

# Engineering Economics



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## 1. Introduction

The course will be an introductory course, whereby the emphasis will be placed on Engineering Economy, the techniques for comparison and evaluation of *alternative solutions* after an analysis of *costs and benefits*. As this course concentrate on the techniques of analysis it is at the *micro-economic level*.

- Economic analysis – economic comparison of projects.
- Engineering economy could be defined as the theory of monetary costs and benefits of works of public utility (civil and agricultural engineering).
- Economic appraisal of civil engineering works.
- Projects with a specific starting and ending point (and therefore different from a business).

### Project Cost - Benefit Analysis (CBA)

The main types of decision guided by cost-benefit analysis are:

- *Investment-type* or *yes/ no decisions*, like whether or not a single project or course of action will be undertaken;
- *Design-type* or *either/ or decisions*, like which of several possible projects should be implemented, or the choice between two or more alternative ways of achieving some technical goal (competing courses of action).

A special kind of design-type decision is the case of *technically mutual exclusive* courses of action. In this case the alternatives are such that only one can be chosen, for technical reasons rather than shortage of resources. For example the choice between a higher and a lower dam at the same site or the choice between methods, one labour-intensive, one machinery-intensive, or one faster than another.

In general both costs and benefits have to be quantified, but in some analyses, called **least-cost** or minimum-cost ones, all the competing courses of action produce the same benefits. Common examples are alternative conveyors for water transfer (long canal, short tunnel, small pipeline with high pumping costs, large pipeline with low pumping costs, etc.). Benefits are identical for all courses of action and do not need to be valued; only the costs vary and are valued.

## **Equipment economics**

Financial analysis: analysis done using market prices

Economic analysis: analysis done using economic values

Techniques of analysis are not different, but the input data are.

### **Time preference**

(Example from 'Cost-benefit analysis for engineers and planners' by Michael Snell.)

Most of us prefer jam today to jam tomorrow. When considering using resources or enjoying benefits, we are not indifferent to timing. Reasons and justifications for this are many and disputed, and some are discussed below, but for the moment let us accept the fact and examine its implications for CBA.

The name of this fact is *time preference*, and it begins with individual time preference. To avoid confusion with monetary ideas like inflation and interest, and to concentrate on resources in a general sense, I will begin by saying that I personally would just as receive 100 jars of jam today as 110 jars of jam over one year. Hence, jars of jam are a proxy for resources in general, and those number happen to describe my personal preference rate. If you make me a credible offer of 111 jars of jam next year in exchange for 100 jars that I have right now, I will accept because that is one jar more than I would regard as equivalent. But if you offer me 109 jars a year hence I will prefer to keep the 100 jars that I have. In the jargon of economists, I am *indifferent* between 100 jars now and 110 next year.

Assuming I am consistent and do not change my attitude to time and jam, I will have the same ratio next year, so that 100 jars next year have the same value to me as 110 jars the year after. This means that 121 jars in two years time has the same value to me as 110 jars next year or 100 jars today.

My reasons might be mixed. I might just be irrationally or irresponsible impatient; I might fear not to be alive next year to eat jam: I might expect to be richer next year and therefore less urgently interested in jam: or I might want to sell some of my 100 jars, put the money in a good investment account, and with the interest buy considerably more jam in the future.

**Interest calculation** - time-value of money  
Mathematics of finance

### **Interest**

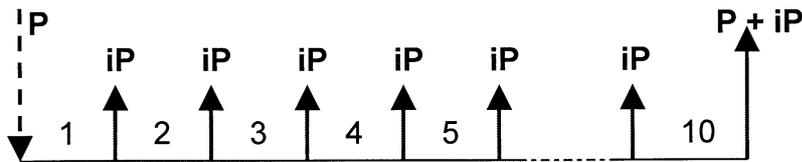
Price or cost of the use of money (or credit). This price is called interest, which is usually computed from an interest rate. The interest rate ( $i$ ) the fraction or % of the principal sum (the amount of money or credit we are talking about) – payable every period at the end of that period. Length of period usual a year; interest rate is then expressed as a percentage of the principal sum per year. Interest rates are more easily comparable with each other than amounts of interest.

### **Nominal value**

The nominal value of money does not change over time: an euro remains an euro.

- What you can buy for it – its purchasing power – its **real value** will, however, change over time.
- We will not deal with problems of **inflation** – Constant prices.

## Simple interest



$P$  is the principal sum,  $i$  the periodic interest rate. The periodic interest  $iP$  is payable at the end of the year as long as the money or credit is available, which is till the end of year 10 in this scheme, when the principal,  $P$ , is returned (amortized). The series of payments  $iP \dots (P + iP)$  from the end of year 1 till the end of year 10 is equivalent to the Principal value,  $P$ , at time 0.

## Compound or composite interest.

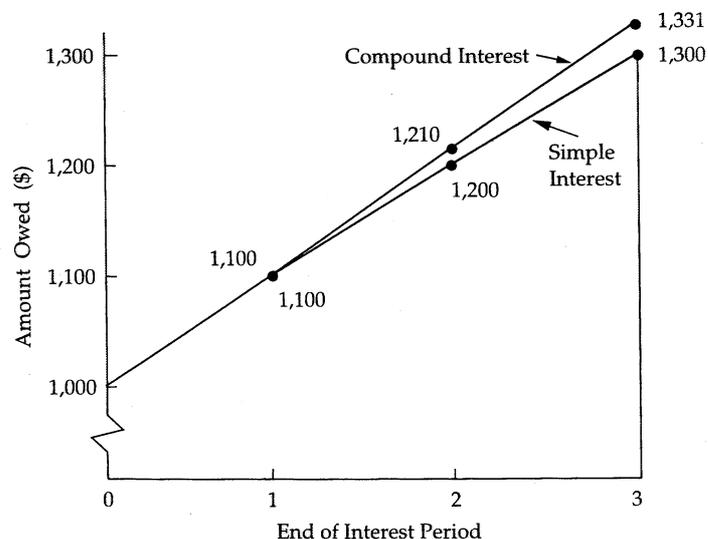
If the interest is not paid but added to the outstanding sum at the end of each period then in future interest will, of course, have to be paid also over the non-paid interest of the previous period, etcetera. Finally the principal sum  $P$  plus all accrued interest will have to be paid to the owner of the money.

See chapter 2. Interest calculations.

The effect of compounding interest can be seen in figure 1.1 for an amount of € 1,000 loaned for three periods at an interest rate of 10 % compounded each period. A total of € 1,331 would be due for repayment at the end of the third period which can be compared with € 1,300 for simple interest (period = year). The difference is due to the effect of *compounding*, which is essentially the calculation of interest on previously earned interest.

Period	Amount owed at beginning of period (1)	Interest amount for period (2) = (1) x 10%	Amount owed at end of period (3) = (1) + (2)
1	€ 1,000	€ 100	€ 1,100
2	€ 1,100	€ 110	€ 1,210
3	€ 1,210	€ 121	€ 1,331

Figure 1.1  
Illustration of Simple versus Compound Interest (for  $i = 10\%$ )



### **Level of the rate i**

Among others a function of:

- solvency of the borrower
- scarcity (or abundance) of the money supply
- expectation of inflation during the period under consideration (the longer the period the higher the rate may be because of more uncertainty)
- cost of handling the money
- the 'profit' the owner of the money wants to make (the compensation he expects for not using the money, not consuming it).

One could say, very roughly, that the real interest rate (without inflation) is the prevailing interest rate minus the prevailing inflation rate. This real rate is – on average – over a long, stable period, usually somewhere between 2 and 6 % per year.

### **Concept of discount rate**

Costs of opportunity of money or resources. Example: investor interested to invest in hydropower. Past history - risk involved - future expectations, say about 10 - 15 % reward of opportunity

Demand and supply graph; equilibrium condition

Discount rate can change.

### **World Bank**

World Bank has a prime rate. As their objective is not commercial (to make money) they use a preferential discount rate for projects which are of social and economic importance - governments give the World Bank cheap money (so kind of subsidized by taxpayer). But the World Bank established some kind of profit making. Prime rate can be in the order of 6 - 8 %

### **Types of discount rates:**

1. Banking interest rate - commercial rates
2. Preferential rates for development projects; lowest rate is called prime rate.
3. Interbank system - uniform interest rate; LIBOR (UK) = London Interbank Operations Rate; loans from bank to another bank; usually short term, but can be years as well.

### **Government**

Social discount rate for Education and Public Health; no matter what it costs; of national importance. Rates are used for comparison of alternatives

### **Economic discount rate**

Also measuring other externalities. Investing in energy. What is the economic value for the regional or national development. Irrigation projects do have a very low return; but food is essential; so social and economic reasons to invest.

## 2. Interest calculation.

### 2.1. Introduction

In all **Benefit - Cost - Analysis** interest calculations play a major role.

Two types of operations are used :

*compounding*

**future value of money**

*discounting*

**present value of money**

In the practice of Benefit- Cost - Analysis it is customary to express benefits and costs in terms of **present value** by applying the appropriate **discount factors**.

$i =$  annual interest rate (used in the formulae).

Point in time method: a discrete point for accounting:

beginning of the year

middle of the year

**end of the year** - generally used = matter of convention.

This means that flows of money that may occur more or less continuously during the year are assumed to occur at one particular point-of-time during that year and that interest will be calculated only at the end of the year.

For compounding this means that at the end of each year the amount at the beginning of that same year, increased by the interest accrued during that year, is carried forward to the next year and at the end of that year the same process is repeated for the year thereafter.

The analysis of the financial, economical or social benefits and costs requires to express them in **comparable terms**.

Costs and benefits occur at different points of time !

In order to make them comparable it is customary to express both in terms of their **present value**. Only then application of criteria.

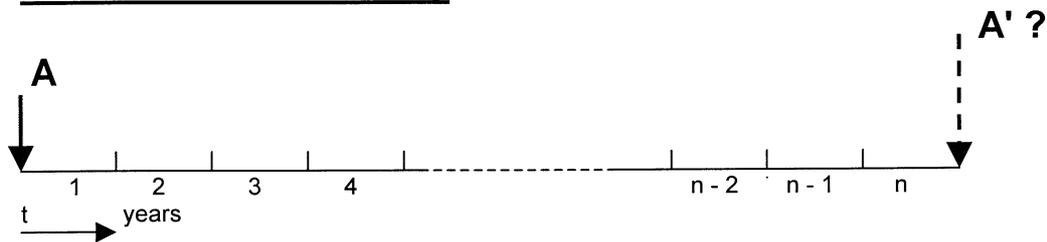
Deposits in instalments.

Periodic compounding: deposits (or withdrawals) can be made annually, usually but not necessarily at the end of the year, or for instance every month, or quarterly or half-yearly. It is also possible to compound continuously.

## 2.2. Single deposit

interest rate:  $i$

### Future value of money



Suppose a deposit  $A$  is made at the time  $t = 0$ .  
What is the value after  $n$  years ?

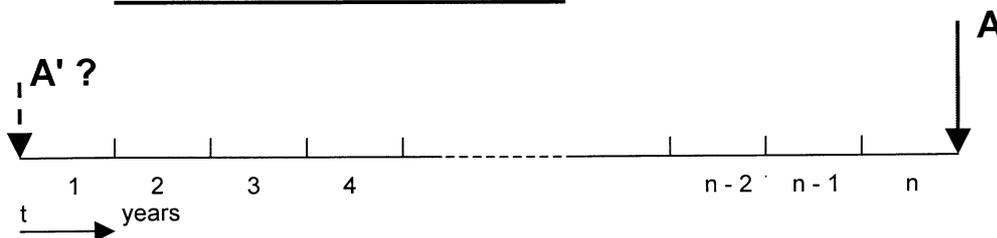
**compounding factor :**  $(1 + i)^n$

$$A' = (1 + i)^n \cdot A$$

Example:  $A = \text{€ } 1,000$  ;  $i = 8\%$  ;  $n = 20$  years

$$A' = (1.08)^{20} \cdot \text{€ } 1,000 = 4.661 \cdot \text{€ } 1,000 = \underline{\underline{\text{€ } 4.661}}$$

### 2.3. Present value of money



Suppose a deposit  $A$  is made at the end of year  $n$ .  
What is the value at the time  $t = 0$  ?

**discounting factor :**  $\frac{1}{(1 + i)^n}$

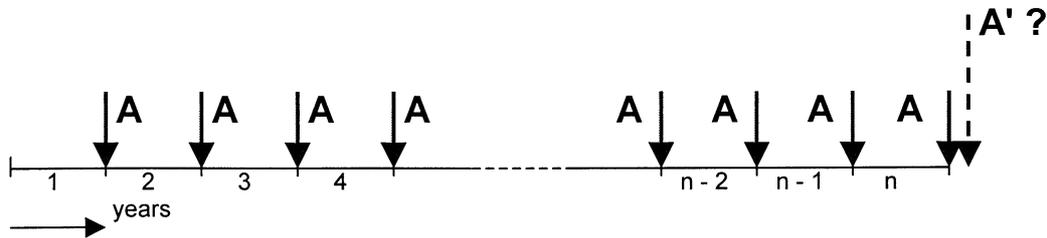
$$A' = \frac{1}{(1 + i)^n} \cdot A$$

Example:  $A = \text{€ } 5,000$  ;  $i = 8\%$  ;  $n = 20$  years

$$A' = \frac{1}{(1.08)^{20}} \cdot \text{€ } 5,000 = \frac{1}{4.661} \cdot \text{€ } 5,000 = 0.215 \cdot \text{€ } 5,000 = \underline{\underline{\text{€ } 1,073}}$$

## 2.4. Annual constant deposit

### Future value of money



Suppose over a period of  $n$  years, a constant annual deposit is made at the end of each year.

What is the total value after  $n$  years ?

$$A' = (1+i)^{n-1} \cdot A + (1+i)^{n-2} \cdot A + \dots + (1+i)^2 \cdot A + (1+i) \cdot A + A$$

compounding factor :  $\frac{(1+i)^n - 1}{i}$

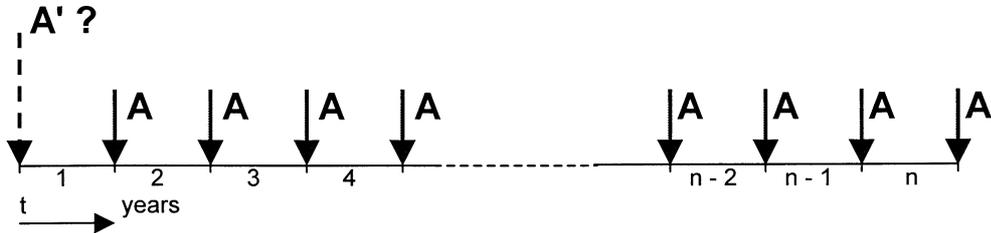
$$A' = \frac{(1+i)^n - 1}{i} \cdot A$$

Example:  $A = \text{€ } 1,000$  ;  $i = 8\%$  ;  $n = 20$  years

$$\begin{aligned} A' &= \frac{(1.08)^{20} - 1}{0.08} \cdot \text{€ } 1,000 = \frac{4.661 - 1}{0.08} \cdot \text{€ } 1,000 \\ &= \frac{3.661}{0.08} \cdot \text{€ } 1,000 = \underline{\underline{\text{€ } 45,762}} \end{aligned}$$

## 2.5. Annual constant deposit

### Present value of money



Suppose over a period of  $n$  years, a constant annual deposit  $A$  is made at the end of each year.

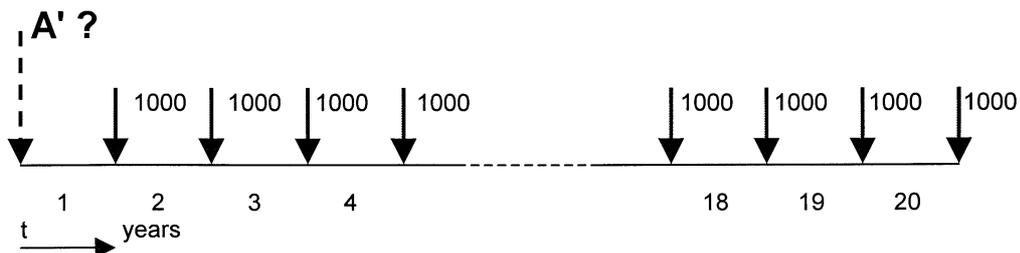
What is the present value of these deposits ?

$$A' = \left\{ \frac{(1+i)^n - 1}{i} \right\} \cdot \left\{ \frac{1}{(1+i)^n} \right\} \cdot A$$

*future value* . *discount factor* → *present value*

**discounting factor** :  $\frac{(1+i)^n - 1}{i \cdot (1+i)^n}$

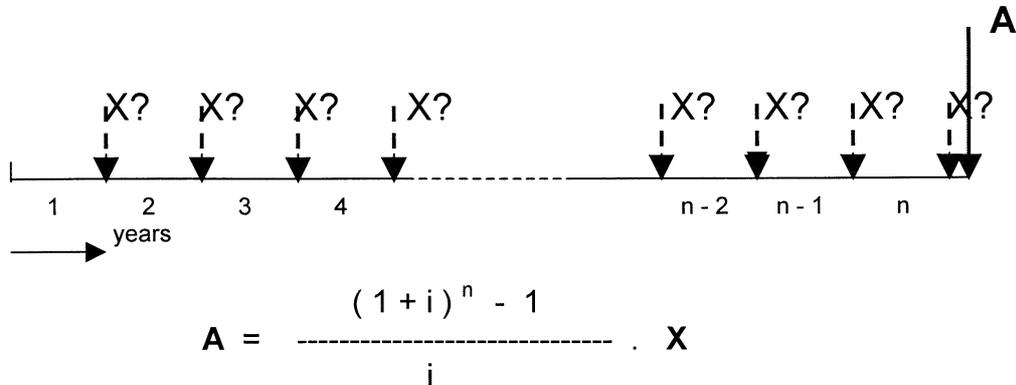
Example :  $A = \text{€ } 1,000$  ;  $i = 8\%$  ;  $n = 20$  years.



$$\begin{aligned} A' &= \frac{(1.08)^{20} - 1}{0.08 \cdot (1.08)^{20}} \cdot \text{€ } 1,000 = \frac{4.661 - 1}{0.08 \cdot 4.661} \cdot \text{€ } 1,000 \\ &= \frac{3.661}{0.373} \cdot \text{€ } 1,000 = \underline{\underline{\text{€ } 9,818}} \end{aligned}$$

## 2.6. Sinking fund factor

Which annual deposit should be made to yield a given amount after a definite number of years ?



<p><b><u>sinking fund factor</u></b> : <math>\frac{i}{(1+i)^n - 1}</math></p>
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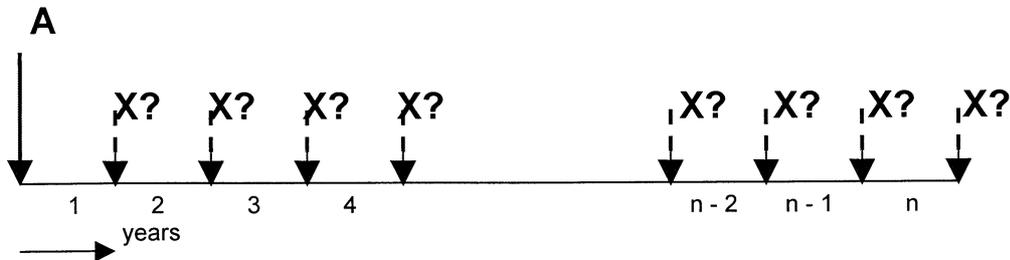
reciprocal value of compounding factor  
for annual constant deposits.

Example:  $A = \text{€ } 50,000$  ;  $i = 8\%$  ;  $n = 20$  years

$$\begin{aligned}
 X &= \frac{0.08}{(1.08)^{20} - 1} \cdot \text{€ } 50,000 = \frac{0.08}{4.661 - 1} \cdot \text{€ } 50,000 \\
 &= \frac{0.08}{3.661} \cdot \text{€ } 50,000 = 0.0219 \cdot \text{€ } 50,000 = \underline{\underline{\text{€ } 1,093}}
 \end{aligned}$$

## 2.7. Annuity factor

Which annual deposit should be made,  
of which the present value is equal to a given amount ?



$$A = X \cdot \frac{(1+i)^n - 1}{i \cdot (1+i)^n}$$

<b><u>annuity</u> :</b> $\frac{i \cdot (1+i)^n}{(1+i)^n - 1}$
---

**Also :**

$$\begin{aligned} \text{Annuity} &= \text{sinking fund factor} \times (1+i)^n \\ &= \text{sinking fund factor} + i \end{aligned}$$

Example: A = € 50,000 ; i = 8 % ; n = 20 years

$$\text{annuity : } \frac{0.08 \cdot (1.08)^{20}}{(1.08)^{20} - 1} = \frac{0.373}{3.661} = \mathbf{0.102} \text{ (0.10185)}$$

$$X = 0.102 \cdot € 50,000 = \underline{\underline{€ 5,090}}$$

sinking fund factor : 0.0219

$$\text{annuity : } 0.0219 \times (1.08)^{20} = 0.0219 \times 4.661 = 0.102$$

$$\text{or } 0.0219 + 0.08 = \mathbf{0.1019}$$

Interest table for equal-payment series (capital recovery factors, CRF)

n	i%	2	4	6	8	10	12	14	16	18	20
1		1.0200	1.0400	1.0600	1.0800	1.1000	1.1200	1.1400	1.1600	1.1800	1.2000
2		0.5151	0.5302	0.5454	0.5608	0.5762	0.5917	0.6073	0.6230	0.6387	0.6545
3		0.3468	0.3604	0.3741	0.3880	0.4021	0.4163	0.4307	0.4453	0.4599	0.4747
4		0.2626	0.2755	0.2886	0.3019	0.3155	0.3292	0.3432	0.3574	0.3717	0.3863
5		0.2122	0.2246	0.2374	0.2505	0.2638	0.2774	0.2913	0.3045	0.3198	0.3344
6		0.1785	0.1908	0.2034	0.2163	0.2296	0.2432	0.2572	0.2714	0.2859	0.3007
7		0.1545	0.1666	0.1791	0.1921	0.2054	0.2191	0.2332	0.2476	0.2624	0.2774
8		0.1365	0.1485	0.1610	0.1740	0.1874	0.2013	0.2156	0.2302	0.2452	0.2606
9		0.1225	0.1345	0.1470	0.1601	0.1736	0.1877	0.2022	0.2171	0.2324	0.2481
10		<b>0.1113</b>	<b>0.1233</b>	<b>0.1359</b>	<b>0.1490</b>	<b>0.1628</b>	<b>0.1770</b>	<b>0.1917</b>	<b>0.2069</b>	<b>0.2225</b>	<b>0.2385</b>
12		0.0946	0.1066	0.1193	0.1327	0.1468	0.1614	0.1767	0.1924	0.2086	0.2253
15		0.0778	0.0899	0.1030	0.1168	0.1315	0.1468	0.1628	0.1794	0.1964	0.2139
20		<b>0.0612</b>	<b>0.0736</b>	<b>0.0872</b>	<b>0.1019</b>	<b>0.1175</b>	<b>0.1339</b>	<b>0.1510</b>	<b>0.1687</b>	<b>0.1868</b>	<b>0.2054</b>
25		0.0512	0.0640	0.0782	0.0937	0.1102	0.1275	0.1455	0.1640	0.1829	0.2021
30		<b>0.0447</b>	<b>0.0578</b>	<b>0.0727</b>	<b>0.0888</b>	<b>0.1061</b>	<b>0.1241</b>	<b>0.1428</b>	<b>0.1619</b>	<b>0.1813</b>	<b>0.2008</b>
40		0.0366	0.0505	0.0665	0.0839	0.1023	0.1213	0.1407	0.1604	0.1802	0.2001
50		<b>0.0318</b>	<b>0.0466</b>	<b>0.0634</b>	<b>0.0817</b>	<b>0.1009</b>	<b>0.1204</b>	<b>0.1402</b>	<b>0.1601</b>	<b>0.1800</b>	<b>0.2000</b>
60		0.0288	0.0442	0.0619	0.0808	0.1003	0.1201	0.1401	0.1600	0.1800	0.2000
70		0.0267	0.0428	0.0610	0.0804	0.1001	0.1200	0.1400	0.1600	0.1800	0.2000
80		0.0252	0.0418	0.0606	0.0802	0.1000	0.1200	0.1400	0.1600	0.1800	0.2000
90		0.0240	0.0412	0.0603	0.0801	0.1000	0.1200	0.1400	0.1600	0.1800	0.2000
100		<b>0.0232</b>	<b>0.0408</b>	<b>0.0602</b>	<b>0.0800</b>	<b>0.1000</b>	<b>0.1200</b>	<b>0.1400</b>	<b>0.1600</b>	<b>0.1800</b>	<b>0.2000</b>
∞		0.0200	0.0400	0.0600	0.0800	0.1000	0.1200	0.1400	0.1600	0.1800	0.2000

A = a<sub>ni</sub> (annuity factor) x P

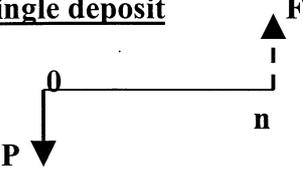
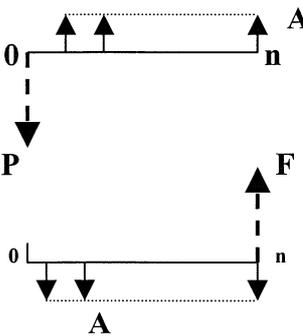
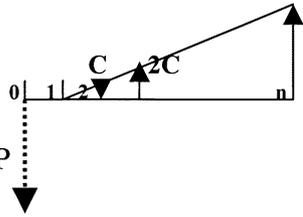
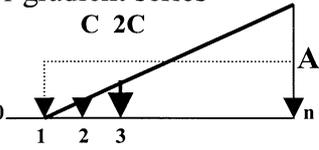
$$a_{ni} = \text{annuity} \frac{i \cdot (1+i)}{(1+i)^n - 1} = \frac{i}{1 - [1/(1+i)^n]}$$

= amount to be paid at the end of every year (or interest bearing period) during n years (or interest bearing periods) to be equivalent to 1 unit now

i = interest rate

n = number of years (or interest bearing periods)

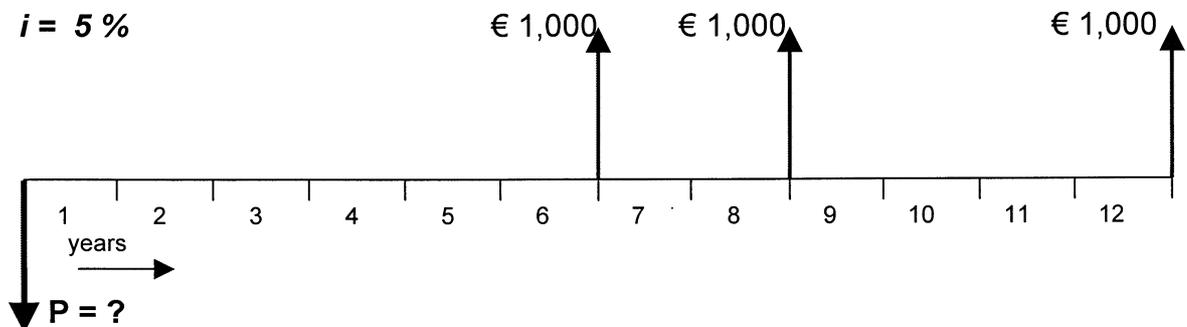
2.8. FORMULA'S ENGINEERING ECONOMY

SUMMARY	Present value - discounting	Future value - compounding
<p><b>Single deposit</b></p> 	$P = \frac{1}{(1+i)^n} \cdot F$	$F = (1+i)^n \cdot P$
<p><b>Annual constant deposit</b></p> 	$P = \frac{(1+i)^n - 1}{i \cdot (1+i)^n} \cdot A$ <p><b>annuity factor:</b> (or: capital recovery formula)</p> $A = \frac{i \cdot (1+i)^n}{(1+i)^n - 1} \cdot P$ <p>for <math>n \geq 100</math> years: <math>A = P \cdot i</math></p>	$F = \frac{(1+i)^n - 1}{i} \cdot A$ <p><b>sinking fund factor:</b></p> $A = \frac{i}{(1+i)^n - 1} \cdot F$
<p><b>Gradient series</b></p>  <p>By constant amount C (first deposit at end of year 2 = C)</p>	$P = \frac{(1+i)^n - (1+i \cdot n)}{i^2 \cdot (1+i)^n} \cdot C$	$F = \frac{(1+i)^n - (1+i \cdot n)}{i^2} \cdot C$
<p><b>Equivalent factor for gradient series</b></p>  <p>(first deposit at end of year 2 = C)</p>	$A = \left\{ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right\} \cdot C$	

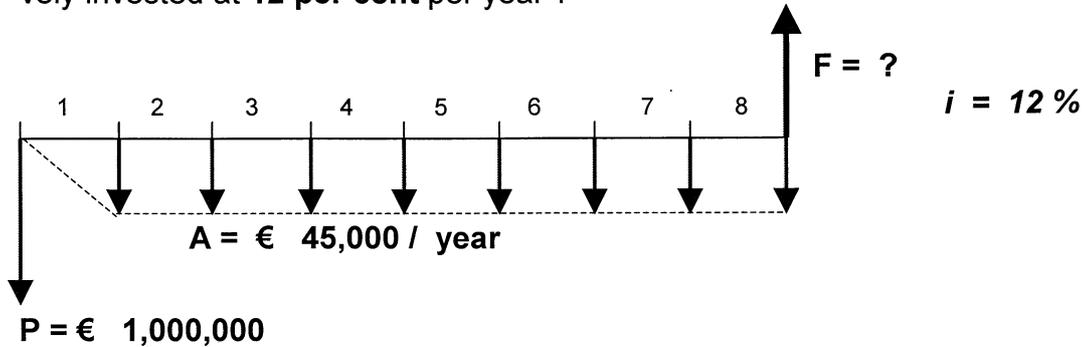
## 2.9. Exercises Interest Calculation

Unless stated otherwise, compounding/ discounting is to be done annually.

