoming value goes wrong and instead of Birr. 100,000 it is Birr. 90,000.
- Thus the new net present worth keeping all other variables same would be

=(-500,000 + 90,000(P/A, 12%, 10) – 5,000(P/A, 12%, 10) + 50,000(P/F, 12%, 10) **=-3632.8**

- We find that the decision taken on net present worth would get reversed.

- The above analysis could be performed in terms of percentage variation in incoming values also.
- Let the percentage variation in incomings be x. We can find the percent change in incomings at which the net present worth becomes zero.
- In this example, if the incoming value decreases by P % the net present worth turns out to be negative.
- The value of incoming at which net present worth is ***zero*** is known as ***break even point.***



Sensitivity graph showing the effect of changes in incomings on net present worth

- Similar analysis can be performed by changing other variables one at a time.
- The slope of the sensitivity line indicates the sensitivity of a particular variable.
- The *steeper the slope*, the *more sensitive the variable* is and the *milder the slope*, the *less sensitive* the variable.
- Next we take the variables: incomings and service life together and study their sensitivity.

- In order to perform the sensitivity analysis of two variables simultaneously for example incomings (R) and life of the equipment (N) we write the expression of net present worth as given below:
- NPW = -500,000+R (P/A,12%,N) – 5,000(P/A,12%,N)+50,000(P/F,12%,N)

. **There are two ways to show sensitivity analysis**

- Family of curves
- Isoquants

} *(read about it…)*

- **Family of curves representation**

  - One variable is kept fixed while another variable is varied.

For example, Assume that service life N is fixed at some value N1 and net present worth is computed for different values of incomings R.

- **Isoquants**

An indifference line is plotted by varying two variables say service life N and incomings R at a time while keeping all the other variables fixed.

- For drawing the indifference line first we try to get points say (N1,R1), (N2,R2),...at which the net present worth is zero?
- At these points the decision maker is indifferent to this proposal that is he is neither for nor against this proposal.
- The indifference line is obtained by joining together these points.
- Thus indifference line divides the acceptance and rejection zone.

Changes in more than two variables at a time

- Conducting scenario (consequence) analysis is the best approach when performing sensitivity analysis involving changes in more than two variables at a time.

- In scenario analysis a number of scenarios such as best scenario, objective scenario, and worst scenario (in some texts these scenarios are referred to as: less favorable estimate, objective estimate, and more favorable estimate)

- The objective of such scenario analysis is to get a feel of what happens under the most favorable or the most adverse configuration of key variables.
- For example the best, normal and the worst scenario for the previous example could be as given below:

Example

Variables	Worst	Normal	Best
Incomings	85,000	100,000	115,000
Outgoings	7,000	5,000	4,000
Salvage value	25,000	50,000	60,000
Interest rate	15%	12%	10%
Service life	8 years	10 years	12 years

Corresponding to each of the above scenarios, the net present worth can be computed.

- Based on the net present worth values for each of the scenarios the decision maker would be in a **better position to take the decision**.

For example,

- If the NPW of the worst scenario is a large negative value, and the NPW of the objective and best scenarios are low positive value, and moderate positive value the decision would be not to acquire the asset.

Advantages and Limitations of Scenario Analysis

Scenario analysis is considered superior to sensitivity analysis because it considers variations in more than two key variables together.

However, scenario analysis has some limitations:

- It assumes that there are few well delineated scenarios which may not be true in many cases.
- There is a huge requirement of data to perform scenario analysis.

For example even for 6 input variables we would require 6 x 3=18 input data altogether corresponding to the best, normal, and worst scenarios.

Sensitivity analysis for more than one alternative

- Here also we can vary one variable, two variables or more than two variables at a time.
- We discuss the variation in one variable only.
- For illustrating the sensitivity analysis corresponding to changes in one variable, we take up the following two alternatives in which the information on acquisition cost, the incomings, the outgoings, salvage values, interest rate, and service life are provided.

