Debre Markos University

Institute of Technology

Construction Technology & Management Academic program

Development and Construction Economics

(CoTM 5271)

March, 2020

Chapter – Two

Cash-Flow Diagrams / Interest

- Any organization involved in a project received and spend different amounts of money at different points in time.
- Cash flow diagram is visual representation of this inflow and outflow of funds with time.
- The cash flow profile should include all cash items of a project. The major items included are:
  - First cost, P, is the sum of the costs required to bring the asset into service
  - Salvage, S, is the net sum realized from the disposal of the asset at the termination of its economic life.
  - Income (revenues), other benefits and expenses.

Cont…,

- Income, other benefits and expenses are identified according to their type of flow over time:
  - Periodic cash flow items occur at specific times

    (e.g., engine overhaul every three years)
  - Uniform series revenues or expenses are equal periodic amounts

    (e.g., property taxes, leasing costs, etc.)
  - Continuous revenues or expenses occurring uniformly over the life of the project

    (e.g., savings realized by replacing obsolete equipment)

Cont…,

- ✓ In practice cash flow does not follow any pattern, however for simplification

  all cash incomings and outgoings are assumed to happen either at the beginning or end of a period.

- ✓ The period could be
  - ✓ day, week, month, quarter, or year.

- ❑ Cash Inflows – Revenues (R), receipts, incomes, savings generated by projects and activities that flow in. Plus sign used
- ❑ Cash Outflows – Disbursements (D), costs, expenses, taxes caused by projects and activities that flow out. Minus sign used

ECONOMIC EQUIVALENCE

Definition: Combination of interest rate (rate of return) and time value of money to determine different amounts of money at different points in time that are economically equivalent.

- Established when we are indifferent between a future payment, or a series of future payments, and a present sum of money .

- Considers the comparison of alternative options, or proposals, by reducing them to an equivalent basis, depending on:
  - ✓ interest rate;
  - ✓ amounts of money involved;
  - ✓ timing of the affected monetary receipts and/or expenditures;
  - ✓ manner in which the interest , or profit on invested capital is paid and the initial capital is recovered.

Cash Flow Diagrams / Table Notation

i = effective interest rate per interest period

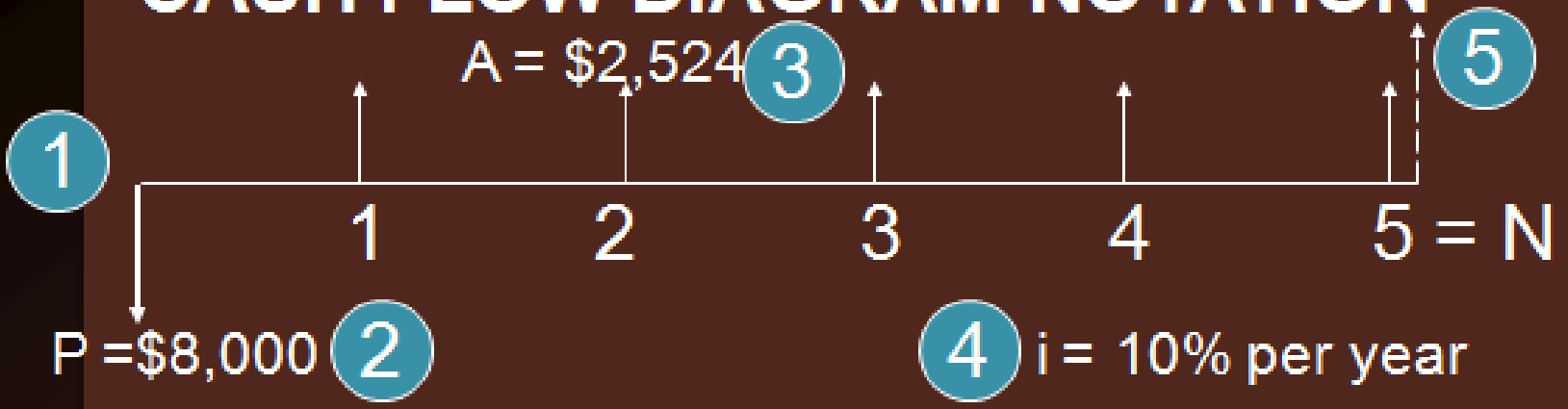
N = number of compounding periods.