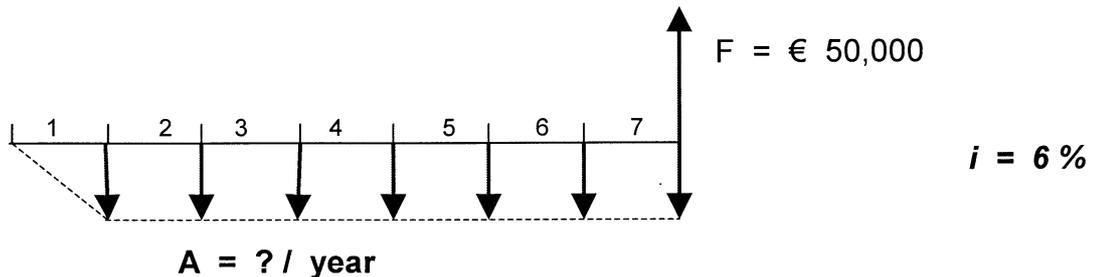
1. You put **(Euro) € 1,000** in a bank account at the end of 2000. The interest rate is **7 %** per year. What amount will you have in your account at the end of 2015 ?
2. What deposit should you make on 1st January 2001, in order to obtain an amount of **€ 100,000** on 31st December 2011? The interest rate is **6 %**.
3. If you make an annual deposit of **€ 2,000** with a bank at the beginning of the year, starting 1st January 2001, what will be the value of all deposits on 31st December 2011, if the interest rate is **6 %** ?
4. If you place **€ 20,000** in a bank account at the end of 2000, yielding **7 %** interest, what equal amounts can you withdraw at the end of every year, starting at the end of the year 2010, and continuing during 5 more years thereafter, so that the account is depleted at the end of 2015 ?
5. What deposit at the end of each month should you make, starting 31st January 2001, in order to obtain an amount of **€ 25,000** on 31st December 2011. Assume an annual interest rate of **6 %** and monthly discounting.
6. How much must a family invest now to provide a lump sum of **€ 1,000** for school fees at the end of each of 6 years, 8 years, and 12 years from now if interest is at **5 per cent**?



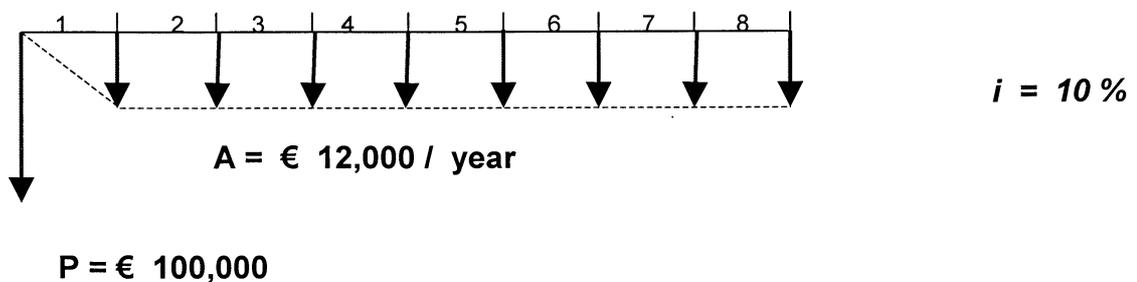
7. A speculator buys a site near the fringe of an industrial area in a large city for € 1,000,000. Annual outgoings on the site for maintenance, fencing, watching, etc., amounts to € 45,000. It is estimated that the site will not be sold for 8 years, at which time the area is due for development. For which minimum price must the site be sold at that time so as to break even on the costs if the original purchase price and the annual outgoings could have been alternatively invested at **12 per cent** per year ?



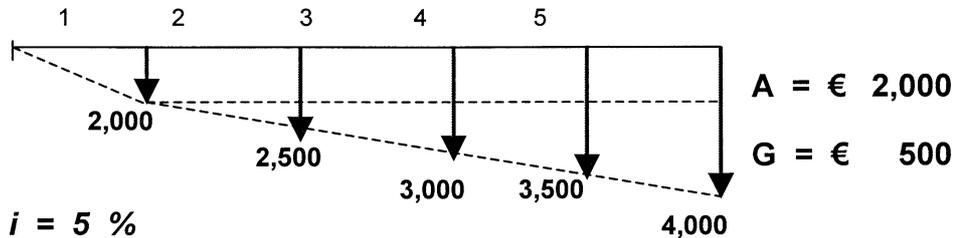
8. A uniform annual investment is to be made into a sinking fund with a view to providing the capital at the end of 7 years for the replacement of a tractor. An interest rate of **6 per cent** is available. What is the annual investment needed to provide for € 50,000 ?



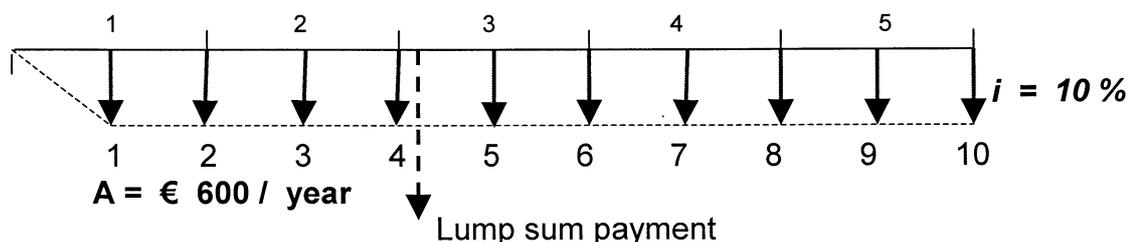
9. A unit of mechanical equipment has an initial cost of € 100,000 and annual maintenance expenditure to average € 12,000 for its 8 years of life. If interest is at **10 per cent** and the equipment has no salvage value, what is its equivalent annual cost, excluding labour, fuels, etc. ?



10. If the maintenance cost of a bulldozer amounts to € 2,000 by the end of the first year of its service, € 2,500 by the end of the second and € 3,000, € 3,500 and € 4,000 by the end of the third, fourth, and fifth year respectively, find the equivalent uniform series cost each year over a period of 5 years. Interest is at **5 percent**.



11. If a mill building is constructed of reinforced concrete (option 1), it will have an estimated initial cost of € 200,000 and no maintenance costs for the first 10 years. A building to serve a similar purpose but erected in structural steelwork and clad in plastic coated metal sheets (option 2) has an initial costs of € 160,000 but the steelwork needs to be painted every 2 years at a cost of € 14,000. With interest at **10 per cent**, which is the cheaper investment considered over the first 10 years of the building's life?
12. If a proposal for the installation of equipment in a factory requires a capital investment now of € 10,000, what saving per year must be shown over the next ten years to justify the expenditure at an interest rate of **5 %** ?
13. A man and wife buy a house and take out a mortgage of € 60,000 to meet part of the cost. They agree to pay off the mortgage over 25 years making monthly payments. Interest on the mortgage is **10.5 per cent** per year. To what will the monthly payment amount? What amount of the original debt of € 60,000 will remain after they made 250 payments ?
14. The purchaser of an automobile is paying for it at the rate of € 600 per half-year, having agreed to make 10 such payments, but after 2 years, when the fourth payment becomed due, decides to make a lump sum payment to settle the account. With an interest rate of **10 per cent**, how much will be needed to do this if there is no rebate of the interest to be charged for the whole of the 5 years?



15. At the **end of 1999**, you have placed **€ 10,000** in a bank account, earning **8 %** interest. What linearly increasing amounts could you withdraw from the account, starting at the **end of 2006** and ending at the **end of 2029**, by which time the account is depleted ? (for instance: first withdrawal € 500.-, next € 1,000.-, next € 1,500.- etc.).

16. You placed at the end of each year in a bank account, with an annual interest rate of **7 %**, an annually linearly increasing deposit:

- at the end of 2000 : € 100.-
- at the end of 2001 : € 200.-
- at the end of 2002 : € 300.- etc.

The **last deposit** is made at the **end of 2015**.

What is the value of all deposits on **1<sup>st</sup> January 2020** ?

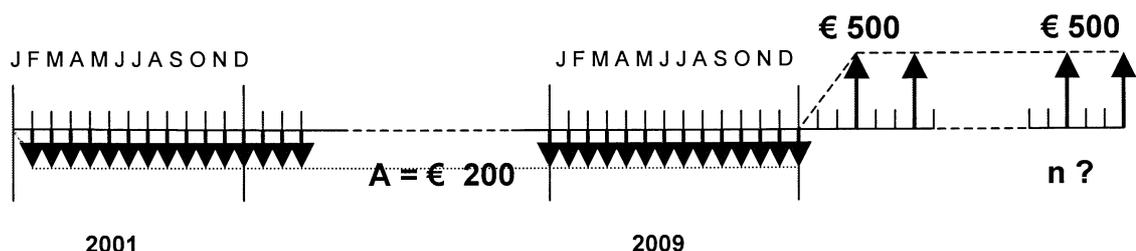
17. How large an amount would you have to place at the **end of 2000** in a bank account, yielding 7 % interest, so that you can make the following withdrawals:

- at the end of 2001 : € 2,000.-
  - at the end of 2003 : € 3,000.-
  - at the end of 2005 : € 4,000.-
  - at the end of 2007 : € 5,000.-
  - at the end of 2009 : € 6,000.-
  - at the end of 2011 : € 7,000.-
- account is depleted.

18. You are making monthly deposits of **€ 200** (at the end of each month), starting 31st January 2001; the last deposit is made 31st December 2009. On 31st March 2010 you start withdrawing money from this account in 3-monthly instalments of **€ 500**.

When can you make your last full withdrawal ?

Assume an annual interest rate of 6 % and monthly discounting.



## 2.10. Answers Exercises Interest Calculations

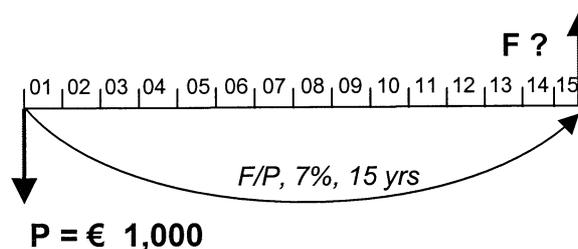
1.  $P = € 1,000$ ,  $i = 7\%$ ,  $n = 15$  years :

compounding factor :

$$(1 + i)^n = (1.07)^{15} = 2.759$$

$€ 1,000$  . (  $F/P$ ,  $7\%$ ,  $15$  years):

$$F = € 1,000 \times 2.759 = \underline{\underline{€ 2,759}}$$



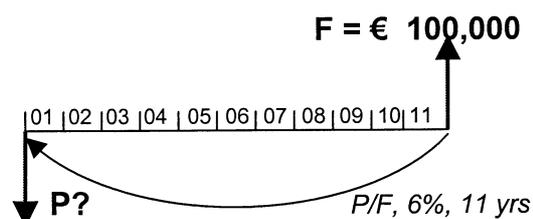
2.  $F = € 100,000$ ,  $i = 6\%$ ,  $n = 11$  years:

discounting factor:

$$\frac{1}{(1 + i)^n} = \frac{1}{(1.06)^{11}} = \frac{1}{1.898} = 0.52679$$

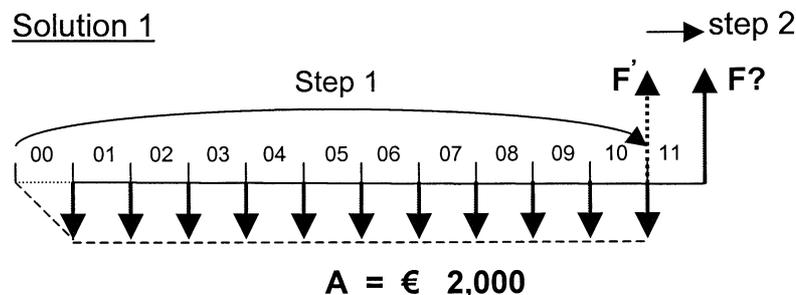
$€ 100,000$  . (  $P/F$ ,  $6\%$ ,  $11$  years):

$$P = € 100,000 \times 0.52679 = \underline{\underline{€ 52,679}}$$



3.  $A = € 2,000$ ,  $i = 6\%$ ,  $n = 11$  years (2000 - 2010) on 1<sup>st</sup> January 2011

Solution 1



$$\text{Compounding factor (on 1<sup>st</sup> January 2011): } \frac{(1 + i)^n - 1}{i} = \frac{(1.06)^{11} - 1}{0.06} = 14.972$$

$$F' \text{ (on 1<sup>st</sup> January 2011)} = 14.972 \times € 2,000 = € 29,944$$

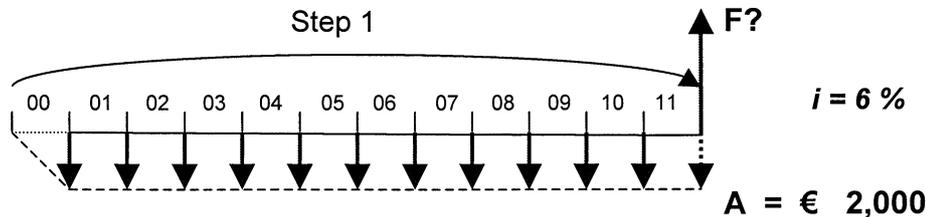
$$F \text{ (on 31 December 2011)} = 1.06 \times € 29,944 = \underline{\underline{€ 31,740}}$$

$$F = € 2,000 \cdot (F/A, 6\%, 11 \text{ years}) \cdot (F/P, 6\%, 1 \text{ year}) = € 2,000 \cdot (14.972) \cdot (1.06) = \underline{\underline{€ 31,740}}$$

**Solution 2**

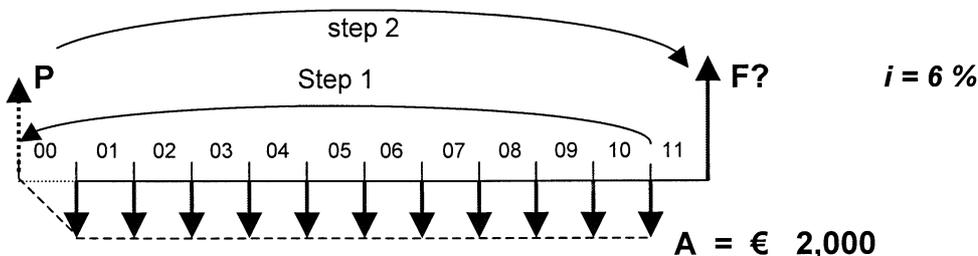
Calculate F on 1<sup>st</sup> January 2012 and deduct the annual deposit of € 2,000

$$F = € 2,000 \cdot (F/A, 6\%, 12 \text{ years}) - € 2,000 = € 2,000 \cdot [16.8699 - 1] = € 2,000 \cdot (15.8699) = \underline{\underline{€ 31,740}}$$



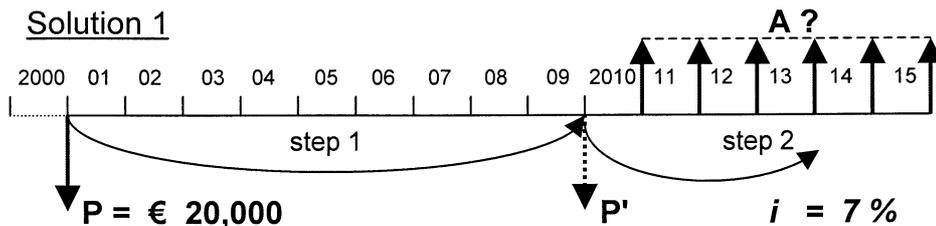
**Solution 3**

Calculate the present value P at the beginning of year 2000 and compound this to the F value at the beginning of year 2012 (which is the same as the end of year 2011).



$$F = € 2,000 \cdot (P/A, 6\%, 11 \text{ years}) \cdot (F/P, 6\%, 12 \text{ years}) = € 2,000 \cdot (7.8867) \cdot (2.0122) = \underline{\underline{€ 31,739}}$$

4. **Solution 1**



$P = € 20,000$ ,  $i = 7\%$ ,  $n = 9$  years (end of 2009) !!!,

F at end of 2009 in order to use the formula for annuity.

$$F = 1.07^9 \times € 20,000 = 1.8385 \times € 20,000 = € 36,769$$

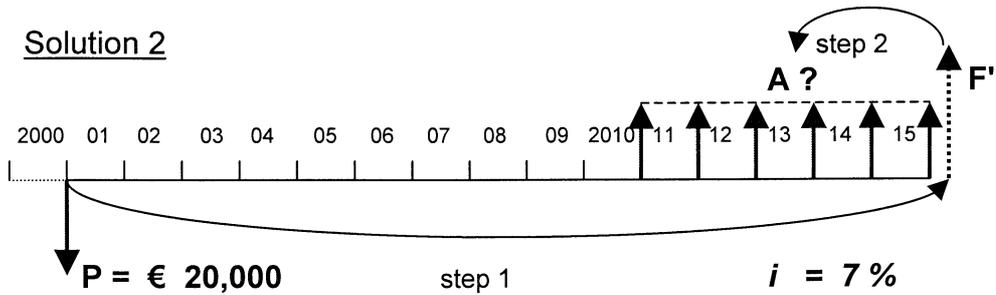
$i = 7\%$ ,  $n = 6$  years (see definition formula annuity)

$$\text{Annuity factor: } \frac{0.07 (1 + 0.07)^6}{(1.07)^6 - 1} = \frac{0.07 \times 1.5007}{1.5007 - 1} = \frac{0.105}{0.5007} = 0.2097$$

$$A = 0.2097 \times € 36,769 \longrightarrow A = \underline{\underline{€ 7,711}}$$

$$A = € 20,000 \cdot (F/P, 7\%, 9 \text{ years}) \cdot (A/P, 7\%, 6 \text{ years}) = € 20,000 \cdot 1.8385 \cdot 0.2098 = \underline{\underline{€ 7,714}}$$

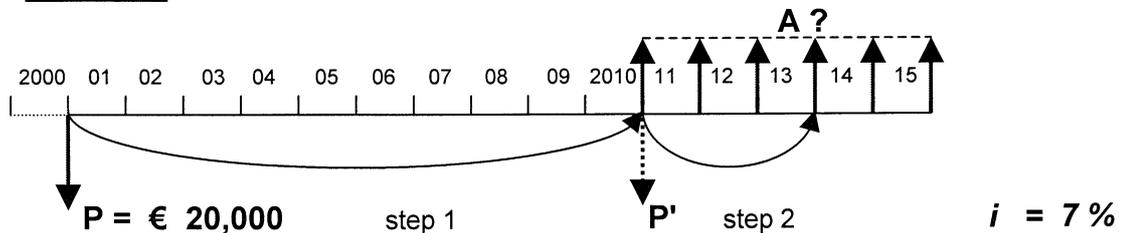
### Solution 2



Calculate F at end of 2015 and use formula for sinking fund factor to find the value for A.

$$A = € 20,000 \cdot (F/P, 7\%, 15 \text{ years}) \cdot (A/F, 7\%, 6 \text{ years}) = € 20,000 \cdot (2.7590) \cdot (0.1398) = \underline{\underline{€ 7,714}}$$

### Solution 3



$$A = [ € 20,000 \cdot (F/P, 7\%, 10 \text{ years}) - A ] \cdot (A/P, 7\%, 5 \text{ years}) = [ € 20,000 \cdot (1.097) - A ] \cdot (0.2439) = 1.2439 A = € 20,000 \cdot (1.097) \cdot (0.2439) \longrightarrow \underline{\underline{A = € 7,714}}$$

5.  $F_{(\text{end of 2011})} = € 25,000$ ,  $i = 0.5\%$  per month (see note),  $n = 12 \text{ month/year} \times 11 \text{ years (2001 - 2011)} = 132$  ( $n = \text{total number of monthly deposits}$ )

$$\text{compounding factor} = \frac{(1+i)^n - 1}{i} = \frac{(1.005)^{132} - 1}{0.005} = \frac{0.9316}{0.005} = 186.32$$

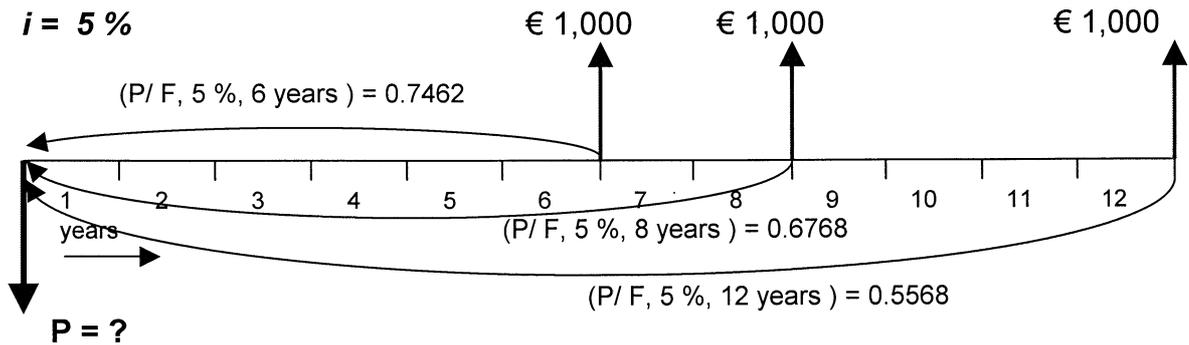
$$A = \frac{€ 25,000}{186.32} = \underline{\underline{€ 134.18}}$$

$$A = € 25,000 \cdot (A/F, 0.005\%, 132 \text{ months}) = € 25,000 (0.005367) = \underline{\underline{€ 134.18}}$$

Note: By dividing the annual interest rate of 6% by 12 months to get the monthly interest rate a small mistake is made as  $(1.005)^{12} = 1.0617 \approx 1.06$ . The actual monthly discount rate is therefore slightly less than 0.005 (0.004862). Check  $(1.004862)^{12} = 1.06$ . Calculating with this monthly interest rate of 0.004862% gives slightly different figures.

$$A = € 25,000 \cdot (A/F, 0.004862\%, 132 \text{ months}) = € 25,000 \cdot (0.00541) = \underline{\underline{€ 135.31}}$$

6.