Example

Description	Alternative 1	Alternative 2
Acquisition cost (First cost)	500,000	400,000
Incomings	100,000 yr. for 10 yrs.	80,000/yr. for 10 yrs.
Outgoings	5,000/yr. for 10 yrs.	10,000/yr. for 10 yrs.
Salvage value	50,000	40,000
Interest rate	12%	12%
Service life	10 years	10 years

Solution

- **The net present worth of alternative 1**

= -500,000+100,000 (P/A,12%,10)

-5,000(P/A,12%,10)+50,000(P/F,12%,10)

= -500,000+100,000*5.6502-5000*5.6502+50,000*0.3220

=52,869

- The net present worth of alternative 2

= -400,000+80,000 (P/A,12%,10)

-10,000(P/A,12%,10)+40,000(P/F,12%,10)

= -400,000+80,000*5.6502-10,000*5.6502

+40,000*0.3220 =8,394

- Now let’s change each of these variables one by one. For example, consider the changes in the variable ‘incoming’.
- In case the ‘incoming’ of alternative 1 changes to 90,000 from the existing 100,000 the new net present worth of alternative 1 changes to - 3633
- We can find that if the incoming value reduces to 90,642.99 the decision is reversed. (net PWV=0)
- Such analysis addresses the questions such as:

At what value of incomings the alternative 1 is preferred to alternative 2?

At what service life of the assets, the alternative 1 is preferred to alternative 2?

limitation of sensitivity analysis

- It is non probabilistic in nature.
- One may recollect that for none of the cases we considered the likelihood of occurrence of a particular variable value.
- It merely shows us what happens to the NPW, AW or ROR when there is a change in some variable(s), without providing any information on the likelihood of the changes.
- Commonly, in sensitivity analysis only one variable is changed at a time which may not reflect the real world situation as variables tend to move together,

- There is subjectivity involved in the sensitivity analysis. Thus the sensitivity analysis may lead one decision maker to accept the proposal while other may reject it.

Benefits of performing sensitivity analysis

(1) It shows how robust or vulnerable(exposed to be harmed) a particular alternative is to changes in the value of different variables,

(2) It enables the decision maker to distinguish the sensitive variables from insensitive variables thus the decision maker can focus its attention in making the estimate of sensitive variables,

BREAK EVEN ANALYSIS

Profit and Loss Terms

- In terms of costs and revenues there are three possible profit and loss points for a business activity.
- **Breakeven**: total revenue = total costs

  Just getting along
- **Profit region**: total revenue > total costs

  Putting money in the bank
- **Loss region**: total revenue < total costs

  Going into debt

Break even analysis

- Another way of performing sensitivity analysis
- Here we are more concerned about finding the value (called the break even point) at which *the reversal of decision takes place*.
- not much emphasis was given on finding this break even value.
- what will happen to the project if the invoice or billing declines or costs increase or something else happens.