P = present sum of money; the equivalent value of one or more cash flows at the present time reference point

F = future sum of money; the equivalent value of one or more cash flows at a future time reference point

A = end-of-period cash flows (or equivalent end-of-period values) in a uniform series continuing for a specified number of periods, starting at the end of the first period and continuing through the last period

CASH FLOW DIAGRAM NOTATION



1. Time scale with progression of time moving from left to right; the numbers represent time periods (e.g., years, months, quarters, etc…) and may be presented within a time interval or at the end of a time interval.
2. Present expense (cash outflow) of $8,000 for lender.
3. Annual income (cash inflow) of $2,524 for lender.
4. Interest rate of loan.
5. Dashed-arrow line indicates amount to be determined.

- standard notation used in equivalence relating one with other

(X/Y, i%, n)

✓ The first letter in the parentheses (X) represent what you "Want to find", while the second letter (Y) represents what is "Given".

✓ For example, F/P means "find F when given P". The i is the interest rate in percent and n represents the number of periods involved.

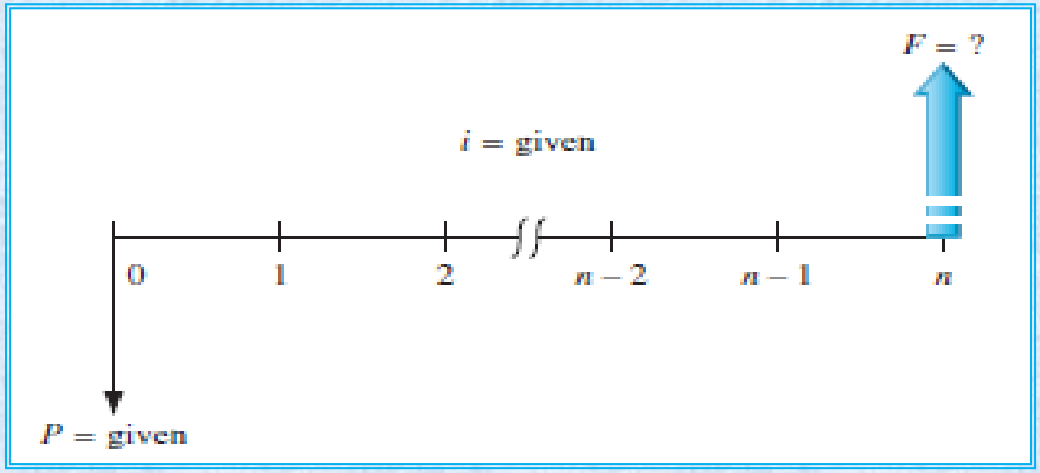
Relating Present and Future Equivalent values of Single Cash Flows

- Finding F when given P:-

  Read as Finding future value when given present value

$$F = P (1+i)^{N}$$

  - $(1+i)^{N}$ single payment compound amount factor
  - functionally expressed as F = P(F / P, i%,N)