Present value (P) of € 1,000 in n years' time =  $1,000 (P/F, 5\%, n \text{ years})$

Present value (P) of € 1,000 in 6 years' time =

$$€ 1,000 \cdot (P/F, 5\%, 6) = 1,000 \cdot (0.7462) = € 746.20$$

Present value (P) of € 1,000 in 8 years' time =

$$€ 1,000 \cdot (P/F, 5\%, 8) = 1,000 \cdot (0.6768) = € 676.80$$

Present value (P) of € 1,000 in 12 years' time =

$$€ 1,000 \cdot (P/F, 5\%, 12) = 1,000 \cdot (0.5568) = € 556.80$$

Total present value to be invested now : **€ 1,979.80**

or:

$$\frac{1,000}{1.05^6} + \frac{1,000}{1.05^8} + \frac{1,000}{1.05^{10}} = \frac{1,000}{1.340} + \frac{1,000}{1.4775} + \frac{1,000}{1.796} = 746 + 677 + 557 = 1,980$$

7. Future value of capital sum =

$$€ 1,000,000 \cdot (F/P, 12\%, 8) = 1,000,000 \cdot (2.4760) = € 2,476,000$$

Future value of annual costs =

$$€ 45,000 \cdot (F/A, 12\%, 8) = 45,000 \cdot (12.2997) = € 553,487$$

Minimum selling price for site in 8 years' time **€ 3,029,487**

8.  $A = € 50,000 \cdot (A/F, 6\%, 7) = \frac{i}{(1+i)^n - 1} \cdot € 50,000 =$

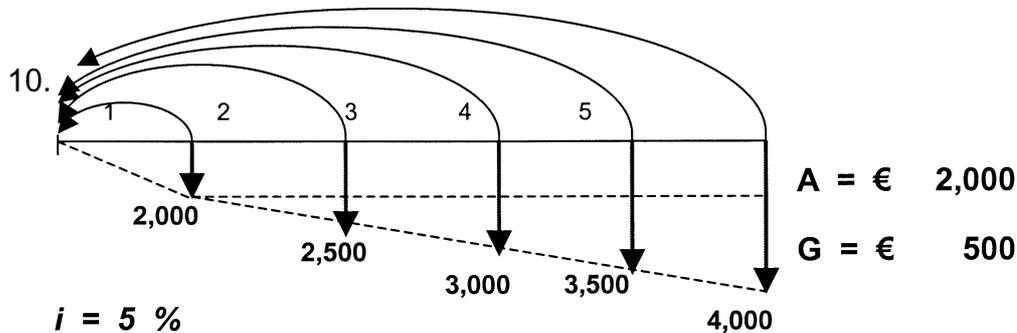
$$\frac{0.06}{1.06^7 - 1} \cdot € 50,000 = 0.1191 \cdot € 50,000 = € 5,955 \text{ per year}$$

9. To convert the capital sum to an equivalent uniform annual series, use the annuity formula  $A = P (A/P, i\%, n \text{ years})$ .

$$A = € 100,000 \cdot (A/P, 10\%, 8) = € 100,000 \cdot (0.1874) = € 18,740$$

Maintenance expenditure is already at annual cost: € 12,000

Total equivalent annual cost : **€ 30,740**



Annual equivalent of increment =  $G \cdot (A/G, i\%, n \text{ years})$   
 arithmetic gradient conversion factor

$$G \cdot (A/G, 5\%, 5) = G \cdot \left[ \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right] = G \cdot \left[ \frac{1}{0.05} - \frac{5}{1.05^5 - 1} \right] =$$

$$€ 500 \cdot \left[ 20 - \frac{5}{0.2763} \right] = € 500 \cdot (20 - 18.096) = € 500 \cdot (1.904) = \underline{\underline{€ 952}}$$

Therefore, uniform series equivalent annual cost of maintenance :

$$= € 2,000 + € 952 = \underline{\underline{€ 2,952}} \text{ for each of five years.}$$

Alternative solution:

Discount the various maintenance costs to  $t = 0$  (Present value)

$$\text{Total P.V.} = \frac{2,000}{1.05} + \frac{2,500}{1.05^2} + \frac{3,000}{1.05^3} + \frac{3,500}{1.05^4} + \frac{4,000}{1.05^5} =$$

$$\frac{2,000}{1.05} + \frac{2,500}{1.1025} + \frac{3,000}{1.1576} + \frac{3,500}{1.2155} + \frac{4,000}{1.2763} =$$

$$1,905 + 2,267.57 + 2,591.56 + 2,879.47 + 3,134.06 = € 12,776.66$$

$$\text{annuity } (A/P, 5\%, 5 \text{ years}): \frac{0.05 \times 1.05^5}{1.05^5 - 1} = \frac{0.05 \times 1.2763}{1.2763 - 1} = \frac{0.0638}{0.2763} = 0.23096$$

$$\text{annual equivalent value: } 0.23096 \cdot € 12,776.66 = \underline{\underline{€ 2,950.93}}$$

11. Option 1 (reinforced concrete):

Present value all costs: € 200,000

Option 2 (structural steelwork):

Present value (P) all costs : € 160,000 + present value of maintenance costs (discounting maintenance costs);

Present value of initial costs		€ 160,000
PV of maintenance costs after 2 years:	$0.8264 \times 14,000 =$	€ 11,570
PV of maintenance costs after 4 years:	$0.6830 \times 14,000 =$	€ 9,562
PV of maintenance costs after 6 years:	$0.5645 \times 14,000 =$	€ 7,902
PV of maintenance costs after 8 years:	$0.4665 \times 14,000 =$	€ 6,531
Total Present Value :		€ 195,565
say:		<u>€ 195,500</u>

Conclusion: Option 2 is cheaper

Note: maintenance costs of option 2 after 10 years is not included in the comparison.

$$€ 160,000 + \frac{14,000}{1.10^2} + \frac{14,000}{1.10^4} + \frac{14,000}{1.10^6} + \frac{14,000}{1.10^8} =$$

$$€ 160,000 + \frac{14,000}{1.21} + \frac{14,000}{1.4641} + \frac{14,000}{1.7716} + \frac{14,000}{2.1435} =$$

$$€ 160,000 + € 11,570 + € 9,562 + € 7,903 + € 6,531 = \underline{\underline{€ 195,566}}$$

	Construction cost	Maintenance cost	Present Value of total cost
<u>Option 1</u> Concrete	€ 200,000	-	€ 200,000
<u>Option 2</u> Steelwork	€ 160,000	€ 14,000 after each 2 years. Present Value: € 35,500	€ 195,500

12. The annuity of  $i = 5\%$  and  $n = 10$  years:  $\frac{0.05 \cdot (1.05)^{10}}{(1.05)^{10} - 1} = 0.1295$

$$A = € 10,000 \cdot (A/P, 5\%, 10 \text{ years}) = € 10,000 \cdot (0.1295)$$

Minimal annual saving to justify the capital investment: **€ 1,295**

13.  $n = 25 \text{ years} \times 12 \text{ months}$  is 300;  $i$  (monthly) = approx.  $\frac{10.5}{12} = 0.875\%$

$$\text{Annuity: } \frac{0.00875 (1.00875)^{300}}{(1.00875)^{300} - 1} = 9.4418 \cdot 10^{-3}$$

$$\text{Monthly payment: } € 60,000 \cdot (9.4418 \cdot 10^{-3}) = \underline{\underline{€ 566.50}}$$

Present value  $P$  of 250 monthly payments  $A$  (of € 566.50)

$$P = € 566.50 \cdot (P/A, 0.875\%, 250) = 566.50 \cdot (101.34) = € 57,409$$

$$\text{Difference (at } t=0) € 60,000 - € 57,409 = € 2,591$$

Future value of this difference after 250 payments:

$$F = € 2,591 \cdot (F/P, 0.875\%, 250) = € 2,591 \cdot (8.8285) = \underline{\underline{€ 22,875}}$$

Alternative solution: Present value of last 50 monthly payments

$$P = € 566.50 \cdot (P/A, 0.875\%, 50) = € 566.50 \cdot (40.357) = \underline{\underline{€ 22,862}}$$

14. Interest rate per half year :  $i = 5\%$  (approx.);

monthly payments : € 600,  $n = 10$

Present value of outstanding payments at  $t = 2$  years ( $n = 6$ )

$$P = € 600 \cdot (P/A, 5\%, 6) = € 600 \cdot (5.0757) = € 3,045$$

Total payment:

fourth payment (€ 600) +  $P$  (at  $t = 2$  years) =

$$€ 600 + € 3,045 = \underline{\underline{€ 3,645}}$$

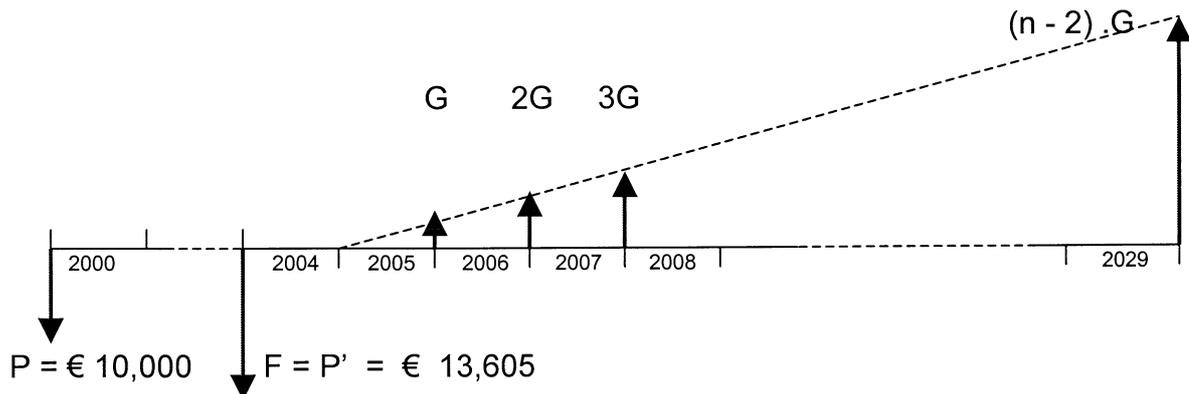
15. Future value  $F$  of  $P$  (end 1999) € 10,000 at the end of 2003 !!

$$F = € 10,000 \cdot (F/P, 8\%, 4 \text{ years}) = € 10,000 \cdot 1.3605 = \underline{\underline{€ 13,605}}$$

$F = P'$

Formula for Present Value ( $P$ ) of an annual linearly increasing amount  $G$

First deposit at end of year 2 !

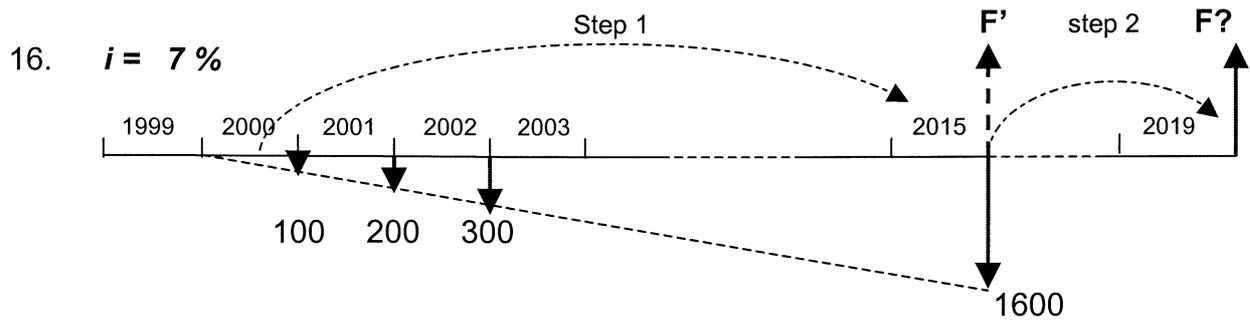


$$\text{discounting factor : } \frac{(1+i)^n - (1+i \cdot n)}{i^2 \cdot (1+i)^n}$$

$i = 8\%$ ,  $n = 26$  years, discounting factor :

$$\frac{1.08^{26} - (1 + 0.08 \cdot 26)}{0.08^2 \cdot 1.08^{26}} = \frac{4.3164}{0.0473} = 91.165$$

$$P' = 91.165 G \longrightarrow G = € 13,605 / 91.165 \longrightarrow \underline{\underline{G = € 149.20}}$$



Future value of annual increasing constant amount  $G$

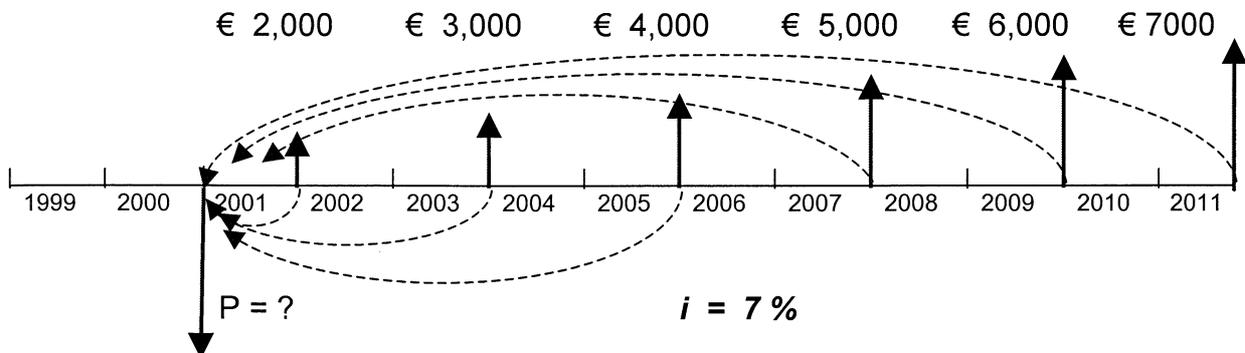
$$\text{compounding factor} : \frac{(1+i)^n - (1+i \cdot n)}{i^2}$$

For  $i = 7\%$ ,  $n = 17$  years, compounding factor : 197.71

$$F_{\text{end } 2015} = G \cdot 197.71 = \text{€ } 100 \cdot (197.71) = \text{€ } \underline{19,771}$$

$$F_{\text{end } 2019} = F_{\text{end } 2015} \cdot (1.07)^4 = \text{€ } 19,771 \cdot (1.3108) = \text{€ } \underline{25,916}$$

17. Solution 1

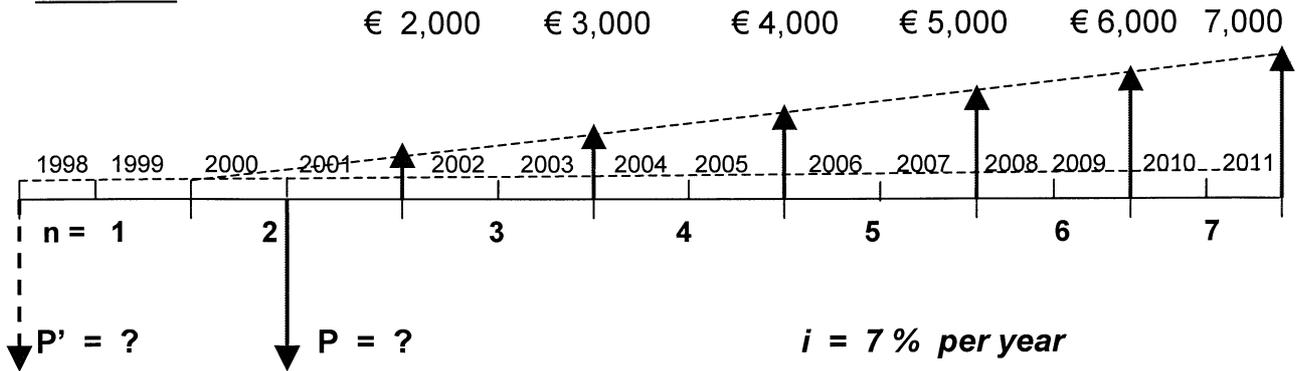


Present value  $P_{\text{end } 2000}$  :

$$\frac{2,000}{1.07} + \frac{3,000}{1.07^3} + \frac{4,000}{1.07^5} + \frac{5,000}{1.07^7} + \frac{6,000}{1.07^9} + \frac{7,000}{1.07^{11}} =$$

$$1,869 + 2,449 + 2,852 + 3,114 + 3,264 + 3,326 = \text{€ } \underline{16,873}$$

Solution 2



Periods of 2 years;  $1 + i = 1.07^2 = 1.1449$ ,  $i = 14.49\%$

Annual constant € 1,000 + annual constant increase of € 1,000

Present value at end of 1997 of annual constant € 1,000

( $n = 7$  periods of 2-years)

$$P'_{\text{end 1997}} = € 1,000 \cdot (P/A, 14.49\%, 7) - € 1,000 / 1.1449$$

$$= € 1,000 \cdot (4.2300) - € 873.44 = \underline{\underline{€ 3,352}}$$

Present value at end of 1997 of annual constant increase of € 1,000

$$P'_{\text{end 1997}} = € 1,000 \cdot (P/G, 14.49\%, 7) = € 1,000 \cdot 10.42 = \underline{\underline{€ 10,420}}$$

Future value at end of 2000 :  $(€ 3,352 + € 10,420) \cdot (F/P, 7\%, 3 \text{ years}) =$

$$(€ 3,352 + € 10,420) \cdot 1.07^3 = € 13,772 \cdot 1.225 = \underline{\underline{€ 16,871}}$$

18.  $A = € 200$ ,  $i = 0.5 \%$  per month ( approx. )  
 $n = 12 \text{ month/ year } \times 9 \text{ years } (2001 - 2009) = 108$  interest bearing periods  
 (  $n =$  total number of monthly deposits)

$$\text{compounding factor} = \frac{(1+i)^n - 1}{i} = \frac{(1.005)^{108} - 1}{0.005} = \frac{0.7137}{0.005} = 142.7$$

step 1  $F_{\text{(at the end of 2009)}} = (F/A, 0.005 \%, 108 \text{ months}) =$   
 $€ 200 \cdot 142.7 = € 28,548$  (becomes P)

step 2  $P_{\text{(at the end of 2009)}} = € 28,548$ ,  $i = 1.5 \%$  per 3 month (approx.),  
 $A = € 500$  per 3 months,  $n = ?$

Formula discounting factor constant deposits (every 3-months):

$$P = \frac{(1+i)^n - 1}{i \times (1+i)^n} \times A = \frac{(1.015)^n - 1}{0.015 \times (1.015)^n} \times € 500 = \underline{\underline{€ 28,548}}$$

**$n = ?$**

$$\frac{(1.015)^n - 1}{(1.015)^n} = 0.015 \times € 28,548 / € 500 = 0.85644$$

$$(1.015)^n - 1 = 0.85644 \times (1.015)^n$$

$$0.14356 \times (1.015)^n = 1$$

$$(1.015)^n = 6.9657 \longrightarrow \underline{\underline{n = 130.4}} \text{ (130 full withdrawals)}$$

or 32.5 years: last withdrawal: **30<sup>th</sup> June 2042**

### 3. Cost estimating of Construction Equipment

#### 3.1. Introduction

An internal cost estimate made by a contractor will form the base for a (price) quotation for a construction project. As the works usually involves construction equipment the weekly operating costs of this equipment - **direct costs** - needs to be calculated. Important elements of the operating costs are depreciation and interest.

#### Type of operating costs:

- Depreciation
  - Interest
  - Maintenance, Overhaul
  - Repair
  - Salaries & wages  
(including costs for leave, food, travel and accommodation)
  - Fuel & lubrication
  - Insurance & franchise
  - Consumables/ stores
  - Administration costs
  - Particular costs (certificates, licence costs etc.)
- } D + I
- } M + R

#### 3.2. Depreciation & interest (D + I)

Large construction equipment and dredging equipment in particular can be very expensive. In the VGBouw "Operating Cost Standards for Construction Equipment", standard values S are mentioned for various types of construction equipment, determined from technical and statistical data valid for The Netherlands (11th revised edition, 1995). See page 44 - 50.

Most companies do not have sufficient own capital to buy such expensive equipment and depends on external financing. The company has to make an investment: money is allocated to a piece of equipment.

The money, required for the purchase of equipment can be obtained as follows (external financing):

- money is borrowed from a bank, which will charge **interest**.
- money is borrowed with the (construction) equipment as security; this is called **mortgage** (money is raised on mortgage); the money-lender receives **mortgage interest**.
- the company can also obtain money by the issue of shares (technical term 'Equity'). The shareholders expect to receive **dividend**.

Borrowing		Equity
Interest	Mortgage interest	Dividend
- interest is percentage of the amount borrowed	- interest is percentage of the amount borrowed	- dividend is coupled to the (financial) results of the total company
- loan may be coupled to a security	- loan is coupled to a security	- is not bound at a security
- redemption arrangement (schedule) (depreciation)	- redemption arrangement (schedule) (depreciation)	- no redemption arrangement

For the determination of **Depreciation and interest** (D + I) the method of borrowing is the easiest to understand. In case a loan is not paid off the annual (or monthly) payable interest (at constant interest percentage) will be a fixed amount.

However the service life (life time, n years) of construction equipment is limited, say 10 years. After these 10 years the equipment will be worn out and the value reduced to 0 or to a residual value (for example 5% of the purchase value). The equipment concerned will be scrapped (or sold) and replaced by a new piece of equipment.

If the market and the technique will not change the same amount of money is needed for this new purchase. One can argue that this amount of money can also be obtained by making a yearly reservation.

The most common reasoning is that the required money is obtained by regularly (once a year or three-monthly) setting aside an amount (to reserve) for the new equipment. One can argue that this setting aside is used for the paying off (redemption) of the loan needed for the existing equipment. This is also called depreciation on machinery.

The depreciation is dependent on:

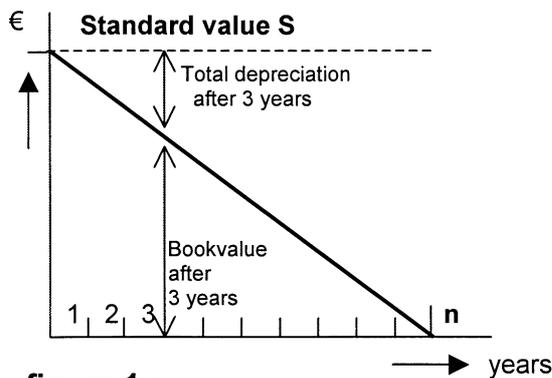
- purchase value (or standard value S)
- life time/ service life n (or: duration of utilisation/ utilisation period)
- residual value (Z).

To understand the concept of **residual value** one can think of a private car, that is bought for € 30,000. - and after six years is sold for € 3,000. -. The € 3,000. - is the residual value, which is received when the car is sold. The remaining € 30,000. - - € 3,000. - = € 27,000.- has to be gradually depreciated over six years (or: to be reserved) to form together with the residual value the € 30,000. -, for which the new car can be bought (procured). Note: in the example of the car only depreciation is being considered, the interest is not yet taken into account; the transformation to present day value of the residual value by discounting (see previous chapter) is therefore not (yet) relevant. The residual value Z at the end of the service life n is usually expressed in % of the purchase or standard value S.

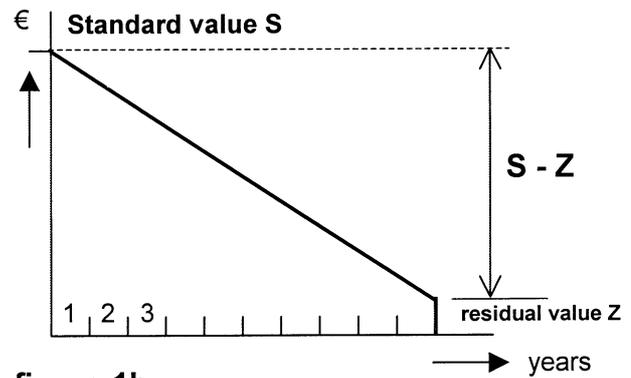
## Linear depreciation

The annual depreciation is determined using the equation:

$$\text{depreciation} = \frac{\text{standard value} - \text{residual value}}{\text{service life}} \quad \text{or} \quad D = \frac{S - Z}{n}$$



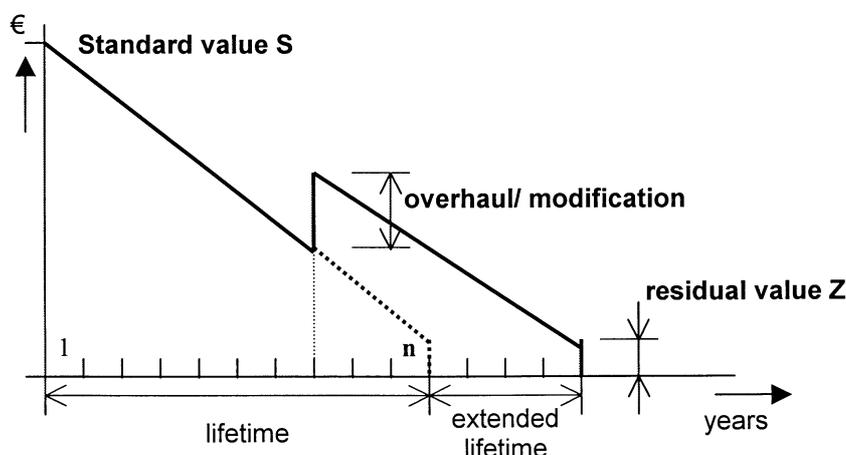
**figure 1a.**  
The most simple method of depreciation: linear from the purchase price  $S$  to 0.



**figure 1b.**  
The depreciation is based on purchase (standard) value  $S$  minus residual value  $Z$ ; and is again linear.

This reasoning is disturbed by the following factors:

- It is not always easy to realize the "residual value"  $Z$  (can the old equipment be sold or scrapped at the supposed residual value ?).
- During the lifetime inflation will occur: the purchase of the same piece of construction equipment 10 years later will be considerably dearer.
- During the utilisation period (lifetime/ service life) of the equipment modifications, improvements or major renovations are carried out. Thus extra investments, which have to be depreciated in a shorter time (the remainder of the lifetime). It is also possible (likely) that these extra investments, such as large hull-(casco-) overhaul for floating equipment, will extend the lifetime of the equipment (figure 2: adjustment of the service life).

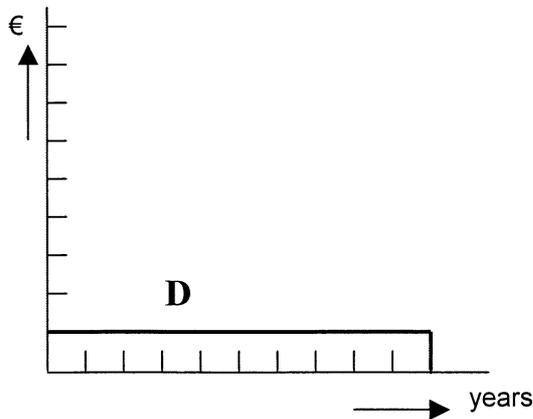


**figure 2.** Value with overhaul/ modification+ extended lifetime.

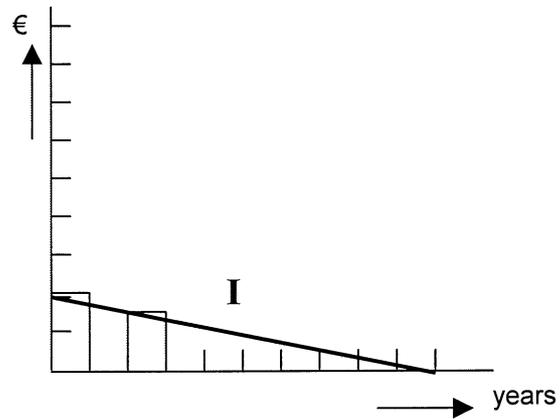
## Interest

The (average) annual interest is given by:

$$\text{interest} = \frac{\text{standard value} + \text{residual value}}{2} \cdot \text{interest rate} \text{ or } \frac{S + Z}{2} \cdot \frac{i}{100}$$



**figure 3.**  
Annual depreciation sum based on linear depreciation (see figure 1a).



**figure 4**  
The annual payable interest for linear depreciation and a residual value of 0.

The interest is paid over the amount borrowed. This amount (or bookvalue) is becoming smaller in time, as repayments (depreciation) are made. Graphically the payable interest is shown in figure 4. With the hatched rectangles is tried to show that the interest to be paid is higher if the depreciation takes place once a year (annually), than monthly (in that case the average interest will follow the straight line closer).