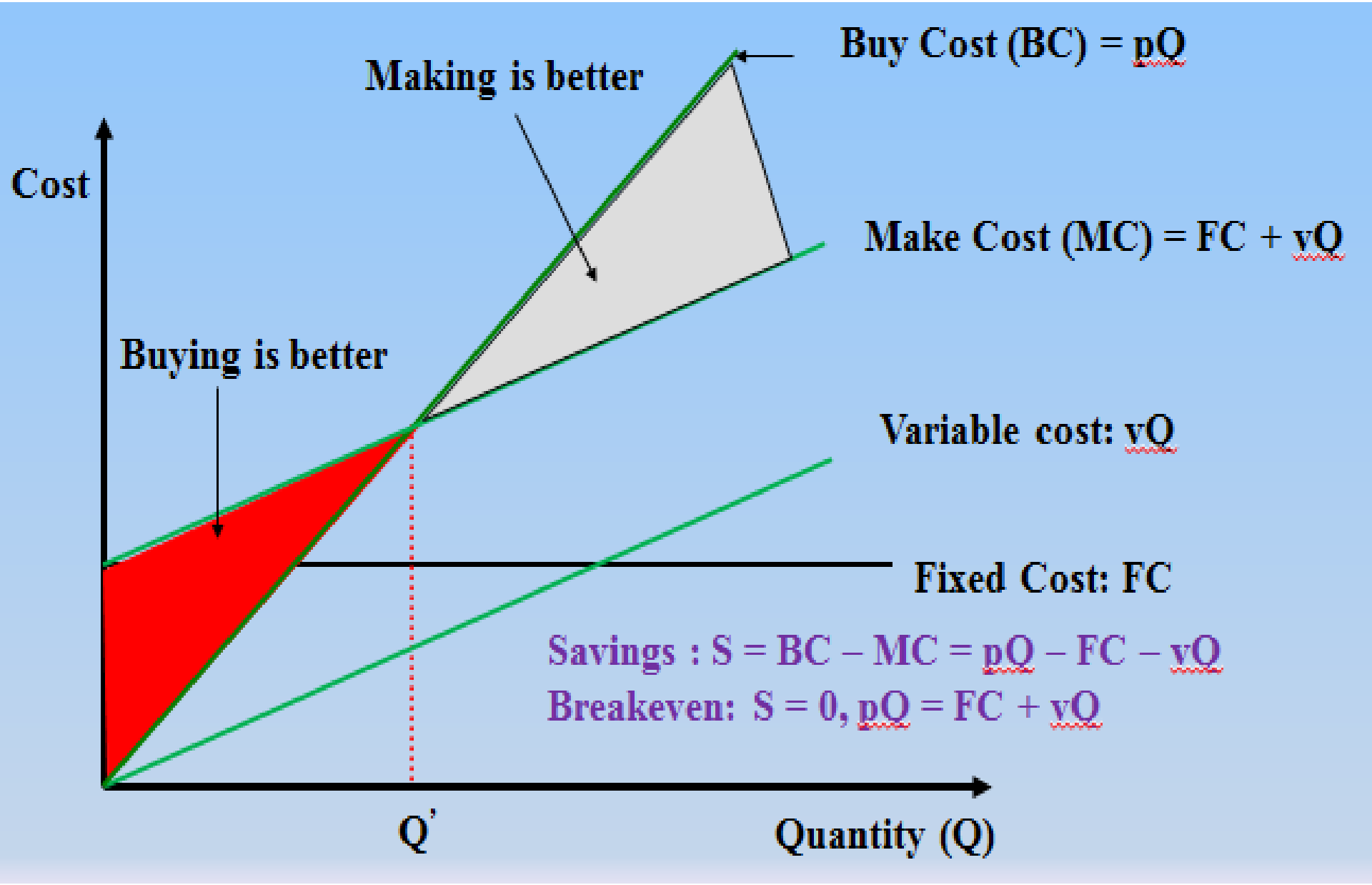
Cont,….

- **how much should be produced and sold at a minimum** to ensure that the project does not 'lose money'.
- The minimum quantity at which loss is avoided is called the break-even point.
- The break even analysis is also referred to as *cost-volume-profit* analysis.

Cont,….

- Addresses the decision of whether to make or buy a product. whether to own or rent an equipment
- Making the product involves two cost elements:
  - Fixed costs
  - Variable costs
- Buying the product involves only one cost element, the selling price.
- However, the price may either be constant or variable based on the quantity.