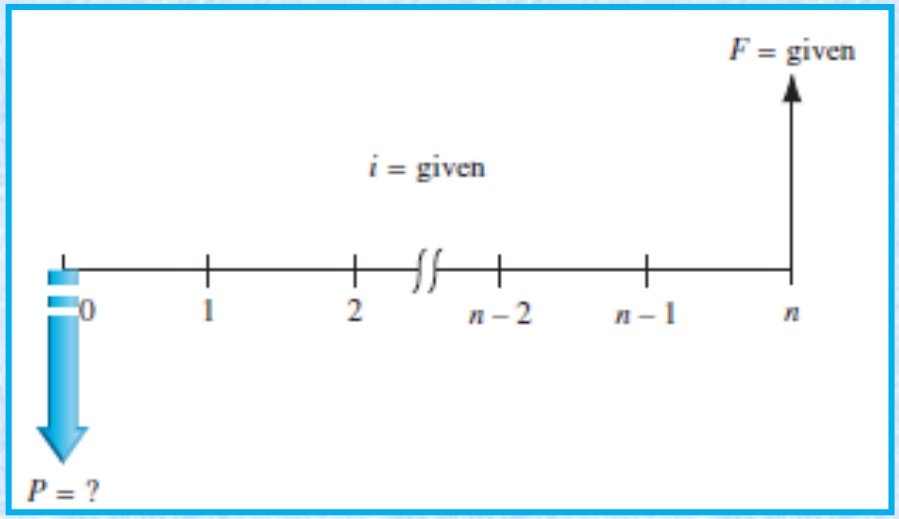


Cont…,

- Finding P when given F:-

  Read as Finding present value when given future value

- $P = F\ (1\ /\ (1\ +\ i\ )^{N}\ )$
  - $(1+i)^{-N}$ single payment present worth factor
  - functionally expressed as P = F (P / F, i%, N)



Uniform Series to Present and Future Equivalent Values

- Finding F given A and Finding A given F:
- Finding future equivalent income (inflow) value given a series of uniform equal Payments

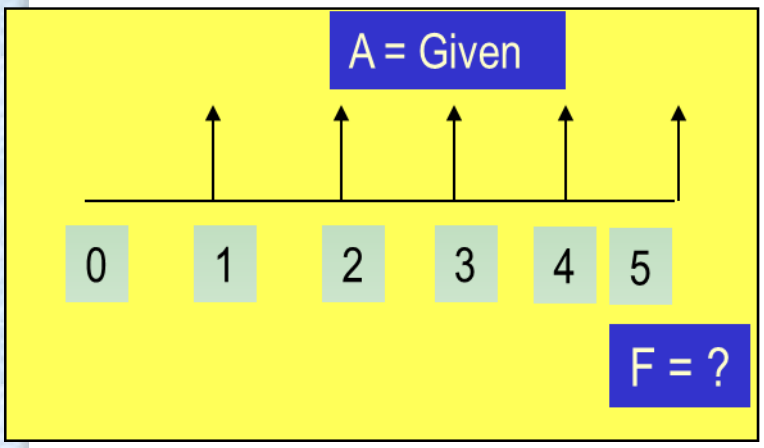
$$F = A\,\frac{(1+i)^n - 1}{i}$$

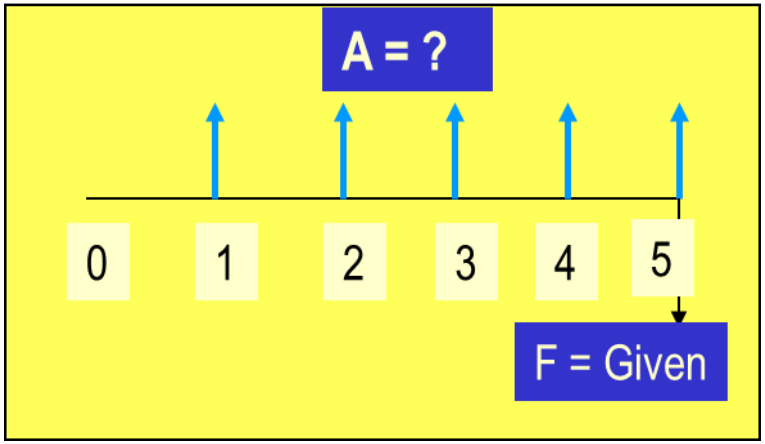
$$A = F\,\frac{i}{(1+i)^n - 1}$$

  - uniform series compound amount factor
  - functionally expressed as

F = A(F/A,i,n)

A = F(A/F,i,n)