The total costs of **Depreciation and interest (D + I)** together are shown in figure 5; (D + I) are an important part of the **fixed costs**. It is clear that these fixed costs (D + I) are initially higher and later lower than the average. A new piece of construction equipment has therefore high fixed costs and can compete less well (for linear repayment the total amount of D + I during the first years is very high). For that reason the **annuity** method is often applied. This method is based on annually constant costs - thus a horizontal line for D + I (see figure 6). An annuity is a constant amount per year that is made up of depreciation and interest together. The formula for annuity has been discussed in the previous chapter.

The VGBouw Operating Cost Standard gives the following formula for  $A_n$  (annuity):

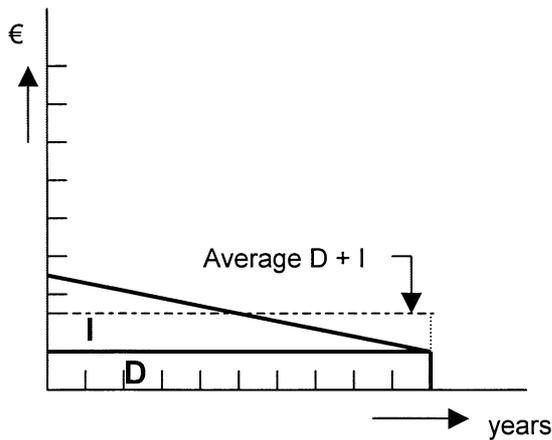
$$A_n = p^n \cdot \frac{p-1}{p^n - 1} \cdot 100$$

$A_n$  = annuity = annual depreciation + interest expressed as a percentage of the standard value S.

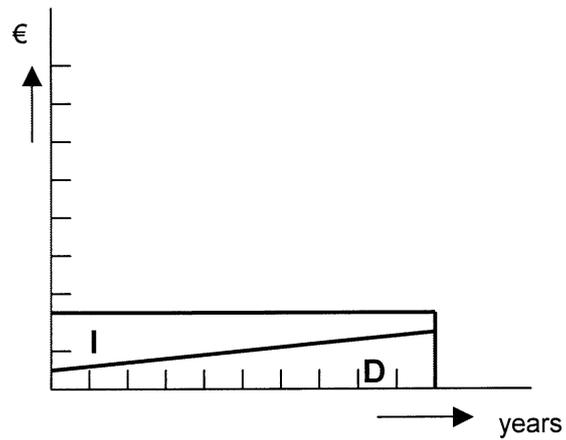
$p$  =  $1 + i / 100$  (for  $i = 7\%$ ,  $p = 1.07$ )

$n$  = service life or utilisation period in years

$i$  = rate of interest (% / year).



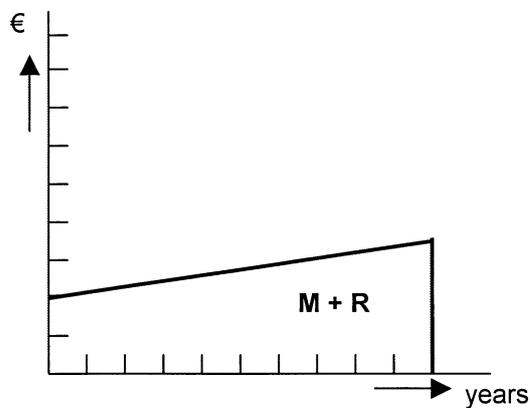
**figure 5**  
*Depreciation and interest ( $D + I$ ) for linear depreciation (fig. 3 + fig. 4).*



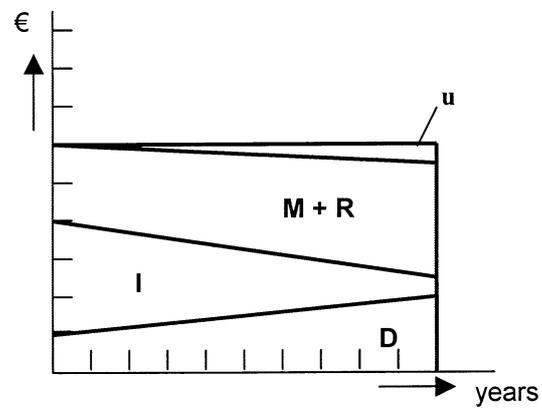
**Figure 6**  
*Depreciation and interest ( $D + I$ ) according the annuity method, whereby ( $D + I$ ) are constant.*

In first instance this seems like a good solution for a uniform cost distribution, however there are some drawbacks:

- older equipment has higher costs for maintenance and repairs ( $M + R$ ) than new equipment (figure 7).
- The degree of utilisation (utilisation  $u$  in weeks/ year) of the new equipment (modern, higher production, more simple operation) will be higher, as its demand will be higher. During the first years of the service life the degree of utilisation will have a lowering effect on the weekly  $D + I$ .



**Figure 7.**  
*Maintenance + Repair ( $M + R$ ) during the service life.*

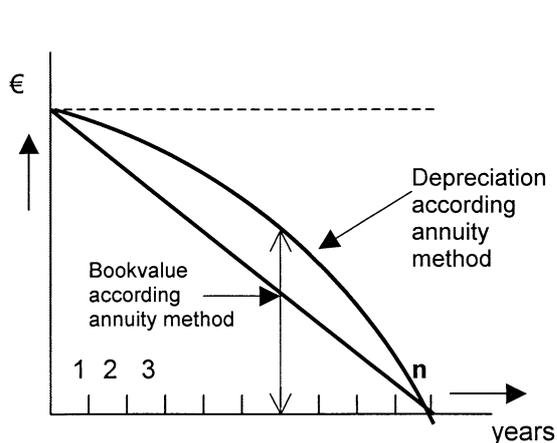


**Figure 8.**  
*Four components; sum is constant.*

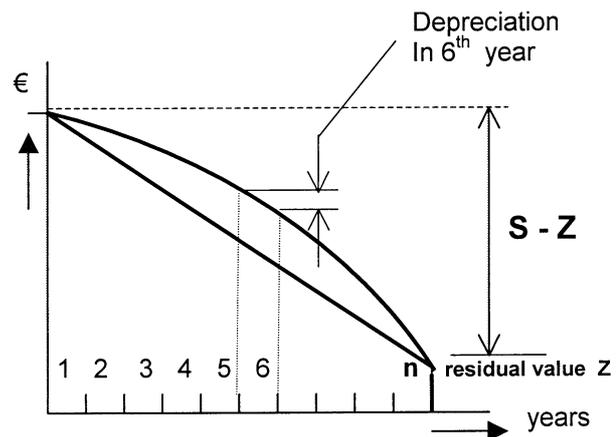
In figure 8 these influences are shown, whereby it appears, that in case of constant annual fixed costs, the linear depreciation method is more or less approximated. It will by now be clear that each owner of construction equipment (contractor) will have its own policy for the distribution of fixed costs. Linear depreciation and depreciation according the annuity method are only two of many possibilities of depreciation. The depreciation policy might even be different for the various pieces of equipment.

However, the following rules do always apply:

- it is not possible to depreciate more than the total investment.
- the amount of depreciation is booked as operating costs in the administration of the company. For high depreciation in the first years the – **taxable!** – profit will be lower. But during the remaining years the depreciation will be lower, which might mean a higher profit and consequently a higher tax (revenue tax) to be paid.
- for the import of equipment in another country the declared value, needed for the determination of the import duties, can be influenced by the allowed amounts of depreciation (for that country).



**figure 9.**  
Linear depreciation and depreciation according annuity method from purchase price  $S$  to  $0$ .



**figure 10.**  
Linear depreciation and depreciation according annuity method based on purchase value  $S$  minus residual value  $Z$

**Fiscal depreciation.** This is the amount of depreciation accepted by the tax collector. These amounts will have no relation with the degree of utilisation in that year (100 % utilized or idle for the whole year).

**Economical depreciation.** The amount of depreciation is the amount that can be obtained by a piece of equipment by its utilization and the associated “hire”.

**Purchase value, replacement value.** In the above a certain value was assumed for the equipment. In particular the dredging industry the determination of the value might be a difficult task, especially by its special character: wear, no standard product. It is possible to use the purchase value (construction costs), but it will be clear that after for example 20 years it is not possible to have an identical dredger constructed for the same costs. Therefore the amounts to be depreciated should be higher in order to have an identical dredger constructed in the future. But how much will the construction costs be in the future?

To solve this problem various methods can be applied, such as the method of **depreciation on replacement value**. This means that each year a new value is determined, which corresponds with the amount needed to acquire a similar piece of equipment. The annual depreciation increases with the same percentage as the price escalation of the equipment.

This is also not easy as the base for comparison for the main equipment (dredgers) is difficult to find. For less costly construction equipment, such as bulldozers, wheel-loaders, mobile cranes etc. this is less complicated. The project manager/ cost estimator should recognize the different aspects of depreciation. It is of course the responsibility of the financial experts of the company to establish and work with these principles and to introduce them administratively.

### **Determination of D + I.**

The determination of the costs for D + I is done in different methods:

- **Own company (internally).**

Internally in the **own company** for all pieces of equipment and installations values are established which form the basis for the determination of these costs.

Sometimes every year tables or lists are produced, so-called "**list of rental tariffs**".

The amount D + I is sometimes called the "**hire**". This is however a rather vague concept and has to be better described for firm agreements or contracts as between various companies and for different circumstances the concept of "hire" can vary widely.

One should think hereby at costs for:

- long-term maintenance (overhaul),
- overheads, administration, storage,
- testing and inspections; certificates
- wear,
- docking costs,
- insurance.

These extra costs, or parts of it might be included in the "hire" rate. It is also possible that in the own company different considerations are developed with regard to the costs of D + I, in relation to the concept of 'fixed costs'. If the concept is explained as follows:

"If the equipment is not working these costs have to be paid anyway + a lot more" it takes a small jump in thoughts to the following statement:

"by not including part of the costs of D + I in the cost estimate the chance of awarding the contract will raise considerably; if awarded the other part will be paid and a lot more". Leaving it in the middle whether this reasoning is healthy for the company, it is mentioned here in order to prepare the project manager/ cost estimator for this philosophy as he might be confronted with it.

- **Between contractors** (combination or joint-venture)

Between various companies the item D + I, or hire, is important as it is needed for the estimate on which the tender is based and for the payment of the "hire" between the joint-venture and the owner of the equipment in case the contract is been awarded.

Again the warning is repeated that the concept of "hire" should be well defined.

The "hire" is relevant in the following situations:

- hire of one or more pieces of equipment by one company to another company
- making available certain equipment within a group of companies, who together have formed a joint-venture
- hire problems (for example during idle time, or more work) by sub-contracting.

### Literature.

Literature exists, where-in values are indicated for the amounts for D + I, which can be calculated for the various categories and capacities of equipment. During discussions between contractor and client these sources are often used as basis (starting-point) for their discussion. It is also possible that the contractor already during the tendering is requested to mention these values.

### VGBouw “Operating Cost Standards for Construction Equipment”.

Publication of the Dutch Federation of Major Contractors, VGBouw. An important source of information is the above mentioned “VGB-book” (English translation included). In this book a wide selection of dredging equipment can be found as well (section 9). The standard values S are listed in tables under one or more data characteristics of the equipment. The standards, which are given, are:

- standard value S in Dutch guilders
- a standard rate of depreciation and interest (D + I)
- a standard rate of maintenance and repairs (M + R)

Other characteristic features of this equipment, such as production capacity and operating coefficient, fall outside the scope of this VGB standard. In separate tables utilisation time (service life) and utilisation degree of construction equipment are mentioned. In the chapter II. “Instructions for use of section 9 – Dredging Equipment” some important rules are given for the calculation of the standard values for cutter suction dredgers and trailing suction hopper dredgers.

These VGB-standards are used in two ways:

- for the determination of the **absolute** standard value for those who have little experience;
- for the **comparison** of standards for D + I and M + R, for instance in case of joint-ventures etc. The VGB-standard provides indication for the determination of standard values of the dredging equipment of the various partners, which are used as basis for the calculation of amounts for D + I and M + R, often by applying a coefficient (usually < 1).

The most important column in this VGB-standard is the standard value (S); these values are the basis of the calculation of the amounts for D + I and M + R (see paragraph 3.7).

### Baugeräte Liste.(Germany)

The “Baugeräte Liste”, is issued by the “Hauptverband der Deutschen Bauindustrie” (Germany), and can be considered as a comprehensive “VGB- standard”:

- the number of listed machines or equipment is very extensive;
- within the categories more attention is paid to the description of the equipment, however this is only true for “dry” equipment; dredging equipment is only marginally treated.

### Hire

the English hire-rates also includes M + R, fuel + lubricants and insurance premiums.

### **Influences.**

It is not correct to apply the figures mentioned in the above standards directly. There are a number of factors which might require an adaptation of these standards, such as:

- Climatic conditions,
- Multiple shift operation (double shift/ continuous),
- Older (outdated), or more modern equipment than meant in the standards,
- Deviating degree of utilisation.

Usually the figures from the literature are used but with applying of **coefficients**, in order to compensate for these deviating circumstances. Except for a few categories of plant, the standard rates of depreciation and interest and of maintenance and repairs are based on operating the plant in a single shift per working week. For overtime and multiple shift charges a coefficient should be applied.

### **Service life (utilisation period)**

The estimated service life is as important as the standard value for the determination of the annual depreciation. One distinguishes:

- a. **technical** lifetime: the period during which the piece of equipment will be technical capable to make the required productions.
- b. **economical** lifetime: the period during which on the basis of cost-benefit considerations (economic reasons) the piece of equipment is efficient.

At the end of the lifetime (service life) it becomes inefficient to continue to keep the equipment operational, either for economic or for technical reasons. Usually the economical lifetime is applied.

### **Degree of utilisation $u$ (weeks / year)**

After the determination of the annual depreciation, the weekly or daily amounts for depreciation and interest have to be determined. To that end it is necessary to know the expected **utilisation degree ( $u$ )**. At any given point in time, equipment is either:

- a. in use on a site including interruptions in work, except for b and c;
- b. taken out of production for overhaul or major repair;
- c. lying idle because of holiday shut-downs, frost, snow and/or ice;
- d. lying idle awaiting allocation to a job.

During the period that the equipment is in status **a**, the annual costs for depreciation and interest have to be charged from the time the equipment arrives on site to the time it is removed. Eight weeks is taken as a standard in the VGB operating costs standard for the time the construction equipment spends in situation **d**. In theory therefore, the equipment could be utilized for a period of 44 weeks per year (equipment available), but a further correction should be made for situations **b**, and **c**, which are allocated 2 weeks and 10 weeks respectively. This means that the equipment is considered to have an actual utilisation time of 32 weeks per year, corresponding to a degree of utilisation of approximately 75%, based on an availability of 44 weeks per annum.

If the utilisation degree  $u$  is known then the weekly depreciation  $K$  can be calculated as follows:

$$K = \frac{An}{u} \cdot \left\{ S - \frac{Z}{p^n} \right\}$$

$K$  = weekly amount of depreciation

$A_n$  = annuity (based on  $n$  = service life in years and  $i$  = rate of interest)

$u$  = degree of utilisation (weeks per year)

$S$  = standard value

$Z$  = residual value (usually 5 % of  $S$ )

In order to solve disputes regarding the following concepts of purchase value (**standard value  $S$** ), **service life  $n$**  and **degree of utilisation  $u$**  the mentioned books are useful and objective means.

### Other values for large construction equipment.

The concept of standard value and residual value have been treated in the above.

Other concepts are (usual for bookkeeping purposes):

**book-value:** purchase price minus de amounts of depreciation

**replacement value:** the value (price) which should have to be paid to acquire the piece of equipment.

### Depreciation as a fixed percentage of the book-value

The annual depreciation is decreasing as each year a fixed percentage (for instance 50 %) of the bookvalue is being depreciated. This method takes into account the reduction in production capacity of a piece of equipment due to wear, extra maintenance and larger repairs. This method also gives some compensation for increasing costs for  $M + R$  (see example 4).

**Note:** a very different aspect of depreciation is the fact that the amounts of depreciation are introduced in the administration as operating costs (re-payment of a loan). This can not be done unlimited; the height of the depreciation percentages has to be agreed by the tax-inspector. The higher the depreciation amount in a certain year the lower the taxable revenues. Sometimes accelerated redemptions are applied purely for the tax advantage. The service life of the equipment however will not change, so at a later stage the amounts of depreciation have to be adjusted (lower values).

If a piece of equipment is worn out faster then expected, for instance due to difficult circum-stances, then this could be a reason for accelerated depreciation. More appropriate would be to raise in such a case the amounts for maintenance, and eventually repairs, if the worn out parts are replaced.

### Summary

For the cost estimate it is recommended to base  $D + I$  on the **annuity cost** of capital and the **replacement value** of the assets (annually indexed purchase value). The actual financing, depreciation and bookvalue should be treated separately. The capital cost component ( $D + I$ ) should be adjusted annually for price escalation (replacement value of the investment).

In practice in very many cases companies and institutions forget about it (on purpose or by oversight). In particular government institutions or para-statal companies in many countries systematically calculate their hire-tariffs without caring for replacement value of the assets. The temptation to do this is clear: the 'hire' remains lower so one can decide to 'overlook' this aspect. The consequence is that if the hire-tariffs are not based on replacement value of the assets, in reality, also cannot be replaced (if reservations for this replacement are being made).

### 3.3. Maintenance and Repair Costs (M + R)

For the cost estimate is the item "Maintenance and Repair" one of the most difficult. In the VGBouw standard and the Baugeräte Liste values for M + R are given. These values which are based on experience have to be considered as indicative only; the figures should never ground-less be used. In the total weekly operating costs M + R forms an important element.

Maintenance and repair (M + R) are defined as all activities which are carried out with the aim of maintaining a system in the technical state necessary for the system to perform properly in respect of the type and extent of its designated functions. To avoid a system that tries to be to perfect and to meet the requirement of standardisation it is assumed that Maintenance and Repair costs of a piece of equipment are a function of lifetime and standard value and can be expressed as a percentage of the standard value (VGB cost standard), similar to the calculation of D + I.

However M + R should **not** be calculated as a percentage of D + I because these are very different quantities. Indeed the depreciation and interest are mainly dependent of the age of the equipment, while the maintenance and repair includes costs for spare parts and various materials. Furthermore the spare parts and/or the various materials might be produced locally or imported. In the VGBouw cost standards the maintenance and repair costs are expressed as a percentage of the standard value. These percentages are based on empirical data for working in The Netherlands under normal conditions!

When work is carried outside The Netherlands account must be taken of a number of cost factors, such as: geographical location and infrastructure, distance from closest port or airport, climatic conditions, local technical facilities, availability of components and technical articles, local price levels, freight costs for spare parts, import duties and facilities, qualifications of local personnel.

### 3.4. Utilisation degree

Influence utilisation degree (u) on hire tariffs of equipment

Number of effective weeks: 40 weeks per year

Utilisation degree (u)	Fixed costs	Variable costs	Required weekly tariff
20 % = 8 weeks	€ 10,000. -	€ 300. -	€ 1,550. -
40 % = 16 weeks	€ 10,000. -	€ 300. -	€ 935. -
80 % = 32 weeks	€ 10,000. -	€ 300. -	€ 612.50

### 3. 5. Examples

#### 3.5.1. Linear depreciation

S	= standard value:	€ 25,000
z	= residual value (% of S):	10 %
Z	= residual value (€):	
n	= service life (years):	8
i	= interest rate (% year)	8
u	= utilisation (weeks/year)	38
M + R	= maintenance & repair (% of S)	8

Calculate the weekly costs of D + I and M + R.

$$Z = S \cdot \frac{z}{100} = € 25,000 \cdot \frac{10}{100} = € 2,500. -$$

$$\text{Depreciation (D)} = \frac{25,000 - 2,500}{8} = € 2,812.50 / \text{year}$$

$$\text{Interest (average) (I)} = \frac{25,000 + 2,500}{2} \cdot 0.08 = €$$

1,100 / year

$$D + I \text{ per week} = \frac{D + I}{\text{utilisation}} = \frac{2,812.50 + 1,100}{38} = € 102.96 / \text{week}$$

$$M + R = 8 \% \text{ of } S = 0.08 \cdot € 25,000. - =$$

€ 2,000. - / year

$$M + R \text{ per week} = \frac{2,000}{u} = \frac{2,000}{38} = € 52.63 / \text{week}$$

$$\text{Total weekly cost (D + I) + (M + R) = € 155.59 / week}$$

#### 3.5.2. Annuity

S	= standard value:	€ 25,000. -
z	= residual value (% of S):	10 %
Z	= residual value (€):	
n	= service life (years):	8
i	= interest rate (% year)	8
u	= utilisation (weeks/year)	38
M + R	= maintenance & repair (% of S)	8

Calculate the weekly costs of D + I and M + R

Annuity (i = 8%, n = 8 years) = 0.1740

$$\text{Present day value of residual value: } 0.10 \cdot S / (1.08)^8 = \frac{2,500}{1.8509} = € 1,350.69$$

$$\text{Standard value - Present Day value of residual value} = € 23,649.31$$

$$D + I \text{ per year} = \text{annuity (A/P, 8\%, 8 years)} = 0.1740 \cdot € 23,649.31 = € 4,114.98 / \text{year}$$

The Future value (F) of the Present Day value (P) of the residual value with the compound-ded interest is the residual value itself and therefore does not play any role in the calculation of D + I.

$$D + I = \text{annuity} = \text{€ } 4,114.98 / \text{year}$$

$$D + I \text{ per week} = \frac{4,114.98}{\text{utilisation}} = \frac{4,114.98}{38} = \text{€ } \underline{\underline{108.29 / \text{week}}}$$

This figure for D + I is higher than the figure found for linear depreciation as could be expected.

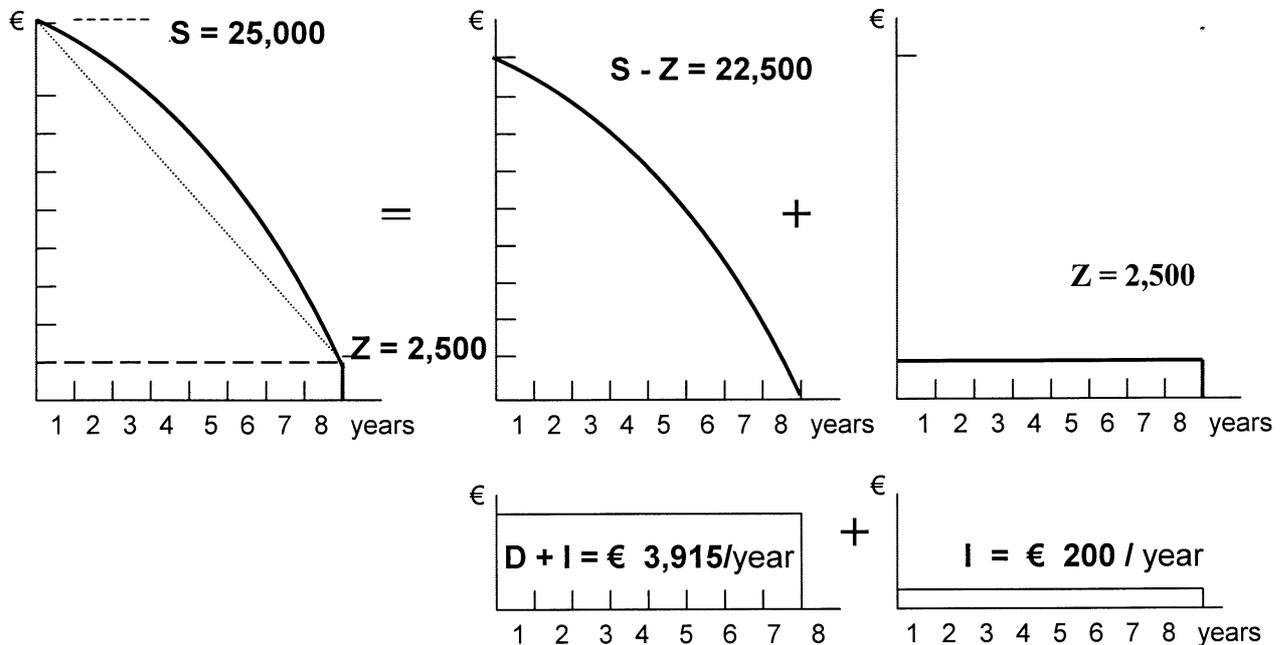
Another approach to this type of problem is to split it into two parts, a part being purchase value (standard value S) minus the residual value (Z) and another part the residual value (Z) itself. Over the first part (S-Z) the annual D + I is calculated by the annuity method while over the second part, the residual value Z, each year interest is being paid (single interest). These two components are added in order to determine to total D + I.

$$0.1740 \cdot (25,000 - 2,500) = 0.1740 \cdot 22,500 = \text{€ } 3,915. -$$

$$0.08 \cdot 2,500 = \text{€ } \underline{\underline{200. -}}$$

$$\text{total annually D + I} \quad \quad \quad \text{€ } \underline{\underline{4,115. -}}$$

which is the same value as found above.