Linear Breakeven Analysis



ILLUSTRATION

- A Ready Mix Concrete (RMC) manufacturer wants to find out the minimum production of concrete which will just be able to recover its total cost incurred in a particular month.
- The total cost (TC) incurred in a month is the sum total of its indirect cost (IC) and direct cost (DC).
- The indirect costs in this example are those costs which are incurred irrespective of concrete production taking place or not.
- However, the direct costs are proportional to the volume or quantity of production.
- By definition the total cost **TC = IC + DC**

- Let the sales price fixed by the RMC supplier is P per unit concrete sold.
- If the quantity of concrete sold is n units, the revenue R would be computed by the expression **R= n x P**.
- The total cost **TC = IC + n x DC**
- Gross profit Z for the period would be defined as:

  Z = R – TC = n x P – IC – n x DC

  = n x (P-DC) – IC.
- The net profit after taking taxes into account is given by **Z'= Z x (1-t)** where t is the tax rate.

- Break even point is defined where profit equals zero. In order to determine the concrete quantity n at which the RMC seller just recovers its total cost we equate total cost to revenue. Thus at break even point,

Total cost **TC = Revenue R**. (Note at this point Profit Z = 0).

- We have,

  TC = R at break even.

  $\Rightarrow$ n x DC + IC = n x P

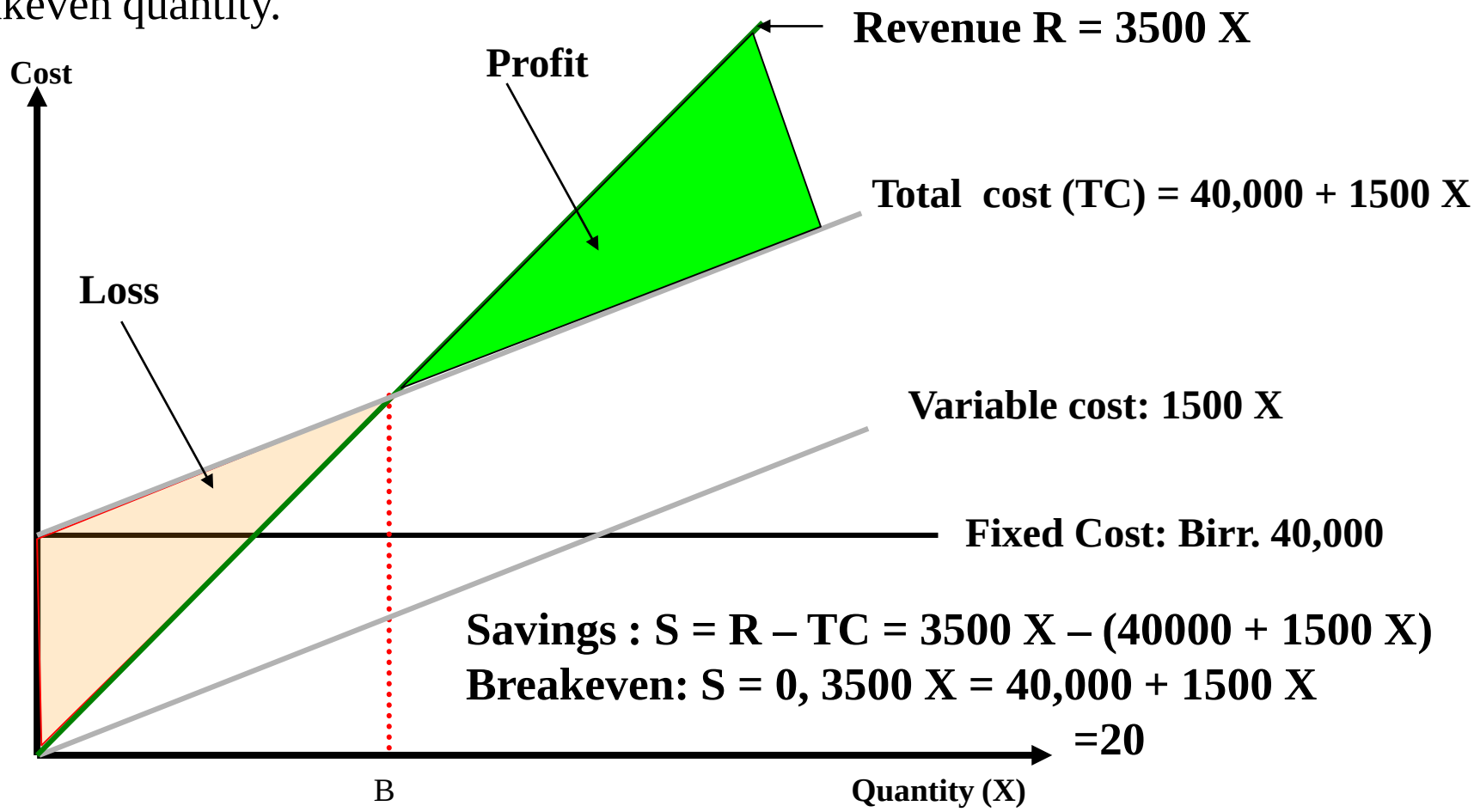
  $\Rightarrow$B x DC + IC = B x P (the quantity produced at Break even point is denoted with B, thus n= B at break even.)