Uniform series to present and future equivalent values

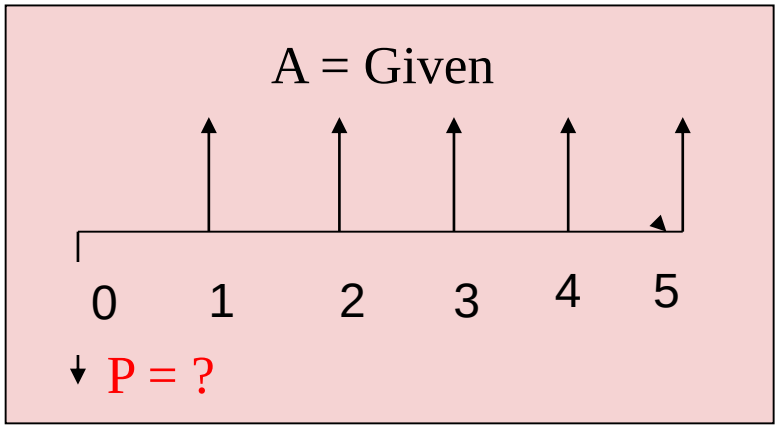
- Finding P given A and Finding A given P:
- Finding present and uniform series equivalent value given a series of equal receipts

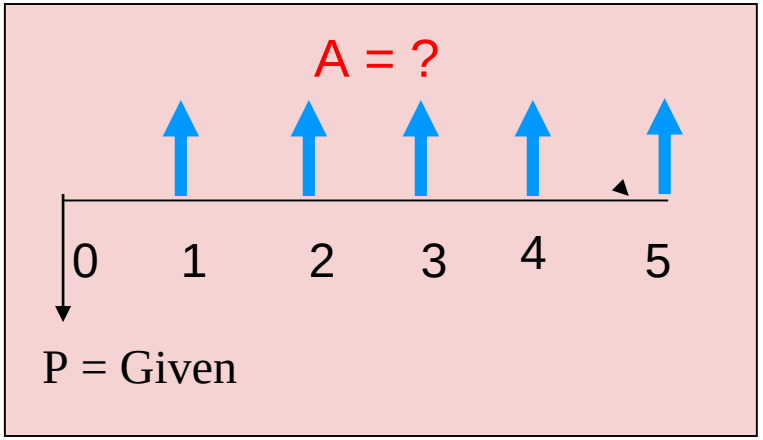
$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$A = P\,\frac{i(1+i)^n}{(1+i)^n - 1}$$

P = A (P / A,i%,N).

A = P (A / P,i%,N)





Examples

1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

P = Rs. 20,000

i = 18% compounded annually

n = 10 years

$F = P(1 + i)^n = P(F/P, i, n)$

$= 20{,}000\ (F/P, 18\%, 10)$

$= 20{,}000 \times 5.234 =$ Rs. 1,04,680

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

2 A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = Rs. 1,00,000

i = 15%, compounded annually

n = 10 years

$P = F/(1 + i)^n = F(P/F, i, n)$

$= 1,00,000\ (P/F, 15\%, 10)$

$= 1,00,000 \times 0.2472$

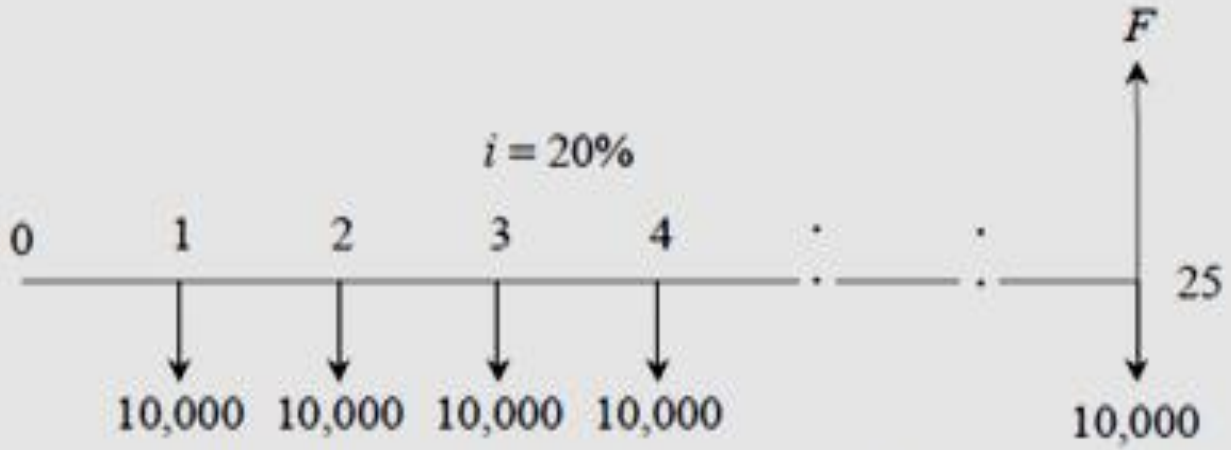
$=$ Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = Rs. 10,000
n = 25 years
i = 20%
F = ?