**Figure 11.**  
Bookvalue and D + I of a standard value S with a residual value Z according the annuity method.

### 3.5.3. Annuity

S	= standard value:	€ 25,000	
z	= residual value (% of S):	5 %	→ Z = residual value (€):
n	= service life (years):	25	
i	= interest rate (% year)	7	
u	= utilisation (weeks/year)	25	
M + R	= maintenance & repair (% of S)	7.625	
p	= 1 + i / 100 =	1.07	
p <sup>n</sup>	= (1.07) <sup>25</sup> =	5.42743	

Calculate the percentage of D + I using the annuity method and the formula:

$$D + I = \frac{i/100}{p^n - 1} \cdot \frac{100}{u} \cdot (p^n - \frac{z}{100}) = \frac{0.07}{p^n - 1} \cdot \frac{100}{25} \cdot (p^n - \frac{5}{100})$$

$$p^n - 1 = 1,07^{25} - 1 = 5.42743 - 1 = 4.42743$$

$$D + I = \frac{0.07}{4.42743} \cdot \frac{100}{25} \cdot (5.42743 - 0.05) = 0.01581 \cdot 4 \cdot 5.37743 =$$

**0.340 % of S/ week**

$$M + R = 7.625 / 25 = \mathbf{0.305 \% \text{ of S / week}}$$

$$\text{Total weekly costs: } (0.340 + 0.305) \cdot 0.01 \cdot \text{€ } 25,000. - = \mathbf{\text{€ } 161,250. - / \text{week}}$$

### 3. 5.4. Depreciation according linear method and according fixed percentage of bookvalue

Purchase price (standard value S):	€ 60,000. -
Residual value Z: (approx.)	€ 7,500. -
Interest rate i:	8 %
Service life n:	3 years

$$\text{Annually depreciation according linear method: } \frac{60,000 - 7,500}{3} = \mathbf{\text{€ } 17,500}$$

The depreciation percentage is  $(17,500 / 60,000) \times 100 = 29.1 \%$

$$\text{Interest 1}^{\text{st}} \text{ year } 8 \% \text{ of } (\text{€ } 60,000 + \text{€ } 42,500) / 2 = \mathbf{\text{€ } 4,100. -}$$

$$\text{Interest 2}^{\text{nd}} \text{ year } 8 \% \text{ of } (\text{€ } 42,500 + \text{€ } 25,000) / 2 = \mathbf{\text{€ } 2,700. -}$$

$$\text{Interest 3}^{\text{rd}} \text{ year } 8 \% \text{ of } (\text{€ } 25,000 + \text{€ } 7,500) / 2 = \mathbf{\text{€ } 1,300. -}$$

$$\text{Total interest costs} \mathbf{\text{€ } 8,100. -}$$

$$\mathbf{D + I (total) = \text{€ } 52,500. - + \text{€ } 8,100. - = \text{€ } 60,600. -}$$

$$D + I \text{ 1}^{\text{st}} \text{ year: } \text{€ } 17,500. - + \text{€ } 4,100. - = \mathbf{\text{€ } 21,600. -}$$

$$D + I \text{ 2}^{\text{nd}} \text{ year: } \text{€ } 17,500. - + \text{€ } 2,700. - = \mathbf{\text{€ } 20,200. -}$$

$$D + I \text{ 3}^{\text{rd}} \text{ year: } \text{€ } 17,500. - + \text{€ } 1,300. - = \mathbf{\text{€ } 18,800. -}$$

$$\text{Total costs (D + I)} \mathbf{\text{€ } 60,600. -}$$

Depreciation with a fixed percentage of the bookvalue:

Calculated is a depreciation percentage of 50 % (in order to arrive to the residual value of € 7,500); the interest costs are as follows:

Interest 1 <sup>st</sup> year 8 % of (€ 60,000 + € 30,000) / 2 =	€ 3,600. -
Interest 2 <sup>nd</sup> year 8 % of (€ 30,000 + € 15,000) / 2 =	€ 1,800. -
Interest 3 <sup>rd</sup> year 8 % of (€ 15,000 + € 7,500) / 2 =	€ 900. -

Total interest costs € 6,300. -

**D + I (total) = € 52,500. - + € 6,300. - = € 58,800. -**

D + I 1<sup>st</sup> year: € 30,000. - + € 3,600. - = € 33,600. -

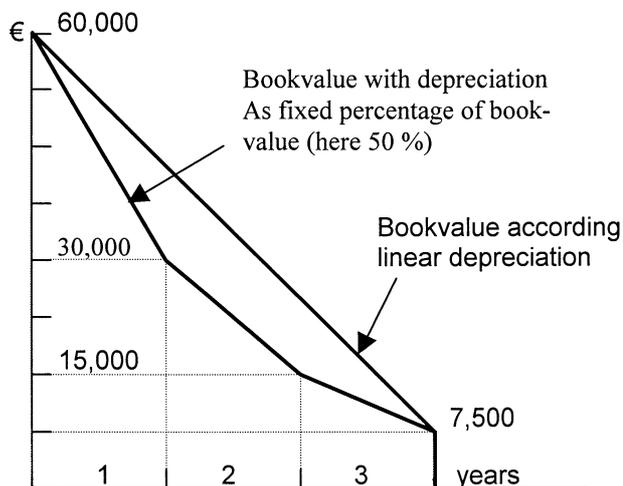
D + I 2<sup>nd</sup> year: € 15,000. - + € 1,800. - = € 16,800. -

D + I 3<sup>rd</sup> year: € 7,500. - + € 900. - = € 8,400. -

Total costs (D + I) € 58,800. -

Although the total costs for D + I are lower in the case of depreciation as a fixed percentage of the bookvalue the question is whether it is possible to charge the high D + I costs of the first year (high hire-rates in the first year). Only if the production capacity during the service life will reduce considerably the weekly costs for D + I will be more or less constant during the utilisation period (service life). For most equipment in the construction industry the production capacity is pretty constant and therefore the depreciation method based on a fixed percentage of bookvalue is not so much used.

If the hourly tariffs are too low or too high this might have a big impact on the competitiveness of the tenderbid.



**Figure 12.**

*Bookvalue according linear depreciation and according fixed percentage (50 %) of book-value.*

### 3. 6. Equivalent annual cost

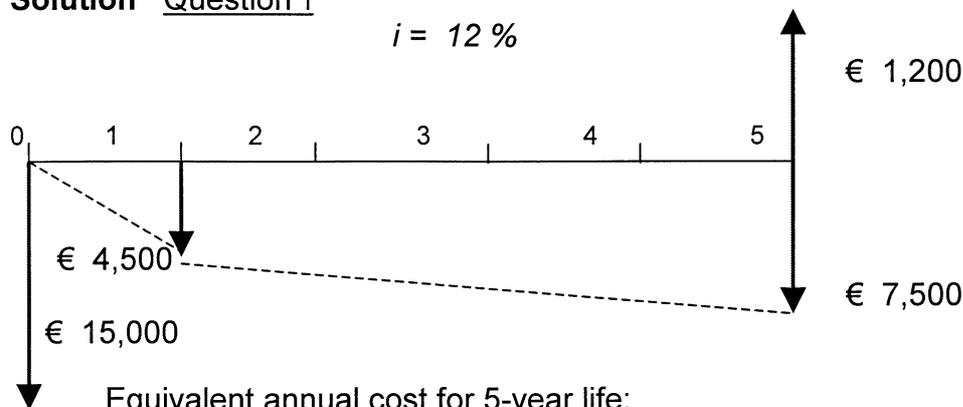
#### Example

A civil engineering contractor operates a fleet of dumpers, and from the past experience has found that a dumper normally has a useful life of 5 years. Such a machine has an initial capital costs of € 15,000. - and at the end of the 5-year period has a salvage value (residual value) of € 1,200. - The cost of maintenance of each dumper amounts to € 4,500. - for the first year, and increases by € 750. - for each succeeding year.

#### Questions

1. If the current interest rate is 12 per cent, what is the equivalent annual cost of owning and maintaining each dumper ?
2. If the contractor can sell the dumpers for € 1,800. - each at the end of the fourth year, should he be advised to do so?

#### Solution Question 1



Equivalent annual cost for 5-year life:

$$\text{Annuity factor for } i = 12\%, n = 5 \text{ years: } 0.27741$$

Present value of investment - salvage value =

$$15,000 - 1,200 (P/F, 12\%, 5) = 15,000 - 681 = \text{€ } 14,319$$

Capital recovery (annuity)

$$14,319 (A/P, 12\%, 5) = 14,319 \times 0.27741 = \text{€ } 3,972 \text{ per year}$$

Present day value of all maintenance costs:

$$\frac{4,500}{1.12} + \frac{5,250}{1.12^2} + \frac{6,000}{1.12^3} + \frac{6,750}{1.12^4} + \frac{7,500}{1.12^5} =$$

$$4017.9 + 4185.3 + 4270.7 + 4289.7 + 4255.7 = \text{€ } 21,019.3$$

$$\text{Equivalent annual cost: } 0.27741 \times \text{€ } 21,019.3 = \text{€ } 5,831.-$$

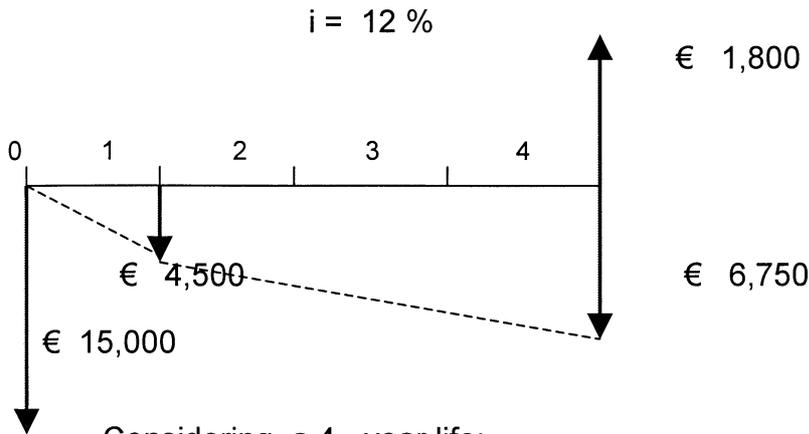
$$\text{Or with formula } \textit{Uniform series equivalent} \quad A = G \left[ \frac{1}{i} - \frac{n}{i} \left( \frac{i}{(1+i)^n - 1} \right) \right]$$

$$4,500 + 750 (A/G, 12\%, 5) =$$

$$4,500 + 750 (1.7745) = 4,500 + 1,331 = \text{€ } 5,831 \text{ per year}$$

$$\text{Therefore, total equivalent annual cost} \quad \underline{\underline{\text{€ } 9,803 \text{ per year}}}$$

Question 2



Considering a 4 - year life:

Equivalent annual cost for 4-year life:

Annuity factor for  $i = 12\%$ ,  $n = 4$  years: 0.32924

Present value of investment - salvage value =

$$€ 15,000 - € 1,800 \cdot (P/F, 12\%, 4) = € 15,000 - € 1,144 = € 13,856$$

Capital recovery (annuity)

$$€ 13,856 \cdot (A/P, 12\%, 4) = € 13,856 \times 0.32924 = € 4,562. - \text{ per year}$$

Present day value of all maintenance costs:

$$\frac{4,500}{1.12} + \frac{5,250}{1.12^2} + \frac{6,000}{1.12^3} + \frac{6,750}{1.12^4} =$$

$$4017.9 + 4185.3 + 4270.7 + 4289.7 = € 16,753.6$$

$$\text{Equivalent annual cost: } 0.32924 \times € 16,753.6 = € 5,516.-$$

Equivalent annual maintenance cost

with formula *Uniform series equivalent*  $A = G \left\{ \frac{1}{i} - \frac{n}{i} \left[ \frac{i}{(1+i)^n - 1} \right] \right\}$

$$€ 4,500 + € 750 \cdot (A/G, 12\%, 4) =$$

$$€ 4,500 + € 750 \cdot (1.3588) = € 4,500 + € 1,019 = € 5,519. - \text{ per year}$$

Therefore, total equivalent annual cost € 10,079. - per year

It will therefore be to the contractor's benefit to keep his dumpers for 5 years.

### 3. 7. Operating Cost standards for Construction Equipment 11<sup>th</sup> revised edition, VGBouw 1995 (The Federation of Major Contractors) Selected tables

#### Standard rates for Depreciation and Interest:

Tables for dredging equipment and earth moving and roadbuilding equipment are based on the **annuity method**, which is tailored to high investment costs in the light of the usually long service lives of this equipment. In the construction industry of foundations and concrete structures, industrial, commercial and public buildings as well as housing projects, the linear method is the customary method for calculating depreciation. For this reason, this method has been chosen for calculating D + I percentages for the cost-standard sections for power supply, cabins & sheds, mechanical equipment, chip and non chip forming machines, statical construction and instruments & communications.

The interest rate for the purpose of calculation has been taken as  $i = 7\%$  annually. Data provided in the tables are based on equipment utilisation which contractors can realistically expect to achieve. Basic utilisation is frequently 32 weeks per year; where there is any deviation therefrom, this is indicated in the tables concerned. Neither the standard rates for depreciation and interest nor the standard rates for maintenance and repair include the costs incurred as a result of use of the plant on site, such as:

- operation;
- energy and water;
- lubricants and fuel;
- loading, unloading and transport;
- assembly and disassembly.

The standard rates likewise do not cover the costs, which must be charged to the site account by the Technical Services Department or the Plant Management Department (see publication General Costs in the Construction Industry of VGBouw). The latter costs include, amongst others:

- supervision and storage;
- insurance;
- special provision;
- modifications needed for the particular project.

With the exception of a few categories of plant (such as some dredging equipment), the standard rates for D + I and M + R are based on operation of the plant in a single shift per working week (for overtime and multiple shift working a pro rata charge should be applied).

Standard value S is expressed in Dutch Guilders (€ 1. - = Dfl. 2.20) and is taken to be the replacement value, i.e. the new purchase price of the production means or the equipment, in the standard design, ex works, yard or importer and exclusive of VAT, in The Netherlands on 1 January 1995.

## Index

VGBouw publishes indexes for the standard values annually for each section. Therefore D + I figures are based on replacement value. The index figures for the calculation of the standard values per 1-1-1998 are as follows:

- section 1 - energy supply	104
- section 2 - cabins and sheds	104
- section 3 - mechanical equipment	105
- section 4 & 5 - chip and non chip forming machines	105
- section 6 - earthmoving equipment	104
- section 6 - road construction equipment	105
- section 7 - statical construction	102
- section 8 - instruments & communication	100
- section 9 - dredging equipment	104 (most dredgers)

Service life n (years) (or utilisation period) is defined as the period between commissioning of a piece of equipment and the time at which it becomes inefficient to keep the equipment in use, either for economic or for technical reasons. If major renovations have been carried out, this must be reflected in an adjustment of the service life.

Maintenance and repair M + R are defined as all activities which are carried out with the aim of maintaining a system in the technical state necessary for the system to perform properly in respect of the type and extent of its designated functions. The standard rates for M + R is expressed as a percentage of the standard value S assuming a standard utilisation of 32 weeks per year. This percentage is based on empirical data for working in The Netherlands under normal conditions.

### # 1011. Power supply generating units

On a sledge, with a diesel engine

Service life: 7 years

Residual value: 5% of S

Utilisation: 32 weeks

D + I 17.28 % of S per year or 0.54 % per week

M + R 14.72 % of S per year or 0.46 % per week

capacity	S standard value	costs per week	
		D + I	M + R
kVA	Dfl	Dfl	Dfl
10	18,000. -	97. -	83. -
30	25,000. -	135. -	115. -
75	33,000. -	178. -	152. -
100	42,000. -	227. -	193. -
200	60,000. -	324. -	276. -
400	105,000. -	567. -	483. -

### # 3000. Lifting jib cranes

Standard design, standard height,

Service life: 12 years

Residual value: 10% of S

Utilisation: 25 weeks

D + I 11.25 % of S per year or 0.45 % per week

M + R 6.25 % of S per year or 0.25 % per week

capacity	hoisting capacity for standard reach		height of boom	S Standard value	costs per week	
	kN	m			D + I	M + R
kNm	kN	m	m	Dfl.	Dfl.	Dfl.
440	15	30	33	243,000. -	1,094. -	608. -
880	22	40	42	437,000. -	1,967. -	1,093. -
1,225	29	42	45	592,000. -	2,664. -	1,480. -
1.760	39	45	54	659,000. -	2,966. -	1,648. -
2,450	55	45	55	950,000. -	4,275. -	2,375. -

### # 3030. Mobile cranes on tires

On rubber tires, standard boom of 7 metres, hydraulic stabilizers

Service life: 12 years

Residual value: 10% of S

Utilisation: 32 weeks

D + I 11.20 % of S per year or 0.35 % per week

M + R 8.00 % of S per year or 0.25 % per week

maximal hoisting capacity	S Standard value	costs per week	
		D + I	M + R
kN	Dfl	Dfl	Dfl
50	227,000. -	970. -	693. -
100	318,000. -	1,113. -	795. -
150	450,000. -	1,575. -	1,125. -
200	500,000. -	1,750. -	1,250. -
400	650,000. -	2,275. -	1,625. -

### #3031. Mobile cranes on tracks

On tracks, without jib

Service life: 12 years

Residual value: 10% of S

Utilisation: 32 weeks

D + I 11.20 % of S per year or 0.35 % per week

M + R 6.40 % of S per year or 0.20 % per week

maximal hoisting capacity	S standard value	costs per week	
		D + I	M + R
kN	Dfl	Dfl	Dfl
350	642,000. -	2,247. -	1,284. -
500	794,000. -	2,779. -	1,151. -
800	1,349,000. -	4,722. -	2,698. -
1000	1,916,000. -	6,706. -	3,832. -
1500	2,380,000. -	8,330. -	4,760. -
2000	3,388,000. -	11,858. -	6,776. -

### # 6000. Bulldozers and angledozers

track- and wheeldozer, including blade and cabin, standard trackwidth  
add 5 % to S for extended tracks

Service life:	7 years
Residual value:	5% of S
Utilisation:	32 weeks
D + I	17.92 % of S per year or 0.56 % per week
M + R dry reclamation	14.08 % of S per year or 0.44 % per week
M + R wet reclamation	28.80 % of S per year or 0.90 % per week

own mass	capacity	S standard value	costs per week		
			D + I	M + R 0.44% dry reclamation	M + R 0.90% wet reclamation
ton	kW	Dfl	Dfl	Dfl	Dfl
12	70	225,000. -	1,260. -	990. -	2,025. -
15	100	260,000. -	1,456. -	1,144. -	2,340. -
19	135	330,000. -	1,848. -	1,452. -	2,970. -
25	150	500,000. -	2,800. -	2,200. -	4,500. -
35	210	780,000. -	4,368. -	3,432. -	7,020. -

### # 6030. Wheelloaders

on wheels, including cabin, closed bucket

Service life:	7 years
Residual value:	5% of S
Utilisation:	32 weeks
D + I	17.92 % of S per year or 0.56 % per week
M + R	16.96 % of S per year or 0.53 % per week

Own mass	capacity	bucket capacity	S standard value	costs per week	
				D + I	M + R
ton	kW	dm <sup>3</sup> = liter	Dfl	Dfl	Dfl
6,5	50	1,000	125,000. -	700. -	663. -
8,5	70	1,500	180,000. -	1,008. -	954. -
10	80	1,750	195,000. -	1,092. -	1,034. -
15,5	140	3,000	260,000. -	1,456. -	1,378. -
20	160	3,500	325,000. -	1,820. -	1,723. -
23,5	200	4,500	425,000. -	2,380. -	2,253. -

### # 6050. Hydraulic excavator on wheels

on wheels, standard, including 2 bucket

Service life: 7 years

Residual value: 5% of S

Utilisation: 32 weeks

D + I 17.92 % of S per year or 0.56 % per week

M + R 11.52 % of S per year or 0.36 % per week

Own mass	capacity	bucket capacity	S Standard value	costs per week	
				D + I	M + R
ton	kW	dm <sup>3</sup> = liter	Dfl	Dfl	Dfl
9	50	500	180,000. -	1,008. -	648. -
12	65	650	220,000. -	1,232. -	792. -
15	80	900	250,000. -	1,400. -	900. -
17,5	100	1,000	280,000. -	1,568. -	1,008. -
20	120	1,200	335,000. -	1,876. -	1,206. -

### # 6051. Hydraulic excavator on tracks

on tracks, standard, including 2 bucket

Service life: 7 years

Residual value: 5% of S

Utilisation: 32 weeks

D + I 17.92 % of S per year or 0.56 % per week

M + R 10.24 % of S per year or 0.32 % per week

Own mass	capacity	bucket capacity	S Standard value	costs per week	
				D + I	M + R
ton	kW	dm <sup>3</sup> = liter	Dfl	Dfl	Dfl
10	55	600	165,000. -	924. -	528. -
17	95	1,000	220,000. -	1,232. -	704. -
22	120	1,250	300,000. -	1,680. -	960. -
25	130	1,500	350,000. -	1,960. -	1,120. -
30	150	1,750	400,000. -	2,240. -	1,280. -
35	160	2,250	480,000. -	2,688. -	1,536. -

**# 9130. Cutter suction dredgers, not under class**

Service life 18 years  
 Residual value 5 % of S-value  
 Utilisation 26 weeks  
 D + I 9.802% of S per year or 0.377% per week

capa- city cutter (C)	capacity pump + jet (P)	own weight (G)	standard value (S)	costs per week		M + R/ week % of S
				D + I	M + R	
kW	kW	t	Dfl	Dfl	Dfl	% of S
30	175	50	920,000	3,468. -	4,296. -	0.467
50	390	80	1,680,000	6,334. -	7,711. -	0.459
70	610	105	2,410,000	9,086. -	10,869. -	0.451
140	725	135	3,300,000	12,441. -	14,586. -	0.442
170	835	470	6,530,000	24,618. -	26,577. -	0.407
294	1275	500	8,320,000	31,366. -	32,282. -	0.388
370	1500	550	9,610,000	36,230. -	36,038. -	0.375
550	1850	620	11,920,000	44,938. -	41,720. -	0.350

D + I and M + R are based on double shift (84 hours/week); For other values M + R than for 84/hours/week, apply factor. If there is additional power installed for a jetpump, this additional power J is to be added to the installed power of the dredgepump(s) P; Different characteristics: S to be calculated according to formula. In case of a different S, interpolate O + R (linear).

$$S = 6,000 \times C + 1,800 \times (P + J) + 8,530 \times G$$

**# 9131. Cutter suction dredgers, under class**

Service life 18 years  
 Residual value 5 % of S-value  
 Utilisation 26 weeks  
 D + I 9.802% of S per year or 0.377% per week

capa- city cutter (C)	capacity pump + jet (P)	own weight (G)	standard value (S)	costs per week		M + R/ week % of S
				D + I	M + R	
kW	kW	t	Dfl	Dfl	Dfl	% of S
550	1850	620	12,520,000	47,200. -	43,069. -	0.344
650	2600	1300	20,930,000	78,906. -	59,651. -	0.285
900	3600	1900	29,930,000	112,836. -	70,036. -	0.234
1250	4400	2300	37,270,000	140,508. -	76,031. -	0.204
1600	5100	2700	44,430,000	167,501. -	81,307. -	0.183
1700	6000	3300	52,350,000	197,360. -	84,807. -	0.162
1800	7300	3700	59,090,000	222,769. -	88,044. -	0.149
1900	8000	4200	65,700,000	247,689. -	89,352. -	0.136

D + I and M + R are based on double shift (84 hours/week); for other values M + R than for 84/hours/week, apply factor. If there is additional power installed for a jetpump, this additional power J is to be added to the installed power of the dredgepump(s) P; different characteristics: S to be calculated according to formula. In case of a different S, interpolate O + R (linear).

$$S = 6,000 \times C + 1,800 \times (P + J) + 9,5000 \times G$$

### # 9000. Bucket dredgers

Service life 25 years  
 Residual value 5 % of S-value  
 Utilisation 20 weeks  
 D + I 8.500% of S per year or 0.425% per week

capa- city bucket	own weight (G)	installed capacity (I)	dredging depth	standard value (S)	costs per week		M + R/ week
					D + I	M + R	
dm <sup>3</sup>	ton	kW	m		Dfl	Dfl	% of S
150	195	150	10	2,700,000	11,475. -	6,966. -	0.258
300	350	300	14	4,900,000	20,825. -	12,005. -	0.245
500	600	500	18	8,380,000	35,615. -	18,771. -	0.224
600	700	600	20	9,800,000	41,650. -	21,168. -	0.216
700	950	700	22	13,100,000	55,675. -	25,938. -	0.198
800	1200	800	24	16,400,000	69,700. -	30,176. -	0.184
900	1450	900	26	19,700,000	83,725. -	33,687. -	0.171
1000	1700	1000	28	23,000,000	97,750. -	36,110. -	0.157

If under class, S-values to be raised by 5 %.

Different characteristics, S to be calculated according to formula.

In case of different s-value, interpolate M + R (linear)

$$S = 12,500 \times G + 1,750 \times I$$

### # 9500. Delivery pipe

with VGBouw Standard flanges; including bolts and packing; standard length 12 m.

Service life 7 years  
 Rest value 5 % of S-value  
 Utilisation 26 weeks  
 D + I 17.966 % of S per year or 0.691% per week  
 M + R Depends on type and quantity of dredged material

Nominal internal diameter	Number of bolts	Wall- thick- ness	Reject thick- ness	S-value (S)	costs per week
					D + I
mm		mm	mm	Dfl / m	Dfl / m
300	16	6	2	130. -	0.90
400	16	8	3	160. -	1.10
500	20	10	4	240. -	1.70
600	20	12	4	320. -	2.20
700	20	16	5	450. -	3.10
800	24	16	5	500. -	3.50
900	28	18	6	580. -	4.00
900	28	20	6	680. -	4.70

Service life and M + R are related to type and quantity of the soil.

D + I percentage is based on the average Dutch conditions during normal use.

Wear and tear costs preferably to be settled on the basis of an in- & out-survey.

### 3.8. Exercises Depreciation and Interest (D + I)

1. For the **maintenance dredging** in the approach and entrance channel of a port the employment of a trailing suction hopper dredger (TSHD) is required to remove the annual siltation, which however varies greatly in quantity. The Port Authority (client) has two options:
  - A. Execution by a dredger which is owned by the Port Authority ('**self-exploitation**').
  - B. One or more 'charter' contracts; in this case all costs are on the account of the owner of the dredger (contractor). The dredger(s) can be chartered on request; the 'hire' begins at the moment of starting the dredging activities and ends at the moment the dredging works are completed. This type of contract is also called a '**charter contract**'.

The expected maintenance dredging requirements for the first seven years are as follows: **10, 5, 10, 5, 15, 5 and 10 million m<sup>3</sup> / year.**

You are requested to make a complete cost estimate for both possibilities for the period of seven years. The following costs are applicable:

	<u>Working / week</u>	<u>Idle/ per week</u>
1. Depreciation	50,000	50,000
2. Interest	37,500	37,500
3. Maintenance	20,000	20,000
4. Repairs	50,000	-
5. Consumables	5,000	-
6a. Salaries 5 crew members (idle)	-	10,000
6b. Salaries 15 crew members (working)		30,000
7. Insurance	12,500	12,500
8. Fuelconsumption	50,000	-
9. Oil & lubricants	5,000	-

Furthermore the following information should be used for the cost estimate:

- For the '**self-exploitation**' option all costs should be increased by a mark-up of 10 % to cover risks of the Port Authority.
- For the **charter contract** all costs are raised by 30 % to cover 'risk and profit' of the contractor.
- For the **charter contract** idle time of the dredger is not charged as this 'risk' is supposed to be covered by the 30% mark-up.
- For '**ad-hoc**' – **rental** of a dredger (only needed for the **self-exploitation** – option), the costs have to be increased by 50 % instead of the 30% for 'risk & profit'.
- The dredger owned by the Port Authority can not be employed elsewhere during periods of idleness.

The weekly production of the trailing suction hopper dredger is 250,000 m<sup>3</sup>  
There are 50 weeks per year of which 40 can be used for dredging (utilisation degree).

#### Questions:

1. Cost estimate for the **self-exploitation option** (for these seven years).
2. Cost estimate for the **charter-contract option** (for these seven years).

3. Your choice from these two options as client (Port Authority); whereby the following questions should be addressed:
- What is the main problem of the 'dredging requirements' for the coming 7 years?
  - What is the main disadvantage of ownership (of the Port Authority) of expensive equipment for the dredging works according the forecasted siltation?
  - How are these observations expressed in the weekly costs?
  - What are the advantages and disadvantages of 'self-exploitation'?
  - Do you have any suggestions to make the 'self-exploitation' more attractive as far as costs are concerned?

## 2. Tower crane

For the construction of a building a towercrane is required for the vertical transportation. According to the planning this tower crane is required for 35 weeks.

There are two alternatives for this crane (both owned by the contractor):

- A tower crane fixed on a movable frame (undercarriage);
- A tower crane fixed on a permanent base (concrete foundation).