  $\Rightarrow$ **B = IC/ (P-DC)**

- The denominator **(P-DC)** in the expression is also known as ‘contribution’.
- For n less than B, the RMC seller is making losses, while for n greater than B, the RMC seller is making profits.
- The point of intersection of the total cost and revenue lines is known as *break even point*.
- The ordinate corresponding to this intersection point gives the break even quantity of concrete to be produced in order to just recover the total cost incurred by the RMC seller.

Linear Breakeven Analysis-Example

A Ready Mix Concrete company sells RMC for a price of Birr. 3500/ cum. If the fixed cost of the company for the production are Birr. 40,000/month and the variable cost associated with per cum production of RMC is Birr. 1500/ cum, determine the breakeven quantity.



- The companies try to lower the break even point by resorting to different means such as
  - (1) increasing the sales price,
  - (2) reducing the total cost of production,
  - (3) increasing the quantity of production.

Utility (use) of break even analysis

- The break even analysis is useful in situations such as:
  - make or buy decision situation,
  - lease or purchase decision and so on.

Assumptions Of Linear Break Even Analysis

- Income is only from the productions under study. In the previous example it was assumed that the ready mix concrete manufacturing company is into concrete production only.
- Whatever quantity is produced is sold out.
- The per unit direct cost, indirect cost, and sales price associated with the production are constant over the study period. These are also constant over the quantity produced.

Another Example

- DC = 7/ UNIT, Sales price P= 12/unit
- Indirect cost IC = 400

$\Rightarrow$ **B = IC/ (P-DC)**

- Break even point = 400/ (12- 7) = 80
- The break even point can be lowered to 40 by
  - Reducing the indirect cost to 200
  - Reducing the direct cost to 2
  - Increasing the sales price to 17

Dumping

- In order to utilize the full capacity some products are sold at price P while the remaining products are sold at lesser price P’.
- Gross Profit Z = n(P-DC) + n’ (P’-DC) –F
- The Dumping may sometime lead to some problems to the company practicing it.
- Countries prevent such practices by levying heavy duty on imports.

Marginal cost

- Marginal cost is the additional cost incurred by the company to produce one extra unit of product.
- For example, suppose that the cost to produce 100 cum of concrete is Birr. 200,000 and the cost to produce 101 cum of concrete is Birr. 201,000,
- The marginal cost would be equal to (201,000- 200,000)/ (101-100)= Birr. 1000.

Marginal revenue

- Marginal revenue is the additional money realized by ***selling <u>one</u>*** <u>extra unit of product</u>.
- For example if the revenue raised by selling 100 cum of concrete is Birr. 250,000 and by selling 101 cum of concrete the revenue is 252,000, the marginal revenue is (252,000-250,000)/(101-100)= Birr. 2,000.
- Using the concept of <u>*marginal cost*</u>, <u>*marginal revenue*</u> and the <u>*principle of calculus*</u>, one can determine the production level at which the firm would be able to ***maximize its profit.***

Thank u 4 your attention!