Cash flow diagram of equal-payment series compound amount.

$$F = A\,\frac{(1+i)^n - 1}{i}$$

$$= A(F/A,\ i,\ n)$$

$$= 10{,}000(F/A,\ 20\%,\ 25)$$

$$= 10{,}000 \times 471.981$$

$$= \text{Rs. } 47{,}19{,}810$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

4 A company has to replace a present facility after 15 years at an outlay of Rs. 5,00,000. It plans to deposit an equal amount at the end of every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15 years.

Solution

F = Rs. 5,00,000
n = 15 years
i = 18%
A = ?

The corresponding cash flow diagram

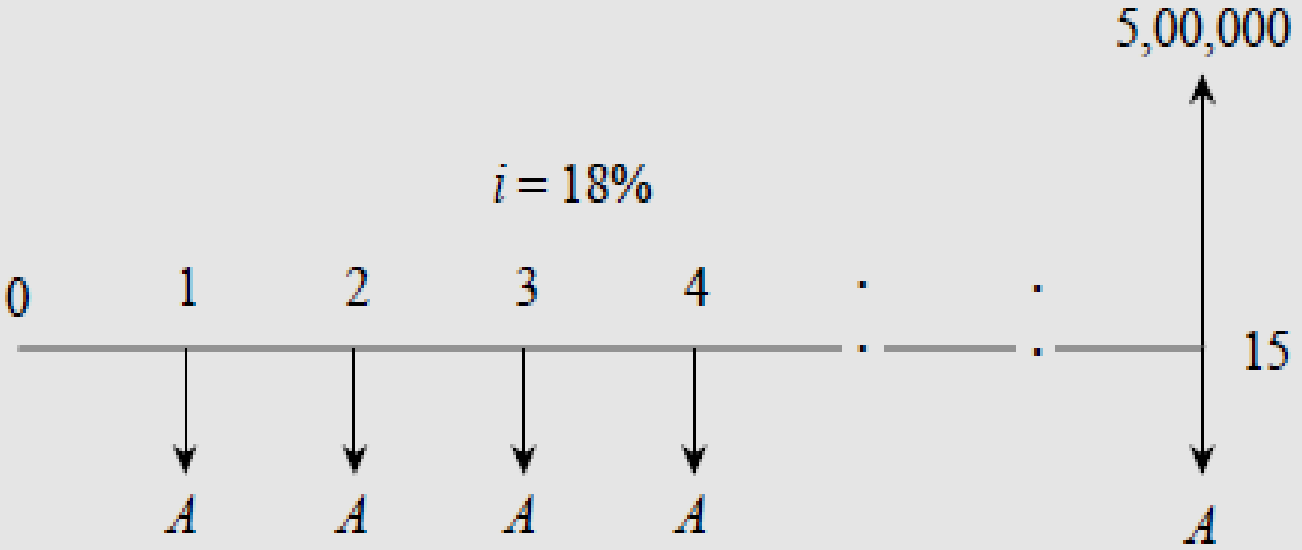


Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

$$= 5{,}00{,}000(A/F, 18\%, 15)$$

$$= 5{,}00{,}000 \times 0.0164$$

$$= \text{Rs. } 8{,}200$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200.