### Data tower crane

purchase value :	€ 250,000.-
residual value :	€ 25,000.-
economical lifetime :	10 years
interest :	6 % per year
insurance :	1.5 % of the bookvalue per year
maintenance & repair :	€ 3,000.-per year
normal occupancy :	40 weeks per year
mobilisation & demob:	€ 8,000.-
(including erection, pull down)	
admin. cost Techn. Department.:	1 % of purchase value.

The tower crane is 3 years old (at start work); depreciation is done according the linear method; the 'average' interest is used over the total depreciation period.

### Data under carriage

purchase value :	€ 75,000.-
residual value :	€ 15,000.-
economical lifetime :	10 years
interest :	6 % per year
insurance :	1.5 % of the bookvalue per year
maintenance & repair :	€ 1,000.- per year
normal occupancy :	30 weeks per year
mobilisation & demob:	p.m.
admin. cost Techn. Department.:	1 % of purchase value.

The under carriage is 5 years old (at start work); depreciation is done according the linear method; the 'average' interest is used over the total depreciation period.

**Data fixed base (concrete foundation)**

purchase base :	€ 8,000.-	(once only)
foundation :	6 piles of ε 125.- each	
dimensions concrete plate :	2 x 2 x 1 m.	
price concrete :	€ 110.- / m <sup>3</sup>	
reinforcement :	250 kg / m <sup>3</sup>	
price reinforcement :	€ 1.25 per kg('all-in')	
formwork or shuttering (lost) :	8 m <sup>2</sup> at ε 10.- per m <sup>2</sup>	
various materials :	€ 30.- / m <sup>3</sup>	
wages :	€ 27.50/manhour	
norm concreting :	1 manhour per m <sup>3</sup>	
norm formwork :	0.7 manhour per m <sup>3</sup>	

As the Technical Department of the contractor has more tower cranes than undercarriages, one may assume that the undercarriage would be employed elsewhere (if not used).

**small tower crane**

purchase value :	€ 150,000.-
residual value :	€ 15,000.-

Other costs (rails, sleepers, stabilisation, mobilisation) can be neglected. The foundation plate (fixed base) does not have to be removed at the end of the works. The construction might also be executed with a smaller tower crane; in that case a heavy mobile crane has to be chartered on an hourly base for the handling of a number of heavy concrete elements.

<b>Data</b> economical lifetime :	10 years
interest :	6 % per year
insurance :	1.5 % of the bookvalue per year
maintenance & repair :	€ 1,400.- per year
normal occupancy :	42 weeks per year
mobilisation & demob:	€ 8,000.- (including erection, pull down)
admin. cost Techn. Departmt.:	1 % of purchase value.

The tower crane is 4 years old (at start work); depreciation is done according the linear method; the 'average' interest is used over the total depreciation period. The small tower crane is also required for 35 weeks (fitted on the same undercarriage or on the same fixed base).

Question 1. Calculate the weekly 'hire', being the 'fixed costs' that includes Depreciation & Interest (D + I) maintenance, insurance and administration costs of the Technical Department. What are the total fixed costs of the large mobile crane for this project? Which costs cannot be calculated with the given data?

Question 2. Calculate the fixed costs for the tower crane fitted on an undercarriage and the fixed costs of the tower crane fitted on a fixed concrete base. Which alternative is the cheapest ?

Question 3. Calculate the weekly 'hire' for the small tower crane. The 'hire' are the weekly fixed costs (and include D + I, maintenance, insurance and administration costs of the Technical Department). Calculate the total fixed costs of the small crane (also required for 35 weeks).

Question 4. In case the 'small' tower crane is installed a number of large, heavy concrete elements have to be hoisted by a heavy mobile crane, which has to be chartered for each hoist. The operating costs of this heavy mobile crane are € 75.- per hour. The hoisting operation will take 6 hours while mobilisation/ demobilisation will need another 2 hours per element (therefore extra costs of € 600.- per element). Calculate - with the given data - the minimum number of heavy concrete elements that justify the permanent installation of the large tower crane.

Question 5. Answer the following questions for the case that 20 heavy concrete elements have to be lifted:

- a. Which tower crane will the project manager (contractor) prefer to install at his site;
- b. Will the Head Office (contractor) agree with this choice or will head office decide to use the other tower crane? Which arguments will be used ?

The following issues play a role in the decision-making process: if the small tower crane is being installed, the large tower crane will be utilized somewhere else, but the Head Office of the contractor expects an idle time of 5 weeks for the large crane. The small crane will be employed elsewhere all the time (in case the large tower crane is installed).

### 3. Equipment cost transport units

Given: A large contractor has (amongst others) a fleet of twelve identical transport units. The following data - per unit - are available:

Purchase value (standard value S)	€ 255,000. -
Rest value (residual value Z = 10 % of S)	€ 25,500. -
Economical service life (lifetime) (n years)	7 year
Technical service life (lifetime)	10 year
Method of depreciation :	fixed percentage of purchase value
Interest, per year	7 %
Insurance premium, per year	€ 4,500. -
Own risk, per case	€ 500. -
Number of insurance claims (normal occupancy)	2
Operator, per hour	€ 45. -
Periodic revision (maintenance) per year	€ 9,300. -
Normal utilisation (occupancy) per year	32 weeks
Maximum service weeks per year	44 weeks
Occupancy per week	5 days of each 8 service hours
Net working time per week	70 %
Repair costs per working hour	€ 10. -
Costs for fuel and lubricants per working hour	€ 6. -
Administration costs (% of fixed costs)	1 %

Question 1. Determine - with the above data - the costs of a transport unit, per week while working. Divide the costs into two groups, **fixed costs** and **variable costs** (two columns) (you act as head of the Equipment Division, responsible for all equipment of the contractor), who is "renting out" the equipment to the various projects under execution by the same contractor.

This division receives a weekly compensation for the use of the equipment from the projects. This compensation is called : "**Hire**". In this exercise the total of "**fixed costs**" is considered to be the amount for "**Hire**". After calculating the weekly costs you are requested to determine the "**hire**" tariff (per week).

Question 2. The transport unit is made available to a certain project for a continuous period of 1.5 year. Which total amount has to be paid by the Project to the Equipment Division, using the "**hire**" tariff.

Question 3. You act as the Project Manager on this project and you need 12 transport units for 1.5 year. You consider submitting a request to the Board of the Contractor to be charged with a reduced "**hire**" tariff.

3a. Which arguments do you have?

3b. Determine a reasonable total "hire"- amount per unit for the total period.

Question 4. Five (5) years after the purchase of the first transport units, the improved version (called "Perfect") costs - € 280,000. - ; the restvalue is estimated to be

€ 28,000.-. The economical service life (lifetime) is estimated to be 8 years and the technical lifetime to be 10 years. Calculate for the transport unit "Perfect" the depreciation costs per year and the total fixed costs per week (linear depreciation method as above).

Question 5. As there are two types of transport means of the same technical specifications (size, etc.) an 'average' (common) hire tariff for both types is considered. Mention a number of advantages and disadvantages of such a common tariff.

Question 6. Draw for the original transport unit as well as the new 'Perfect' type diagrams (lifetime on the horizontal axis) for:

- a) Annual interest
- b) Annual depreciation
- c) Book-value

Question 7. An original transport unit is being overhauled completely after 8 years (costs € 50,000.-), whereby the economical lifetime is extended to 11 years. Make a graph of (over the lifetime) :

- a) Interest (annual)
- b) Depreciation (annual)
- c) Book value.

for the total lifetime with a residual value of € 30,000. - .

Question 8. What is the internal hire tariff of transport means (old type) of 8 years old (not revised). What is the lowest acceptable hire tariff?

### 3.9. Answers

#### Exercise 1. Maintenance dredging

##### Weekly costs

Trailing suction hopper dredger	Working	Idle
Depreciation	50,000	50,000
Interest	37,500	37,500
Maintenance	20,000	20,000
Repairs	50,000	-
Consumables	5,000	-
Salaries 15/ 5 crewmembers	30,000	10,000
Insurance	12,500	12,500
Fuelconsumption	50,000	-
Oil & lubricants	5,000	-
<b>Weekly costs total</b>	<b>260,000</b>	<b>130,000</b>

<b>Self-exploitation risk</b>	<b>10 %</b>	<u>26,000</u>	<u>13,000</u>
<b>total</b>		<u><b>286,000</b></u>	<u><b>143,000</b></u>

Note: Weekly rates for 'Depreciation & interest (D + I)', 'Maintenance & repairs (M + R)' and 'Insurance' are based on an utilization of 40 weeks per year; therefore idle rates should be charged for a maximum of 40 weeks except for salaries (idle 2- rates).

<b>Charter contract 30 % risk &amp; profit</b>		<u>81,000</u>	-
<b>total</b>		<u><b>351,000</b></u>	

<b>'Ad-hoc' – hire 50 % risk &amp; profit</b>		<u>135,000</u>	-
<b>total</b>		<u><b>405,000</b></u>	

Maximum 40 'net' production weeks per year.  
 $250,000 \text{ m}^3 / \text{week} \times 40 \text{ weeks} = 10 \text{ million m}^3 / \text{year}.$

Year	1	2	3	4	5	6	7	total
Quantity ( $10^6 \text{ m}^3$ )	10	5	10	5	15	5	10	60
Dredging weeks	40	20	40	20	60	20	40	240
Idle 1 weeks	-	20	-	20	-	20	-	60
Idle 2 weeks	10	10	10	10	10	10	10	70
Ad-hoc hire	-	-	-	-	20	-	-	20

Idle 1 is time the dredger is available for work, but there is no work, therefore the dredger is not being utilized or productive. Idle 2 is time (10 weeks per year) that the dredger is being overhauled (maintenance) etc. and the dredger is not available for production; the costs of this overhaul is recovered by the item 'maintenance' which is being charged that the dredger is productive.

**Question 1. Self-exploitation option**

240 weeks working at 286,000 =	<b>68,640,000</b>
60 weeks idle 1 at 143,000 =	<b>8,580,000</b>
70 weeks idle 2 at 10,000 =	<b>700,000</b>
20 weeks 'ad-hoc' hire at 405,000 =	<b><u>8,100,000</u></b>

**total** **86,020,000**

**Question 2. Charter contract**

240 weeks working at 351,000 =	<b><u>84,240,000</u></b>
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**Question 3**

The charter-contract option is more economical. The large variation in forecasted dredging requirements over these seven years is the main problem. The trailing suction hopper dredger owned by the Port Authority is idle on one hand for half of the possible utilization time (20 weeks/ year) for three out of seven years while on the other hand the large siltation in year 5 requires the additional 'ad-hoc' charter of a second dredger. Only for three out of seven years the match of required dredging work and available production capacity is good. In the cost estimate this can be clearly observed by the large amounts for 'idle-time' as well as 'ad-hoc'-rental. Even if the siltation is evenly spread over the seven years (8.57 million m<sup>3</sup> per year) there is an overcapacity of 1.43 million m<sup>3</sup> per year (16,7 %).

The main disadvantage of 'self-exploitation' is the idle-time which cannot be made productive (the contractor is usually in a much better situation to employ his dredgers elsewhere); the other disadvantage is that the Port Authority owns one trailing suction hopper dredger of a certain size only, while the contractor might have the choice between various sizes of dredgers and employ one depending on the rate of siltation in that particular year.

The advantage of 'self-exploitation' is that the Port Authority has a dredger on stand-by, which can start the maintenance dredging works immediately; this might not be the case for a charter-contract. The 'self-exploitation' option can be made more attractive by overdredging during the years that the owned dredger is not working to its full capacity; a buffer is created by deepening or widening the port approach/ entrance channel beyond the normal dredging levels. The aim of the buffer is to reduce the large dredging requirement of year 5 and thereby the necessity of 'ad-hoc'-rental.

## Exercise 2. Tower crane

Question 1. Fixed costs large tower crane (3 years old):

Depreciation	: $\frac{250,000 - 25,000}{10} = \frac{225,000}{10} =$	€ 22,500.-
Interest (average)	: $\frac{250,000 + 25,000}{2} \times 6\% = 137,500 \times 0.06 =$	€ 8,250.-
Bookvalue:	purchase value = € 250.000	
	depreciated: $3 \times 22,500 =$ € 67,500	
	present bookvalue = € 182.500	
Insurance:	1,5 % x € 182.500	€ 2,737.50
Maintenance costs:		€ 3,000.-
Administration costs:	1 % of € 250.000	€ 2,500.-
	Fixed costs per year	€ 38,987.50
Normal utilisation :	40 weeks	
Hire rate per week:	$\frac{38,987.50}{40} =$ € 974.69 say	€ 975.-
Fixed costs for the project: 35 weeks x € 975.- =		€ 34,125.-
Mob/ demob		€ 8,000.-
		€ <u>42,125.-</u>

### Extra costs (variable costs)

- operator (wages) (number of 'service' hours /week)
- fuel/ lubricants (number of 'working' hours /week)
- small maintenance/ repairs (number of 'working' hours /week)
- consumables (cables etc)(number of 'working' hours /week)

Question 2. Costs undercarriage (5 years old)

Depreciation	: $\frac{75,000 - 15,000}{10} = \frac{60,000}{10} =$	€ 6,000.-
Interest (average)	: $\frac{75,000 + 15,000}{2} \times 6\% = 45,000 \times 0.06 =$	€ 2,700.-
Bookvalue:	purchase value = € 75.000	
	depreciated: $5 \times € 6,000 =$ € 30,000	
	present bookvalue = € 45,000	
Insurance:	1,5 % x € 45,000	€ 675.-
Maintenance costs:		€ 1,000.-
Administration costs:	1 % of € 75,000	€ 750.-
	Fixed costs per year	€ 11,125.-
Normal utilisation :	30 weeks	
Hire rate per week	$\frac{11,125}{30} =$ € 370.83 say	€ 371.-
Costs for the project:	35 weeks x € 371.- =	€ <u>12,985.-</u>

Note: The utilisation degree for the undercarriage is higher for this year than the average or normal utilization.

### Costs of fixed foundation

Purchase frame:		€ 8,000. -
Piles:	6 x 125. -	€ 750. -
Concrete:	2 x 2 x 1 = 4 m <sup>3</sup> à € 110. -/m <sup>3</sup>	440. -
Reinforcement:	250 x 4 x € 1.25/ kg	€ 1,250. -
Formwork:	8 m <sup>2</sup> x € 10. -/ m <sup>2</sup>	€ 80. -
Various materials:	4 m <sup>3</sup> x € 30. -/ m <sup>3</sup>	€ 120. -
Pouring:	1 x 4 x € 27.50 / hour	€ 110. -
Formwork:	0,7 x 8 x € 27.50 / hour	€ 154. -
Total		€ <u>10,904. -</u>

The fixed frame is cheaper than the undercarriage.

### Question 3. Costs small tower crane (4 years old)

$$\text{Depreciation} : \frac{150,000 - 15,000}{10} = \frac{135,000}{10} = € 13,500. -$$

$$\text{Interest (average): } \frac{150,000 + 15,000}{2} \times 6\% = € 82,500 \times 0.06 = € 4,950. -$$

Bookvalue:	purchase value	= € 150,000
	depreciated: 4 x € 13,500	= € 54,000
	present bookvalue	= € 96,000

Insurance:	1,5 % x € 96,000	€ 1,440. -
Maintenance costs:		€ 1,400. -
Administration costs:	1 % of € 150.000	€ 1,500. -
Fixed costs per year		€ <u>22,790. -</u>

Normal utilisation: 42 weeks / year

$$\text{Hire rate per week} \frac{22,790}{42} = € 542,62 \text{ say } € 543. -$$

Costs for the project:	35 weeks x € 543. - =	€ 19,005. -
	Mob/demob	€ 8,000. -
	Total costs	€ <u>27,005. -</u>

### Question 4. Cost comparison from the project manager point of view:

Costs large tower crane:	€ 42,125. -
Costs small tower crane:	€ 27,005. -
Difference:	€ 15,120. -

$$\text{Costs mobile crane per element: } (6 + 2) \times € 75. - = € 600. -$$

$$\text{Critical number of elements: } 15,120 / 600 = 25.2$$

As from 26 elements the large tower crane is cheaper.

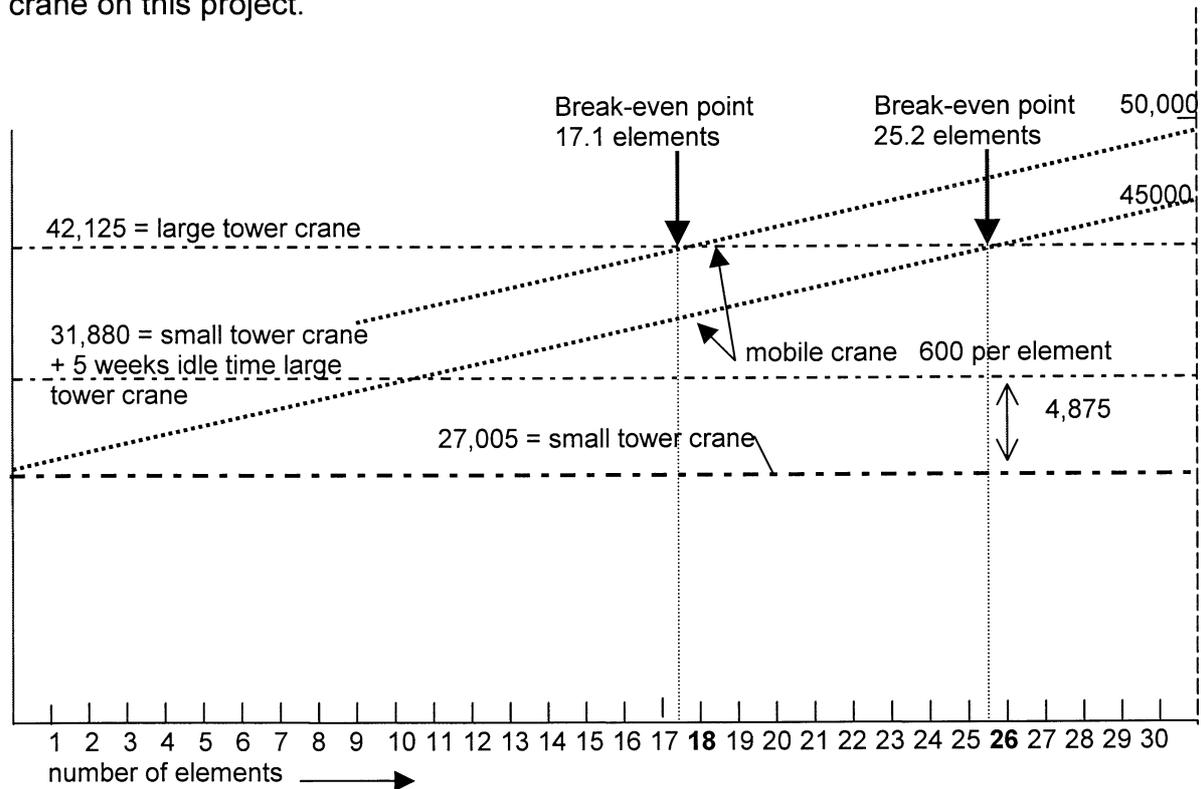
### Question 5. Cost comparison Head Office point of view:

+ 5 weeks idle time large tower crane:

Costs:	5 x € 975. - (= weekly hire rate) =	€ 4,875. -
Difference:	€ 15,120 - € 4,875 =	€ 10,245. -

$$\text{Critical number: } \frac{10,245}{600} = 17.1 \text{ elements therefore 18 elements.}$$

For 20 heavy concrete elements the project manager will select the small tower crane + mobile crane, while the Head Office will decide to employ the large tower crane on this project.



Break-even points (number of elements) between small tower crane (fixed costs) + mobile crane (variable costs) and large tower crane only (only fixed costs).

### Exercise 3. Transport units

#### Question 1

Fixed costs	€ / year	€ / week
Depreciation (linear): $(255,000 - 25,500) / 7$ (D)	32,786. -	
Interest (I)		
Average bookvalue: $(255,000 + 25,500) / 2 = €$		
140,250	9,909. -	
Average annual interest (I) 7 % of € 140,250	4,500. -	
Insurance premium (given)	9,300. -	
Maintenance (O) (periodic costs of revision)		
Subtotal	€ 56,495. -	
Administration costs: 1 % of fixed costs	565. -	
Total	€ 57,060. -	<b>Hire</b>
Normal (average) utilisation (occupancy): 32 weeks		<b>€ 1,783. -</b>

- Remarks:
- Depreciation over economic service life (being the shortest)
  - Take 1 % over subtotal for admin. costs to calculate total fixed costs
  - Own risk (insurance) is considered to be a variable cost
  - Determination of weekly hire based on normal degree of utilisation.

Variable costs	€ / (working) hour	€ / week
Operating costs: 40 hours/ week	45. -	1,800. -
Repair costs: 28 working hours per week (70 % of 40 hrs)	10. -	280. -
Fuel costs: 28 working hours per week	6. -	168. -
Own risk: 2 x / year = 2 x € 500. - / 32 per week		31. -
<b>Total variable weekly costS</b>		<b>2,279. -</b>

Total weekly costs: 'Hire' (fixed costs) + variable costs: € 1,783 + € 2,279 =  
**€ 4,062. -**

### Question 2

Available period for hire per week: 44 weeks (the other 8 weeks per year are needed for periodic maintenance / holidays/ weather etc.).

1.5 year x 44 weeks = 66 weeks

Total 'hire' to be paid to the owner (equipment division, internal):

66 weeks x € 1,783 = **€ 117,678. -**

### Question 3a. Arguments:

1. The 'hire' is based on an average (normal) **utilisation** of 32 weeks per year (approx. 73%) - with an annual availability of 44 weeks - during the service life (economic lifetime). In the first year the equipment has an utilisation degree of 100 % (or 44 weeks); the project (project manager) is being charged by the mother company, who is the owner of the equipment, with a much higher annual 'hire amount' than the actual costs for D + I , M + R and insurances of the mother company. This argument is much less applicable for the second year, because the utilisation degree over the total 2<sup>nd</sup> year (and in particular the second part of it) is not known. The mother company (owner) will argue that the utilisation degree of 73% is based on experience of long-lasting projects followed by long idle periods. Therefore the mother company has to charge more now in order to allow for the fixed costs which needs to be paid during the idle periods when the equipment is not used (rented out) and does not generate income.
2. **'Quantity' discount** (or Quantum discount). The project uses a large number (12) of transport means for a long time, therefore the calculation of the 'internal hire' for this project might eventually not be based on a normal but on a somewhat higher degree of utilisation.
3. The transport-means might be hired cheaper **elsewhere** (third parties; this third party is prepared to give discount for long lasting rental periods for a large number of units).
4. Argument against: the total number of insurance claims might be higher in the first year due to the high utilisation degree, but as the 'own-risk' cost component is placed under the variable costs this will not make much difference.

**Question 3b. Reasonable Hire sum** (to be proposed by the project manager).

First year: based on 100 % utilisation degree

Hire per week € 57,060 / 44 = € **1,297. -**

Second year: hire based on utilisation degree of 73% but with 'quantity-discount' :

Proposal an amount between € 1,297. - and € 1,783. - say around € 1,550. -

Total hire amount per transport unit:

44 weeks @ € 1,297. - =	€57,068
16 weeks @ € 1,550. - =	€ <u>24,800</u>
total	€ 81,868 x 12 = € <b>982,416</b>

**Question 4. Transport mean "Perfect"**

Fixed costs	€ / year	€ / week
Depreciation (linear): $(280,000 - 28,000) / 8$ (D)	31,500. -	
Interest (I) Average bookvalue: $(280,000 + 28,000) / 2 = € 154,000$	10,780. -	
Average annual interest (I) 7 % of € 154,000	4,941. -	
Insurance premium: pro rata $280,000 / 255,000 \times 4,500 =$	10,211. -	
Maintenance (O) pro rata $280,000 / 255,000 \times 9,300 =$		
Subtotal	€ 57,432.	
Administration costs: 1 % of fixed costs	- 574. -	
Total normal (average) utilisation (occupancy): 32 weeks	€ 58,006.	€ <b>1,813.</b>

**Question 5. Advantages and disadvantages average unit rate for 'hire':** An average unit rate is applied (weighed average) for example: € 1,800. - / per week. For this case the differences are rather small but for large differences the following arguments are valid:

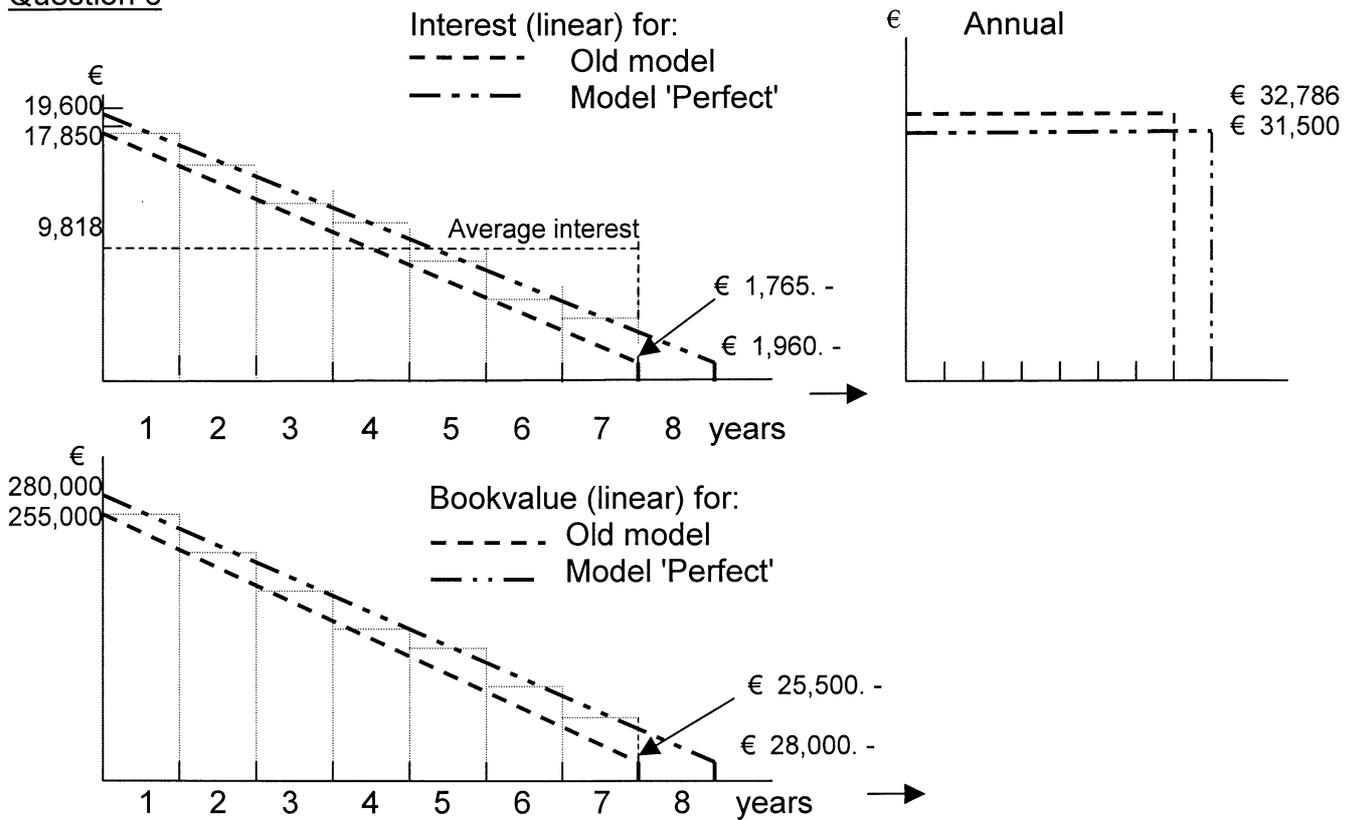
Advantages:

1. Simple cost estimate by the estimating department; beforehand it is not always clear which transport-means will be available at the moment the execution of the works do start;
2. Simple administration of the incoming hire sums (only number is important, not which type);
3. Also more expensive transport-means have a change to be rented-out (in particular true for large difference), cheaper equipment subsidies more expensive equipment;
4. Good maintenance of the old equipment (old model) is now very important, because they are being rented for the same hire tariff as the new model Perfect and should therefore perform equally good.