5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

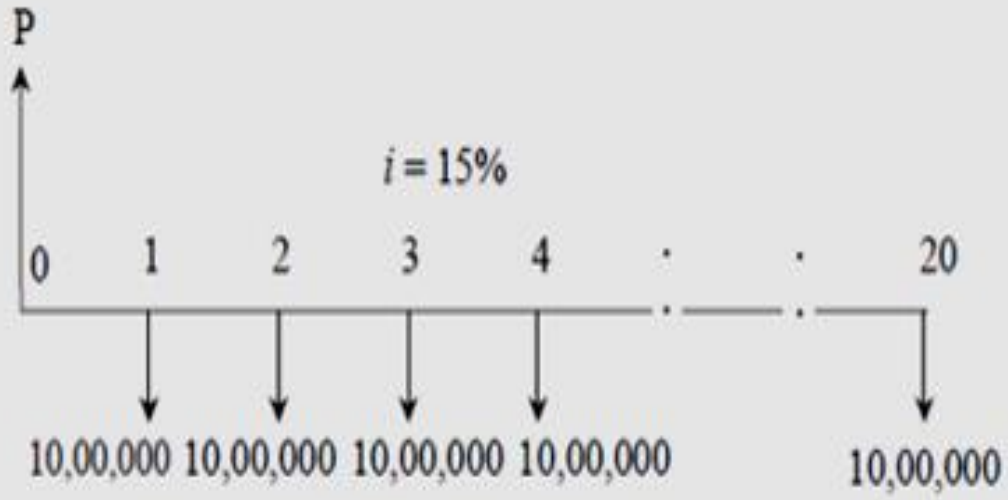
Solution

A = Rs. 10,00,000

i = 15%

n = 20 years

P = ?



Cash flow diagram of equal-payment series present worth amount.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

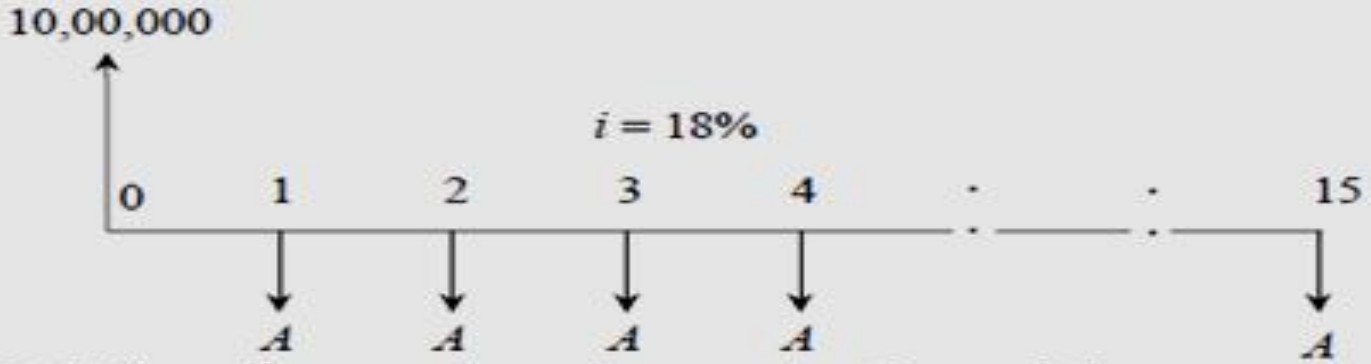
$$= 10{,}00{,}000 \times (P/A, 15\%, 20)$$

$$= 10{,}00{,}000 \times 6.2593$$

$$= \text{Rs. } 62{,}59{,}300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300.

6 A bank gives a loan to a company to purchase an equipment worth Rs. 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the installment amount that the company has to pay to the bank.



Cash flow diagram of equal-payment series capital recovery amount.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n)$$

$$= 10,00,000 \times (A/P, 18\%, 15)$$

$$= 10,00,000 \times (0.1964)$$

$$= \text{Rs. } 1,96,400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

Additional Examples

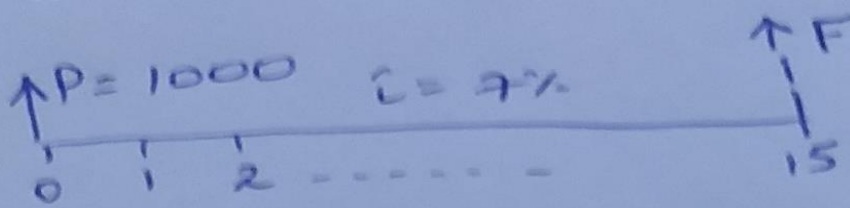
1- You put 1,000 Birr in the Bank account at the end of 2000. the interest rate is 7% per year. What amount will you have in your account at the end of 2015?