Disadvantages:

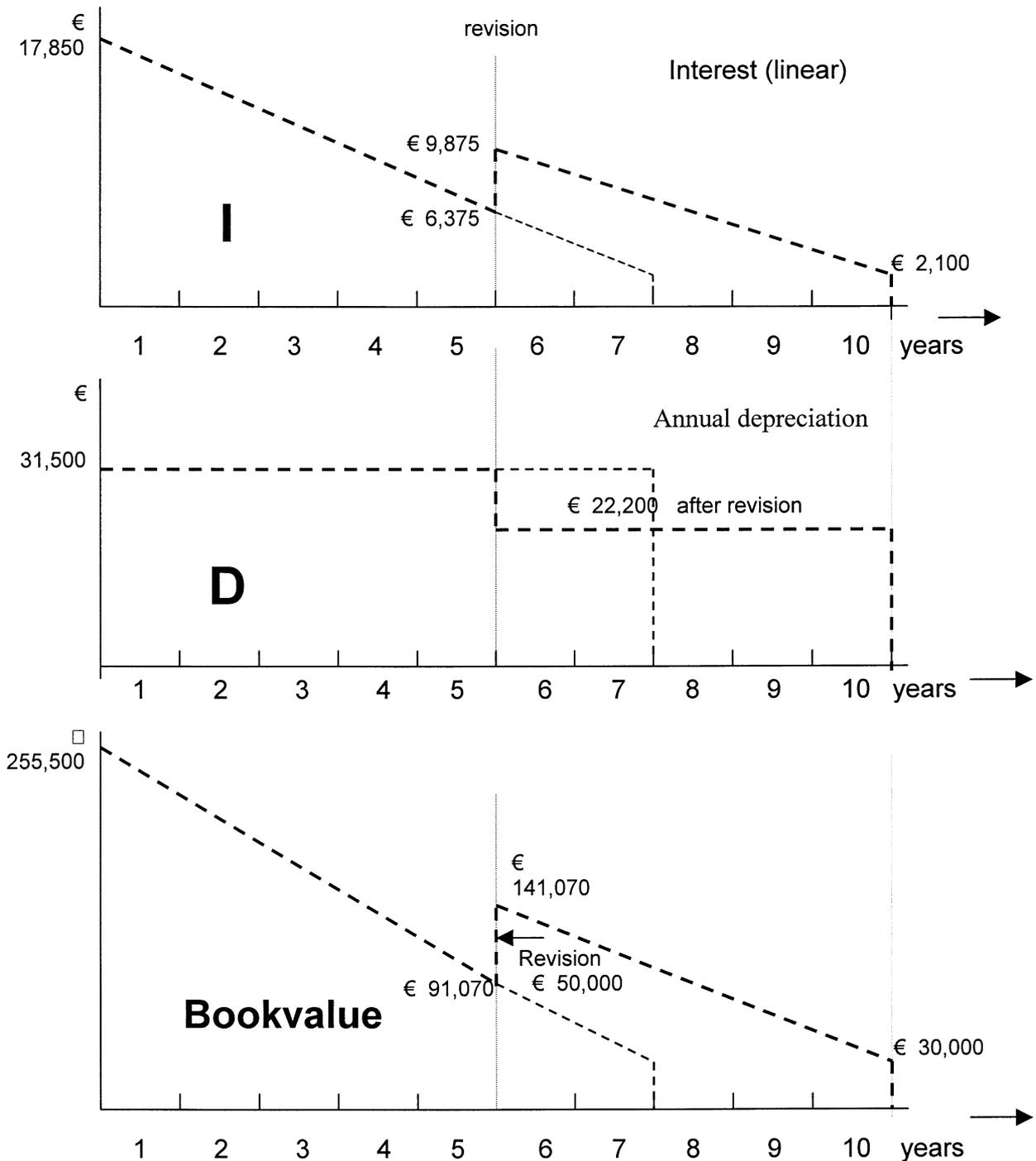
1. Those who hire the equipment will be selective and will prefer the newer transport-means 'Perfect' for the same weekly hire rate; therefore a higher utilisation degree of the newer type could be expected.
2. The old model will be idle for longer periods and this could lead to the decision to scrap or sell the equipment while the economical lifetime has not been reached (at a loss?).
3. The old model becomes more expensive after a number of years (not logical?) and becomes less competitive with other rental companies.

Question 6



**Question 7.** Transport mean 'old-type' overhauled (revised).

Starting point: the revision has not been foreseen at the date of purchase and the costs of the revision or overhaul has nor been taken into account (bookvalue, depreciation and, interest) until the moment the revision took place.



**Question 8.** Hire depreciated transport means. After seven years the old type transport means has been fully depreciated to residual (scrap) value ( $D = 0$ ) and the interest of 7% is now based on the residual value only (€ 1,785.-); also the insurance will be less but  $M + R$  provision will certainly goes up ( $> € 9,300$ ), while the utilisation degree is likely to decrease. Say  $\varepsilon 15,000 / 30$  weeks = € 500. - per week minimal.

### 3.10. Literature: (Dutch)

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## 4. PROJECT BENEFIT COST ANALYSIS

### 4.1. Least cost

This chapter deals with the comparison of alternative strategies or alternative solutions. To illustrate this the following example is discussed.

An rather old excavator, which will be replaced after two years, needs repairing (or overhaul) at an estimated costs of € 10,000 (investment). If the overhaul is not carried out it is anticipated that the operating expenses (M + R) for the excavator will involve an increase of € 5,600 for each of the two remaining years of its service life. It might appear that this is a good investment because an expenditure of € 10,000 leads to a total saving of € 11,200 over the next two years. This is, however, incorrect as it overlooks the time value of money.

The question is whether the **present value** of the anticipated savings on running costs is greater or less than the investment costs for overhaul of € 10,000. An alternative, but equivalent, way of framing the question is: "would the owner of the excavator earn more or less than the € 5,600 per year offered by the savings on running costs by placing the initial € 10,000 into an alternative investment (for example in stocks or bonds)". The first approach focuses upon the discounting of future euro's into present value, whereas the second approach concentrates on the translation of present euro's into a time stream of euro's (compounding). In principle these approaches are equivalent. The problem can only be solved if a certain interest rate is assumed.

#### **Present value concept** (discounting).

For an interest rate of **10 %** the present value of the savings in M + R costs is:

$$\frac{5,600}{1.10} + \frac{5,600}{1.10^2} = € 5,091 + € 4,628 = \underline{\underline{€ 9,719}}$$

When the present values of the savings on M + R expenses is compared with the overhaul costs of € 10,000 it is obvious that the overhaul investment is not a wise decision. If, however, the interest rate is only **5 %** then the present value of the savings on M + R expenditure is:

$$\frac{5,600}{1.05} + \frac{5,600}{1.05^2} = € 5,333 + € 5,080 = \underline{\underline{€ 10,413}}$$

In present euro's, which is more than the initial investment of € 10,000, making the repair at the beginning the better option.

#### **Compounding approach.**

Suppose that there is an opportunity to invest the € 10,000 in a stock or a bond offering an annual rate of return equal to **10 %** (after costs etc.). The future value is the initial investment plus the compounded interest earned. At the end of the first year the future value will be € 10,000 plus € 1,000 in interest earning (10%) = € 11,000. Assume (for simplicity) that the M + R extra costs are only paid once a year (at the end of the year); the balance after paying the bill for the extra M + R costs will be € 11,000 - € 5,600 = € 5,400.

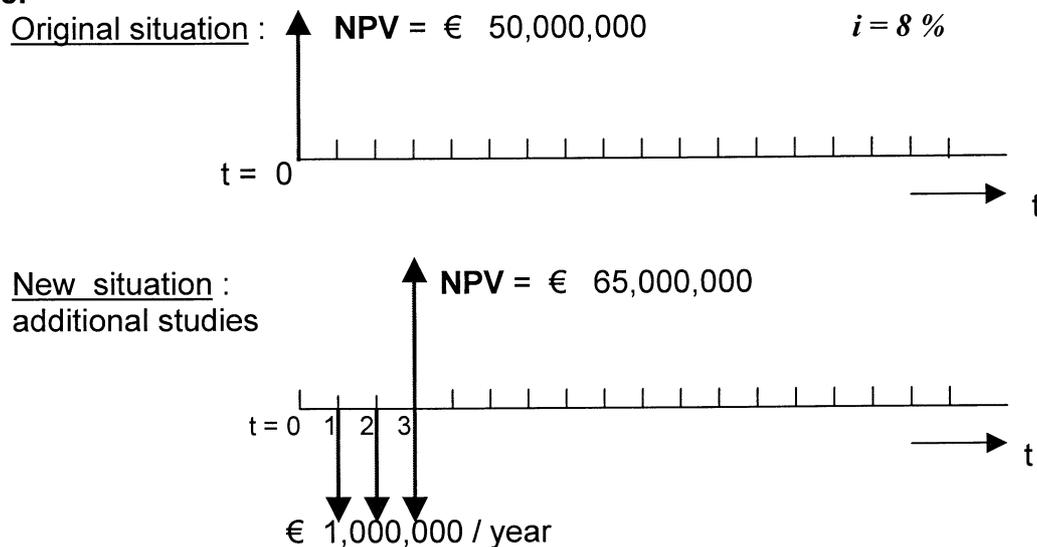
If the balance is reinvested at 10% the future value at the end of the second year is € 5,400 x 1.10 = € 5,940 or a surplus of € 340 remains after payment of the second extra costs for M + R. Clearly the alternative investment is a stock or a bond is more attractive. Alternatively if the available return is 5 % rather than 10 % the future value at the end of the first year will be € 10,000 x 1.05 = € 10,500 or € 4,900 after payment of the extra € 5,600 expenses for M + R. By the end of the second year there would only be € 4,900 x 1.05 = € 5,146 available from the original € 10,000 to pay the bill of € 5,600 for the extra M + R costs of the second year or a deficit of € 354. In this case the investment in the overhaul is justified.

The following comparison of options is exactly the same as the discussed example above. An office building is to be replaced by a new building at the end of two years. Should the insulation in this old building be improved at a cost of € 10,000 if the anticipated savings in heating and cooling cost is € 5,600 for each of the remaining years in the building's life ?

### Example

The expected **Nett Present Value NPV** (see next paragraph) of some project is € 50,000,000. Collecting more data and doing some additional studies during a period of 3 more years are expected to increase the NPV to a value of € 65,000,000 (discounted value at the end of the 3-years study period). The costs of these additional activities is € 1,000,000 at the end of each of these 3 years. Should the extra data collection and studies be done? The discount rate is 8 %.

### Answer



Express new situation in **Present Value (P.V.)** (  $t = 0$  ):

$$\text{discount factor } 1 / (1.08)^3 = 0.794 \text{ (for NPV of € } 65,000,000\text{)}$$

$$\text{discount factor } [(1.08)^3 - 1] / [0.08 \times (1.08)^3] = 2.577$$

$$\text{NPV } \text{€ } 65,000,000 \times 0.794 - \text{€ } 1,000,000 \times 2.577 = \underline{\underline{\text{€ } 49,023,000}}$$

As the NPV for the new situation is lower than the original schedule the conclusion is that additional studies should not be carried out !

## Capitalized costs

In construction works the precise life of an asset may be difficult to access with accuracy. An initial capital investment is made in order to shape the natural ground; the life of these earth works (for example a canal, a road cutting) may be very long or even forever. In such cases the computation of capital recovery takes a similar form to the computation of simple interest.

If the value of  $n$  increases, so the term  $\frac{(1+i)^n}{(1+i)^n - 1}$  approaches to 1.

and the capital recovery formula becomes approximately :  $A = i \cdot P$  (for  $n = 100$  years).

If the lifetime of an asset is considered to be 100 years and not in perpetuity, there will be only a very small difference in the resulting calculation between using the appropriate capital recovery factor itself and using the relevant interest rate.

The term **capitalized costs** is commonly used by engineers in cases where comparisons of costs are made over periods of time in perpetuity and annual costs are assumed to be incurred on a perpetual basis.

The Present Value of the annual costs becomes:  $P = \frac{A}{i}$

## Optimum of initial cost and maintenance cost

### Example

A dike is proposed for river protection. The higher the dike the greater the costs, and the lower the risk of flooding. Estimated data are indicated in the following table:

Height of dike (m)	2	3	4	5	6	7	8
Cost of dike (€ 1,000)	10	25	43	67	100	150	225
Risk of flooding (times per year)	4	2	1	0.5	0.1	0.05	0.01

If the damage by flooding is estimated at € 10,000 each time it occurs, what design height should be selected if money can be borrowed at (a) **10 %**; and (b) **20 %** ?

### Answer:

Capital costs: the dike would be everlasting, so only annual costs need to be considered

a. For **10 % interest rate**.

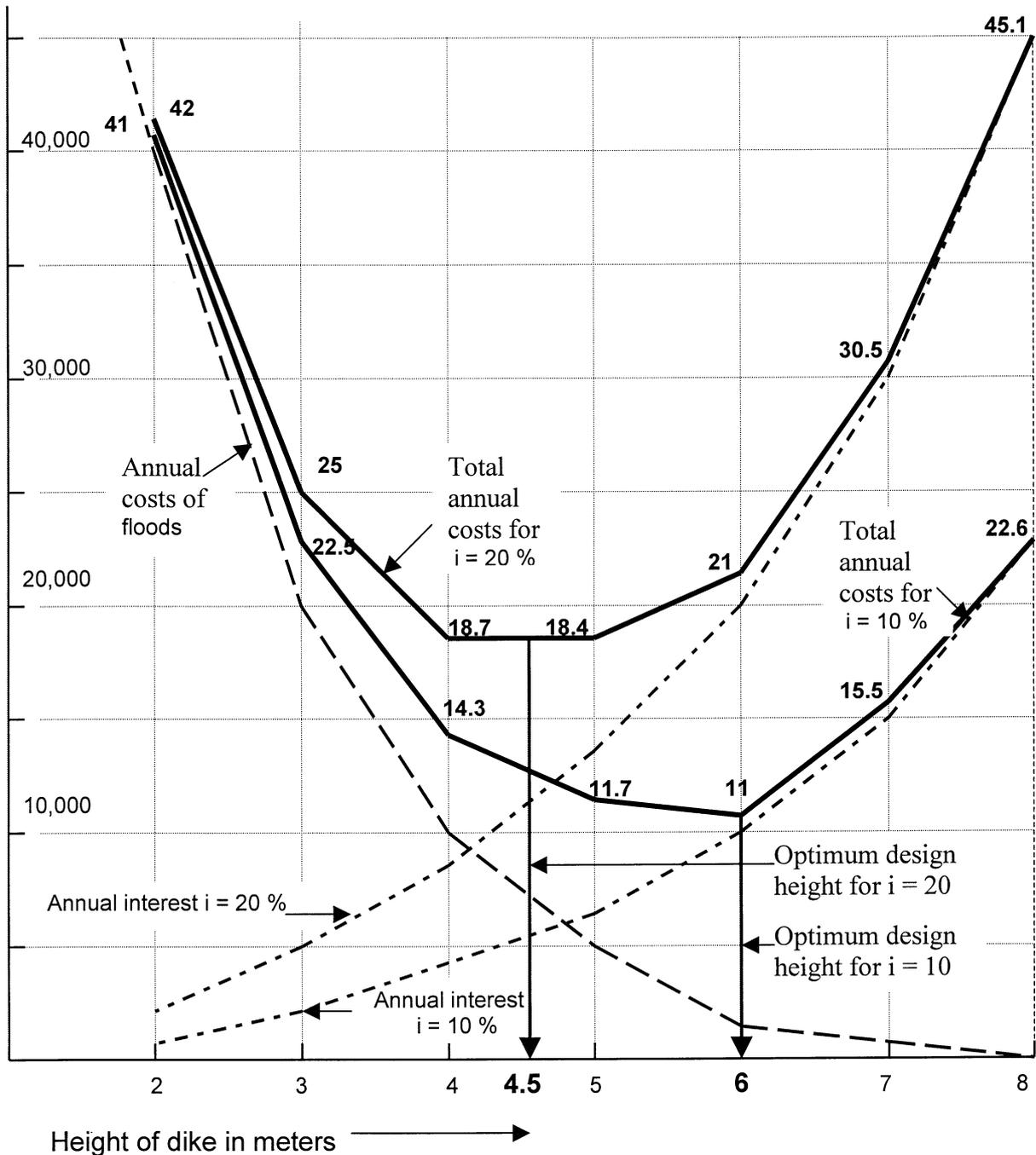
Height of dike (m)	2	3	4	5	6	7	8
Annual interest (€ 1,000)	1.0	2.5	4.3	6.7	10.0	15.0	22.5
Annual costs of Floods (€ 1,000)	40	20	10	5.0	1	0.5	0.1
Total annual Costs (€ 1,000)	41	22.5	14.3	11.7	11	15.5	22.6

The total annual costs are minimal, at 10 % interest rate, for a design height of **6 metres**.

b. For **20 % interest rate**.

Height of dike (m)	2	3	4	5	6	7	8
Annual interest (€ 1,000)	2.0	5.0	8.7	13.4	20.0	30.0	45.0
Annual costs of Floods (€ 1,000)	40	20	10	5.0	1	0.5	0.1
Total annual Costs (€ 1,000)	42	25	18.7	18.4	21	30.5	45.1

The total annual costs are minimal, at 20 % interest rate, for a design height of **4.5 metres**.



Optimum design height of a river dike for flood protection, optimisation of capital costs and the cost of flooding for different rates of interest ( $i = 10\%$  and  $i = 20\%$ ).

## 4.2. Net Present Value (NPV)

The method of appraising alternative capital investment projects by the **net present value** (NPV) is long established and well tried. The net present value method is alternative known as the present value, the present worth or the net present worth method. The basis of this method is that all future costs and benefits concerned with an investment project are converted (discounted) to present value, using a selected interest rate.

$$\text{NPV} = \sum \frac{B_t}{(1+i)^t} - \sum \frac{C_t}{(1+i)^t} \quad \begin{array}{l} B = \text{Benefits} \\ C = \text{Costs} \end{array}$$

In all cases the NPV is uniquely defined. It is widely used in the selection of projects. If the NPV is positive, the project is considered to be profitable: it yields benefits and exceeds investments, operating costs and taxes. It is frequently more convenient and certainly more conventional to express all euro estimates in terms of present value.

For example consider two alternative projects, A and B, either of which would cost € 10,000 today and yield benefits over a four-year period as follows:

Year	1	2	3	4
Project A	€ 6,000	€ 2,000	€ 16,000	€ 4,000
Project B	€ 8,000	€ 1,000	€ 12,000	€ 4,800

Which of these projects is preferable? From a mere comparison of the annual benefits it is impossible to determine the answer, as project A is to be preferred for the 2<sup>nd</sup> and 3<sup>rd</sup> year, while project B is better for the first and last year.

Once a discount rate is selected these benefits can be converted into present values and a comparison made. Present value of the benefits for an interest rate of **5 %**:

$$\begin{aligned} \text{Project A: } & \frac{6,000}{1.05} + \frac{2,000}{1.05^2} + \frac{16,000}{1.05^3} + \frac{4,000}{1.05^4} \\ & = \text{€ } 5,714 + \text{€ } 1,814 + \text{€ } 13,821 + \text{€ } 3,291 = \underline{\underline{\text{€ } 24,641}} \end{aligned}$$

$$\begin{aligned} \text{Project B: } & \frac{8,000}{1.05} + \frac{1,000}{1.05^2} + \frac{12,000}{1.05^3} + \frac{4,800}{1.05^4} \\ & = \text{€ } 7,619 + \text{€ } 907 + \text{€ } 10,366 + \text{€ } 3,949 = \underline{\underline{\text{€ } 22,841}} \end{aligned}$$

Project A is superior to project B.

Furthermore both projects have a positive NPV and are therefore economically feasible.

Another meaning is that if € 24,641 is put in a bank today at **5 %** interest it would be possible to withdraw € 6,000, € 2,000, € 16,000 and € 4,000 in the first, second, third and fourth year respectively before the account would be depleted.

**Example**

Two different tenders have been received for works. Both quote a total price of € 50 million, but they demand **different payment schedules**:

Tenderer A demands the following schedule:

- initial payment (t=0): € 5 million,
- thereafter 9 equal 6-month instalments, each € 5 million.

The works will be completed at the end of year 5.

Tenderer B demands the following schedule:

- initial payment (t=0) € 2.5 million
- after 6 month € 10 million
- after 12 month € 15 million
- after 18 month € 5 million
- after 24 month € 5 million
- after 36 month € 5 million
- after 48 month € 7.5 million

The works will be completed at the end of year 4.

Which tender is to be preferred if:

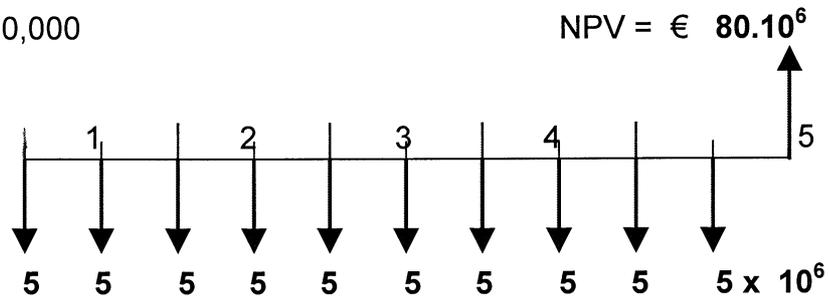
- a. The criterion of **least cost** is applied;
  - b. The criterion of **maximum NPV** is applied; the net benefits of the project, discounted at the moment of completion, are estimated at € 80 x 10<sup>6</sup>.
- The discount rate is **10 %**.

**Answer**

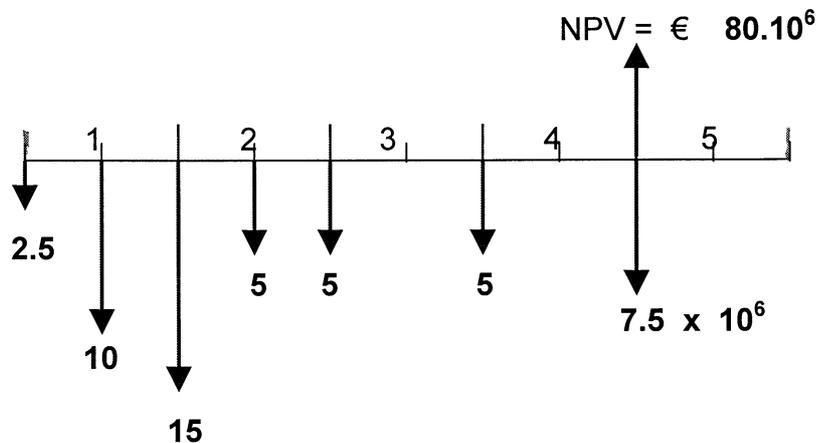
a. Criterion of least costs

tenderprice € 50,000,000  
i = 10 %

**Tender A**  
completion  
end of year 5



**Tender B**  
completion  
end of year 4



Simplify (not fully correct) : half-yearly interest : 5 % and bring half-yearly payments to the beginning of the year:

**Tender A**

$$\text{Total payment every year: } \text{€ } 5,000,000 + \text{€ } \frac{5,000,000}{1.05} = \text{€ } 9,762,000$$

Present Value of all payments:

$$\left\{ 1 + \frac{1}{1.10} + \frac{1}{1.10^2} + \frac{1}{1.10^3} + \frac{1}{1.10^4} \right\} \times \text{€ } 9,762,000 =$$

$$= 4.169 \times \text{€ } 9,762,000 = \text{€ } \underline{\underline{40,700,000}}$$

**Tender B**

Present Value of all payments:

$$\left\{ 2.5 + \frac{10}{1.05} + \frac{15}{1.10} + \frac{5}{1.05 \times 1.10} + \frac{5}{1.10^2} + \frac{5}{1.10^3} + \frac{7.5}{1.10^4} \right\} \cdot 10^6$$

$$= \{ 2.5 + 9.524 + 13.636 + 4.329 + 4.132 + 3.757 + 5.123 \} \cdot 10^6$$

$$= \text{€ } \underline{\underline{43,001,000}}$$

**Conclusion: Tender A is cheaper if criterion of least costs (cheapest) is applied.**

b. Criterion of maximum NPV

**Tender A**

$$\text{Present Value of net benefits of the project at } t = 0: 80 \cdot 10^6 \times \frac{1}{1.10^5} = \text{€ } 49.67 \cdot 10^6$$

$$\text{Net Present Value} = \text{€ } 49.67 \times 10^6 - \text{€ } 40.7 \times 10^6 = \text{€ } \underline{\underline{8,970,000}}$$

**Tender B**

$$\text{Present Value of net benefits of the project at } t = 0: 80 \cdot 10^6 \times \frac{1}{1.10^4} = \text{€ } 54.64 \cdot 10^6$$

$$\text{Net Present Value} = \text{€ } 54.64 \times 10^6 - \text{€ } 43.0 \times 10^6 = \text{€ } \underline{\underline{11,640,000}}$$

**Conclusion: Tender B is cheaper if criterion of maximum NPV is applied.**

### 4. 3. Equivalent annual cost method

In using the equivalent annual cost method for the purpose of comparison, all payments (costs) and receipts (benefits), are converted to their equivalent uniform annual costs. Again it is necessary to make an assumption about the required rate of return (freely interchangeable with the interest  $i$ ) before it is possible to convert variable cash flows to an uniform series of payments over the life of an investment proposal. The following example illustrates the application of the equivalent annual cost technique:

#### Example

To cross a river, a timber bridge has been designed, at an estimated cost of € 8 million. The lifetime of the bridge is estimated at 25 years and the annual costs for maintenance at 2.5 % of the construction costs. It is believed that a concrete bridge, with a lifetime of 50 years and annual costs for maintenance of 0.5 % of the construction costs, could be a better alternative. What are the maximum cost of a concrete bridge, in order to make this a viable alternative? The discount rate is **7.5 %**; the residual value is in both cases zero.