#1 Solution

Given P = 1000
i = 7%
n = 15 Years (2000-2015)

Required F = ?

CFD (cash flow Diagram)



$F = P(1+i)^n$ /(F/P, i%, n)

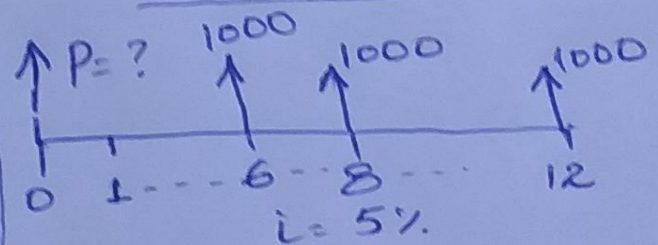
$F = 1000\ (1+0.07)^{15}$

$= 2759.03$

2- How much must a contractor invest NOW to provide a lump sum of Birr 1,000 to pay at the year 6 years, 8 years & 12 years from now. If interest is 5%?

#2 Solution

Cash flow Diagram



Given $i = 5\%$

$F_6 = 1000$

$F_8 = 1000$

$F_{12} = 1000$

Required $P = ?$

$F = P(1+i)^n \Rightarrow P = F/(1+i)^n$

functionally $(P/F, i\%, n)$

$$P = \frac{1000}{(1+0.05)^6} + \frac{1000}{(1+0.05)^8} + \frac{1000}{(1+0.05)^{12}}$$

$$= 746.215 + 676.839 + 556.837$$

$$= 1979.891$$

3- Suppose you make an annual contribution of $5,000 to your savings account at the end of each year for five years. If your savings account earns 6% interest annually, how much can be withdrawn at the end of five years.

#3 Solution

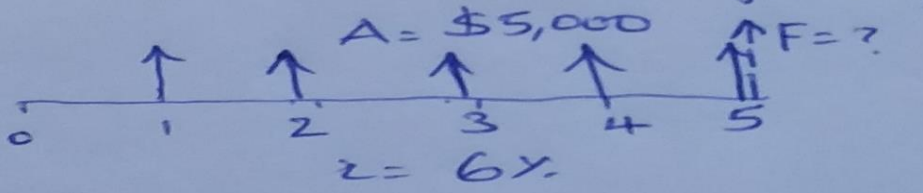
Given $A = \$5,000$

$n = 5$ years

$i = 6\%$

Required $F = ?$

CFD (Cash flow Diagram).

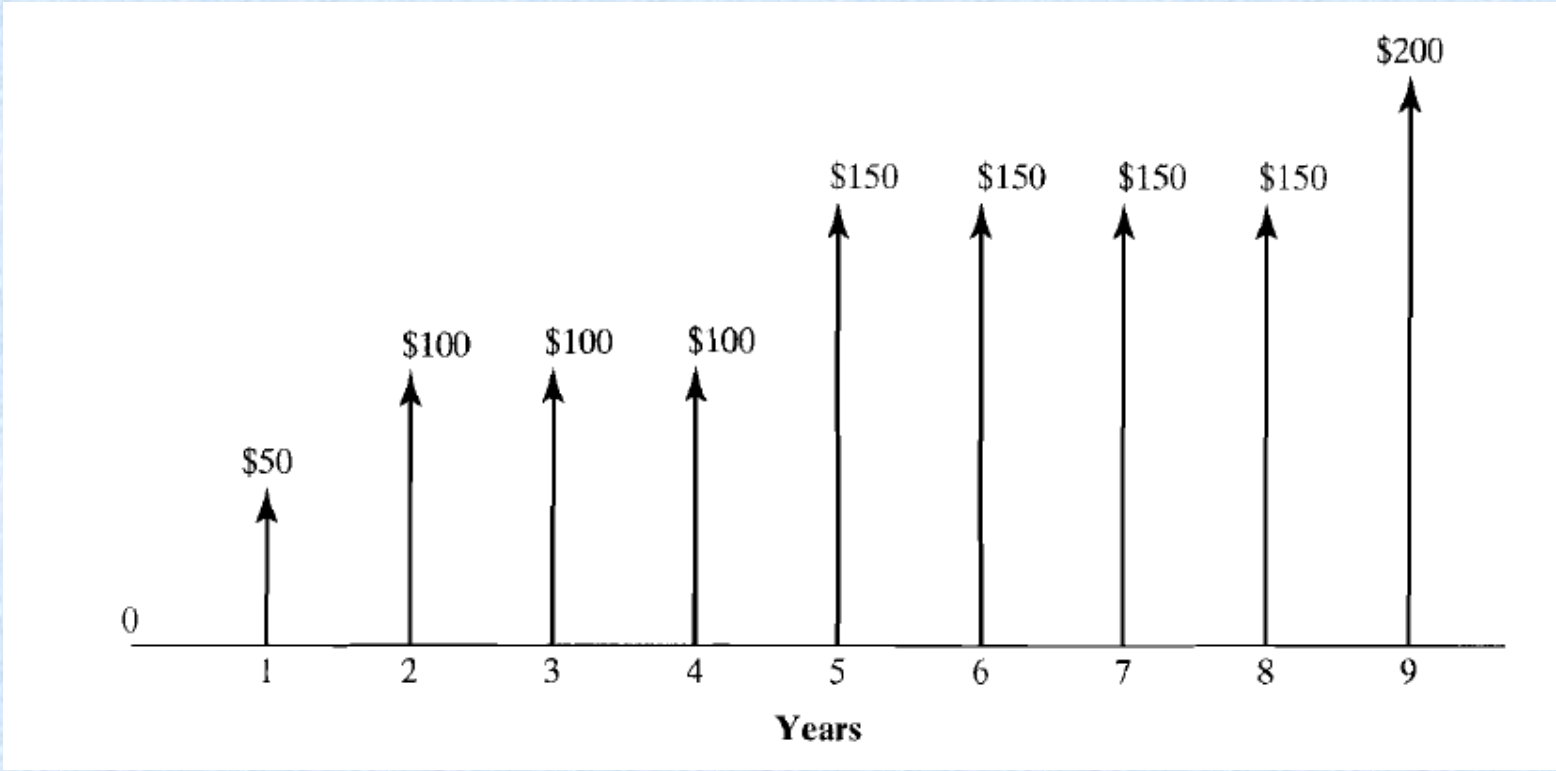


$$F = A\left(\frac{(1+i)^n - 1}{i}\right) / (F/A, i, n)$$

$$F = \$5,000 * \frac{(1+0.06)^5 - 1}{0.06}$$

$$= \$28,185.46$$

4- find the present worth for the following CFD at 15% per year.



#4 Solution

Method 1 - Brutal force approach (P/F)

" 2 - Grouping approach

Group 1 → $50 (P/F, 15%, 1)

$P_1 = \$50 (1+0.15)^1 = \$43.48$

Group 2 → $100 (P/A, 15%, 3) (P/F, 15%, 1)

$$P_2 = \$100 \frac{(1+0.15)^3 - 1}{0.15(1+0.15)^3} * \frac{1}{(1+0.15)^1}$$

$= \$198.54$

Group 3 → $150 (P/A, i%, 4) (P/F, 15%, 4)

$$P_3 = \$150 \frac{(1+0.15)^4 - 1}{0.15(1.15)^4} * \frac{1}{(1.15)^4}$$

$= \$244.85$

Group 4 → $200 (P/F, 15%, 9)

$P_4 = \$200 \left(\frac{1}{1.159}\right) = \$56.85$

Present worth $= P_1 + P_2 + P_3 + P_4$

$= \$543.72$

Cont…,

5- What is the present equivalent value of 50000 birr, 5 years from now at 14% compounded semiannually?

Solution

i_{eff} = **$(1+.14/2)^2-1$ = 14.49%**

Present equivalent P = 50000(P/F, i_{eff}, 5 yrs)

= 50000(.5085)=25417.46birr

Alternatively

Present equivalent P = 50000(P/F, $i_{nom}/2$, 10)

= 50000/$(1+0.07)^{10}$= 50000 x0.5085)

=25417.46birr


Thank
you