#### **Answer**

The two designs represents mutually exclusive projects with identical benefits and constant annual costs the comparison can be made on annual costs basis.

#### Timber bridge

Annuity $[A/P, 7.5 \%, 25] =$	0.0897 (say 9 %)
Depreciation & interest	9 %
Maintenance:	<u>2.5 %</u>
Total annual costs:	<b>11.5 %</b> of € $8 \times 10^6$

#### Concrete bridge (maximum construction cost X)

Annuity $[A/P, 7.5 \%, 50] =$	0.0771 (say 7.7 %)
Depreciation & interest:	7.7 %
Maintenance:	<u>0.5 %</u>
Total annual costs:	<b>8.2 %</b> of $X \cdot 10^6$

$$\text{Therefore:} \quad 0.082 \cdot X \cdot 10^6 < 0.115 \times \text{€ } 8 \times 10^6 \quad \underline{\underline{X < \text{€ } 11.22 \cdot 10^6}}$$

#### Remark

If the NPV Method would have been used it has to be realised that the **service life** of the timber bridge is shorter than the concrete bridge or with other words the two bridges do not offer the same 'service'. The timber bridge only provides a connection for 25 years, while the concrete bridge provides the same 'service' but for 50 years. So in order to make a correct comparison the timber bridge should be renewed after 25 years in order to provide the same duration of 'service'. In this example this is rather simple but usually the lifetime of one alternative is not equal or a multiple value of the other alternative. This problem is avoided by using the equivalent annual cost method.

Example. Equivalent annual cost comparison

A flood control pumping station is being designed. Three possible pumping stations are proposed and the relevant costs are shown in the table.

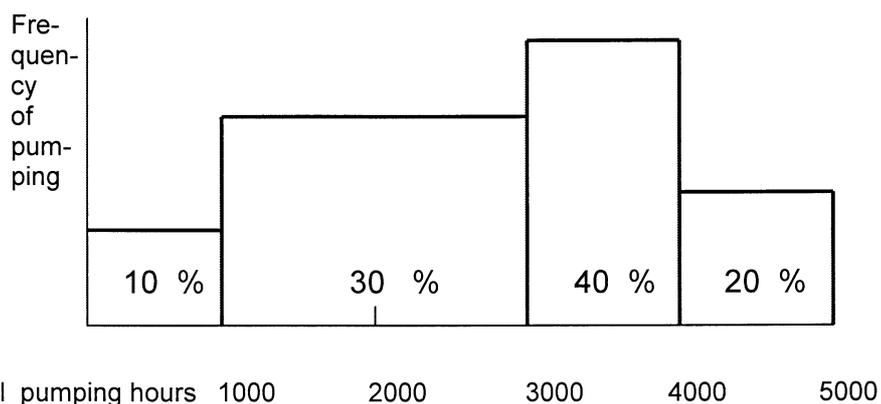
The cost of capital may be taken as **19 %**.

Scheme	A	B	C
Cost of pumps (€)	12,000	18,000	28,000
Life (years)	15	15	20
Maintenance Per annum (€)	1,000	1,500	1,500
Cost of pipes (€)	22,000	18,000	12,000
Life (years)	30	30	30
Cost of pumping (€ per hour)	1.20	0.90	0.80

*Table: Costs of alternative schemes.*

Questions

1. What is the most economic range of pumping times in hours/ year for each scheme (demonstrate your answer by a graph).
2. What is the most economical scheme if the expected frequency of pumping is according the following figure:



*Figure: Frequency of pumping demand*

### Answer

The solution is to plot the equivalent annual costs of each scheme for different pumping demands and determine the range of pumping demands which are cheapest for each scheme.

Convert cost of installation of the pumps and the pipes to an annual cost by the annuity factor (or capital recovery factor), where  $i = 19\%$  and  $n = 15$  or 20 years (pumps) or  $n = 30$  years (pipes).

annuity for  $i = 19\%$  and  $n = 15$  years: 0.20509

annuity for  $i = 19\%$  and  $n = 20$  years: 0.19604

annuity for  $i = 19\%$  and  $n = 30$  years: 0.19103

The maintenance cost of the pumps is already expressed in annual costs.

Calculate the annual 'fixed' costs, which are independent of the number of hours pumping.

#### Scheme A

The equivalent annual costs of installation and maintenance costs of the pumps and pipes =  $\text{€ } 12,000 \times 0.20509 + \text{€ } 1,000 + \text{€ } 22,000 \times 0.19103 = \underline{\underline{\text{€ } 7,663.74}}$

#### Scheme B

The equivalent annual costs of installation and maintenance costs of the pumps and pipes =  $\text{€ } 18,000 \times 0.20509 + \text{€ } 1,500 + \text{€ } 18,000 \times 0.19103 = \underline{\underline{\text{€ } 8,630.16}}$

#### Scheme C

The equivalent annual costs of installation and maintenance costs of the pumps and pipes =  $\text{€ } 28,000 \times 0.19604 + \text{€ } 1,500 + \text{€ } 12,000 \times 0.19103 = \underline{\underline{\text{€ } 9,281.48}}$

The annual 'variable' cost depending on the number of hours pumping for each scheme are:

	<i>Scheme A</i>	<i>Scheme B</i>	<i>Scheme C</i>
Pumping hours			
0	0	0	0
1000	1,200	900	800
5000	6,000	4,500	4,000

These pumping costs vary linearly between 0 and 5000 hours.

Taking the 'fixed' equivalent annual cost and the 'variable' pumping cost the following figure can be plotted (see next page).

Economic break-even point between Scheme A and Scheme B at

X pumping hours.

$$\text{€ } 7,663.74 + X \text{ hours} \times \text{€ } 1.20 = \text{€ } 8,630.16 + X \text{ hours} \times \text{€ } 0.90$$

$$X \text{ hours} \times \text{€ } 0.30 = \text{€ } 966.42 \quad X \longrightarrow \underline{\underline{\text{3,221 pumping hours.}}}$$

**Question 2**

For the given frequency of pumping demand the 'average' pumping hours is:

$$0.10 \times 500 + 0.30 \times 2,000 + 0.40 \times 3,500 + 0.20 \times 4,500$$

= 2,950 pumping hours; therefore Scheme A is the most economical solution.

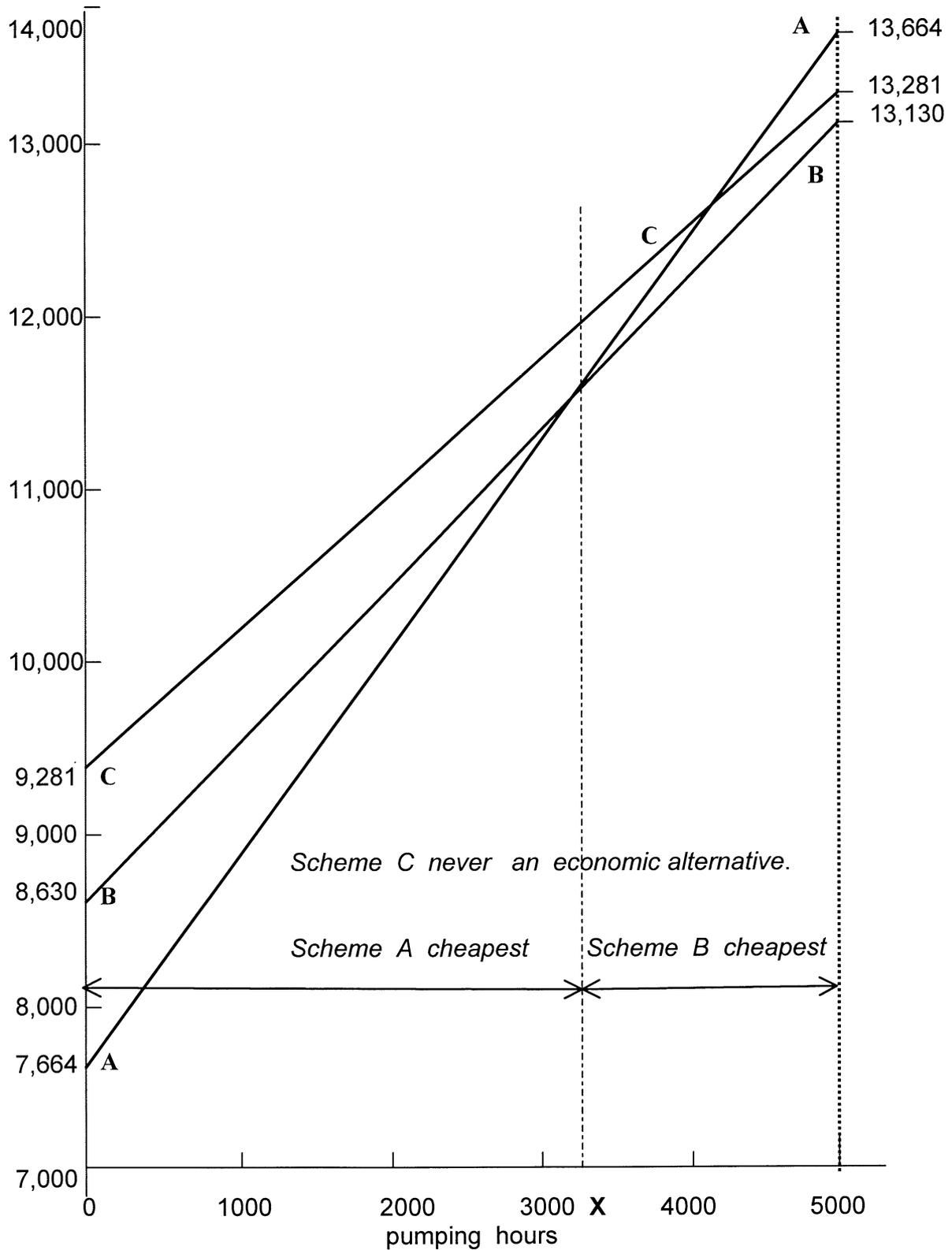


Figure: Annual costs versus pumping demand.

#### 4.4. Internal Rate of Return (IRR)

The IRR is defined as the **discount rate** at which the present value of benefits equals the present value of costs, or at which the **NPV = 0**.

Whereas the determination of the NPV is straightforward, the IRR as a rule cannot be calculated easily. Usually the IRR has to be determined by trial and error: by assuming some values for  $i$ , the NPV can be calculated and by way of interpolation the value of  $i$  can be determined, for which the  $NPV = 0$ , thus yielding the IRR. Nowadays, various pocket calculators are programmed to determine quickly the IRR.

The IRR is a measure for the return on the investments that the project yields. Any project with an IRR exceeding the market rate of interest, i.e. the interest rate at which investible funds can be obtained, is acceptable. As such it can be used with other investment opportunities and in particular with the prevailing market rate of interest. The underlying assumption in the calculation of the IRR is that revenues generated by the project, can be re-invested against the same (high) rate as the IRR itself. This may be too an optimistic assumption, particularly if the IRR is high. There may not be other opportunities for investments which yield the same high returns.

##### Example

The construction of a water supply project is under construction and will be completed on January 1, 2006. The expenditure during construction are as follows:

January 1, 2002	€ 150,000
January 1, 2003	€ 200,000
January 1, 2004	€ 250,000
January 1, 2005	€ 300,000
January 1, 2006	€ 200,000

A final payment to the contractor will be made on January 1, 2007 of € 100,000.

The useful life of the project is assumed at 20 years. The residual value of the project at the end of this period is nil. The interest that has to be paid on the borrowed capital is 7 %. The annual cost of operation and maintenance at the end of every year is expected to be:

€ 50,000	per year during the first five years,
€ 100,000	per year during the second five years
€ 150,000	per year during the last ten years.

It is expected that the sale of the water will be as follows:

1,000,000 m <sup>3</sup>	per year during the first ten years,
2,000,000 m <sup>3</sup>	per year during the second ten years.

##### **Question a:**

At what constant price should the water be sold in order to be able to liquidate the project at the end of the 20 years without debt, or profit ?

##### **Question b:**

The end of years receipts are assumed to be as follows:

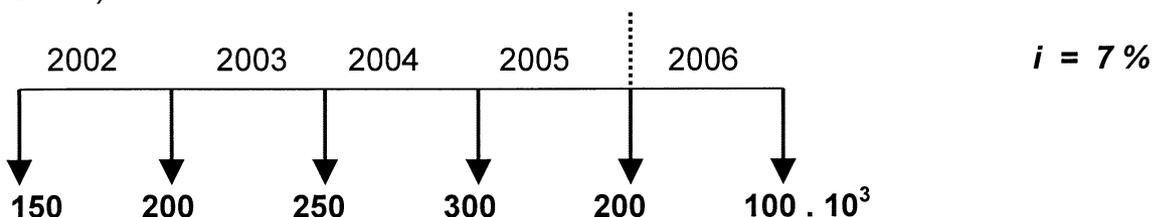
€ 120,000	per year during the first five years
€ 180,000	per years during the years 6 – 10
€ 250,000	per year during the years 11 – 15
€ 390,000	per year during the years 16 – 20

Determine the B/C ratio and the Net Present Value (NPV) (for 7 % interest).  
 Determine the maximum interest rate for which the money could be borrowed whereby the project still is economically viable (IRR = Internal Rate of Return).

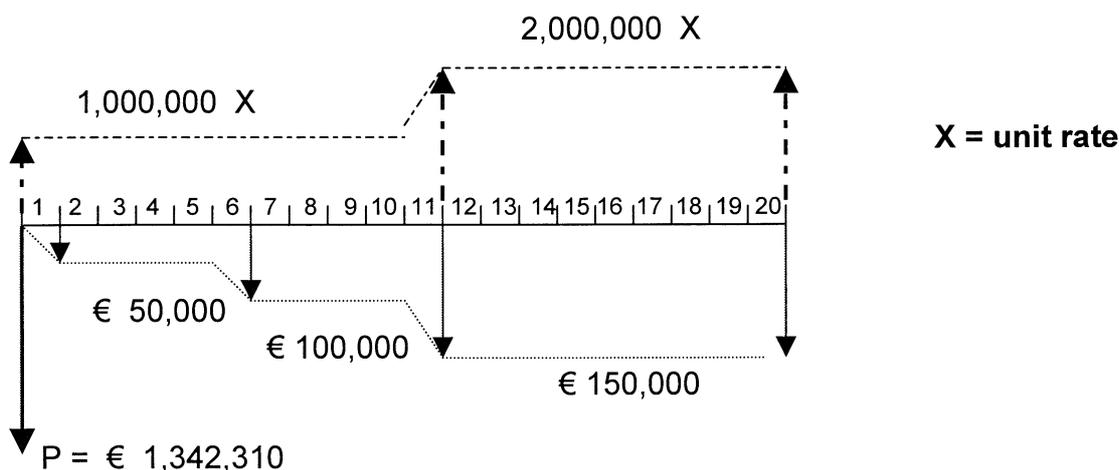
**Answer**

Question a

Present value at start of project (Jan. 1, 2006) of construction costs (in thousand euro's)



$$\begin{aligned}
 & 150 \cdot [F/P, i, 4] + 200 \cdot [F/P, i, 3] + 250 \cdot [F/P, i, 2] + 300 \cdot [F/P, i, 1] + 200 + 100 \cdot [P/F, i, 1] \\
 & = 150 \cdot 1.3108 + 200 \cdot 1.2250 + 250 \cdot 1.1449 + 300 \cdot 1.07 + 200 + 100 \cdot 0.9346 \\
 & = 196.62 + 245 + 286.23 + 321 + 200 + 93.46 = \underline{\underline{\text{€ } 1,342.31 \cdot 10^3}}
 \end{aligned}$$



**P.V. of Benefits :**  $\text{€ } 1,000,000 X [P/A, i, 10] + \text{€ } 2,000,000 X [P/A, i, 10] \cdot [P/F, i, 10]$   
 $= \text{€ } 1,000,000 X \cdot 7.0236 + \text{€ } 2,000,000 X \cdot 7.0236 \cdot 0.5083$   
 $= \underline{\underline{\text{€ } 14,157,224 X}}$

**P.V. of Costs :**  $1,342,310 + 50,000 [P/A, i, 5] + 100,000 [P/A, i, 5] [P/F, i, 5] +$   
 $150,000 [P/A, i, 10] [P/F, i, 10] =$   
 $1,342,310 + 50,000 \cdot 4.1002 + 100,000 \cdot 4.1002 \cdot 0.7130 +$   
 $150,000 \cdot 7.0236 \cdot 0.5083 =$   
 $1,342,310 + 205,010 + 292,338 + 535,565 = \underline{\underline{\text{€ } 2,375,223}}$

**Unit rate X :**  $2,375,223 / 14,157,224 = \underline{\underline{\text{€ } 0.168 \text{ per m}^3}}$

Question b

**P.V. of Benefits :**  $120,000 [P/A, i, 5] + 180,000 [P/A, i, 5] \cdot [P/F, i, 5] +$   
 $250,000 [P/A, i, 5] \cdot [P/F, i, 10] + 390,000 [P/A, i, 5] \cdot [P/F, i, 15]$   
 $= 120,000 \cdot 4.1002 + 180,000 \cdot 4.1002 \cdot 0.7130 + 250,000 \cdot$   
 $4.1002 \cdot 0.5083 + 390,000 \cdot 4.1002 \cdot 0.3624 =$   
 $492,024 + 526,220 + 521,033 + 579,506 = \underline{\underline{\text{€ } 2,118,783}}$

**P.V. of Costs :** € 2,375,223

**B/C ratio :** **0.89**

**NPV:**  $2.118,783 - 2,375,223 =$  (say) € 256,000 (for  $i = 7\%$ ) (negative !)

**IRR:** **try 5%:**

**PV benefits:**  $\{120,000 + 180,000 \cdot 0.7835 + 250,000 \cdot 0.6139 + 390,000 \cdot$   
 $0.4810\} \cdot 4.3295 = \text{€ } 2.606.770$

**PV costs:** **Construction costs:** € 1,300,000

**Operation and maintenance:**

$50,000 \cdot 4.3295 + 100,000 \cdot 4.3295 \cdot 0.7835 + 150,000 \cdot$

$7.7217 \cdot 0.6139 = \text{€ } 1,366,744$

**NPV (5%)**  $\text{€ } 2,606,770 - \text{€ } 1,300,000 - \text{€ } 1,266,770 =$  (say)  $+\text{€ } 39,000$

**IRR** therefore slightly above 5% (by interpolation approximately 5.3%).

#### 4.5. Benefit-cost ratio

The B/C - ratio has been widely used in the early stages of benefit- cost analysis. It is defined as:

$$\text{B/C-ratio: } \frac{\sum \frac{B_t}{(1+i)^t}}{\sum \frac{C_t}{(1+i)^t}}$$

Present value of B = Benefits  
Present Value of C = Costs

the ratio of the present value of benefits to the present value of costs.

If the B/C ratio has a value of more than 1, then the project was considered to be attractive; if the value was less than 1, then the project could not earn back the inputs applied, and thus was not recommended for execution (for a certain value of i). For nearly 60 years the B/C ratio method has been the accepted procedure for making go / no-go decisions on independent projects and for comparing alternative projects in the public sector, even though the other methods as discussed will lead to identical recommendations, assuming all these procedures are properly applied.

$$\text{Conventional B/C-ratio: } \frac{\sum \frac{B_t}{(1+i)^t}}{I + \sum \frac{O \& M_t}{(1+i)^t}}$$

Present Value of B = Benefits  
Present Value of O & M =  
Operation and Maintenance Costs  
Present Value of I = Initial Costs

$$\text{Modified B/C-ratio: } \frac{\sum \frac{B_t}{(1+i)^t} - \sum \frac{O \& M_t}{(1+i)^t}}{I}$$

The resulting B/C-ratios will give consistent results in determining the acceptability of a project (B/C > 1 or B/C < 1 or B/C = 0). The magnitude however of the B/C ratio will differ between conventional and modified B/C. Therefore is the B/C – factor not used internationally anymore , for a number of reasons:

1. Without further information, the B/C ratio is not well-defined: are the benefits net of running costs, or are gross benefits considered?
2. The project with the highest B/C ratio does not always yield the highest value for other indicators (NPV, IRR), used in project appraisal.

### Example

For the extension of the runway of an airport land needs to be purchased for € 350,000. Construction cost for the runway are projected to be € 600,000 and the additional annual maintenance cost for the extension are estimated to be € 22,500. If the runway is extended, a small terminal will be constructed at a cost of € 350,000. The annual extra operating and maintenance cost for this terminal is estimated at € 75,000. The operational cost of the airport itself will increase by € 100,000 for additional air traffic controllers to cope with the increased number of flights. The annual benefits of this extension, consisting of extra income from airlines leasing, airport tax, convenience benefit, additional tourism, is estimated at € 490,000. Apply with a study period of 20 years and 10 % interest rate

$$\begin{aligned} \text{Conventional B/C-ratio: } & \frac{\text{€ 490,000} \cdot (P/A, 10\%, 20 \text{ years})}{\text{€ 1,200,000} + \text{€ 197,500} \cdot (P/A, 10\%, 20 \text{ years})} = \\ & \frac{\text{€ 490,000} \cdot 8.5136}{\text{€ 1,200,000} + \text{€ 197,500} \cdot 8.5136} = 1.448 \end{aligned}$$

### Modified B/C-ratio:

$$\frac{\text{€ 490,000} \cdot (P/A, 10\%, 20 \text{ years}) - \text{€ 197,500} \cdot (P/A, 10\%, 20 \text{ years})}{\text{€ 1,200,000}} = 2.075$$

The difference between conventional and modified B/C –ratios is essentially due to subtracting the equivalent present value of operating and maintenance from both the numerator and the denominator of the B/C-ratio. Subtracting a constant (the present value of O & M costs) from both numerator and denominator does not alter the relative magnitudes of the numerator and denominator but the ratio is not the same.

An additional issue of concern is the treatment of *disbenefits* in benefit/cost ratio analysis. In the example of the runway extension project the increased noise level from commercial planes will be a serious nuisance to people living nearby the airport. The annual disbenefits of this 'noise pollution' is estimated at € 100,000. Taken this into account the conventional B/C-ratio's will change as follows:

Disbenefits considered as reduced benefits:

$$\frac{[\text{€ 490,000} - \text{€ 100,000}] \cdot (P/A, 10\%, 20 \text{ years})}{\text{€ 1,200,000} + \text{€ 197,500} \cdot (P/A, 10\%, 20 \text{ years})} = 1.152$$

Disbenefits treated as additional costs:

$$\frac{[\text{€ 490,000}] \cdot (P/A, 10\%, 20 \text{ years})}{\text{€ 1,200,000} + [\text{€ 197,500} + \text{€ 100,000}] \cdot (P/A, 10\%, 20 \text{ years})} = 1.118$$

#### 4.6. Exercises Cost Benefit Analysis

1. At a long-term strip-mining coal site it is proposed to maintain temporary haulage roads serving the excavation by using hand labour. The annual wage bill is estimated to be € 105,000. With other associated expenses, the total cost of labour to the contractor will be € 145,000 per year. The production of coal on the site is expected to last for 6 years, and alternative methods of constructing and maintaining haulage roads need to be investigated.

The first alternative is to buy a motor-grader for € 95,000 and, as a consequence, reduce the labour force. Maintenance of the grader is estimated to average € 4,000 per year for the 6 years, after which it will have a salvage value or resale value of € 20,000. The labour costs associated with the use of the grader amount to € 80,000 per year.

The second alternative is to lay more substantial roads in the first instance, extending these after 2 years and again after 4 years. Initial costs are then € 80,000, with further investments of € 40,000 and € 37,000 after 2 and 4 years respectively. Total labour costs in this scheme amount to € 64,000 per year.

If the return of at least 10 % is desirable on the capital invested, which is the most economic scheme ?

Make the comparisons based on :

- a. the equivalent annual cost method and
- b. the present value method.

2. The erection of a building for storage is under consideration. There are two technical acceptable alternatives: a reinforced concrete shell roof structure having an initial cost of € 2,700,000 and a steel-framed structure with brick cladding for an initial cost of € 1,800,000. The life of the concrete building is estimated to be 60 years and, while there will be no maintenance costs for the building during the first 10 years, there will thereafter be an annual maintenance cost of € 35,000. The life of the other building is estimated to be 20 years with, an equivalent annual maintenance cost from completion of construction of € 40,000. The salvage value of the concrete building is estimated at € 80,000 and that of the steel-framed building at € 27,000. An acceptable rate of return is assessed at 10 %.

Which is the better economic proposition ?

Make the comparisons based on :

- a. the equivalent annual cost method and
- b. the present value method.

3. A specialized piling rig is purchased by a contractor for one project only. The duration of the project is two years. The economic life of the rig is 10 years, but it is sold at the end of the project, that is after 2 years, then the contractor will be able to get half the purchase value. If the rig costs € 75,000 and the required rate of return is 10 %, what is the annual cost of the rig to the contractor if operating expenses are ignored ?

4. A pumping scheme being developed has three different possible systems of pumps and pipeworks. If the life of the scheme is **20 years**, which scheme should be recommended as the most economic ?

Scheme	Pipe diameter (mm)	Installation cost (€)	Annual running cost (€)
A	500	24,000	9,500
B	600	26,000	6,000
C	700	31,000	5,200

Use **10 %** to represent the cost of capital. If the cost of capital was **6 %**, would the recommendation alter ?

5. A hydroelectric project, if completely developed now, will cost **€ 100,000,000**. Annual operation and maintenance charges will amount to **€ 5,000,000** per year. Alternatively, **€ 55,000,000** may be invested in the project now and the remainder of the work carried out in 12 years' time at a cost of **€ 53,000,000**. In this alternative case annual operation and maintenance charges will be **€ 3,400,000** per year for the first 12 years and **€ 5,600,000** per year thereafter. Both schemes are assumed to have perpetual life. Compare their equivalent annual costs with an interest at **12 %**.
6. Water for an irrigation scheme can be supplied either by gravity (Alternative A) or by pumping (Alternative B).  
Alternative A requires a relatively long canal with intake from a reservoir. The total investment is estimated at **€ 300,000**. The annual costs for maintenance and operation are estimated at **€ 10,000**. Useful service life is estimated at 30 years.  
Alternative B requires a pumping station with an intake from a nearby river. The investments are estimated at **€ 90,000** for the civil engineering structures with a service life of 30 years and at **€ 25,000** for the mechanical and electrical equipment with a service life of 15 years. The annual costs for maintenance and operation are estimated at **€ 20,000**.  
 The net salvage of all investments at the end of their service life is assumed to be zero.
- Determine the most economic alternative for an interest rate is 6 %
  - Determine the most economic alternative for an interest rate is 4 %
  - Determine the unit cost per  $\text{m}^3$  for an interest rate of 6 % if the estimated consumption is 1.5 million  $\text{m}^3$  / year during the first 6 years and 2 million  $\text{m}^3$  / year during the remaining 24 years.
7. In an economic assessment concerned with the alignment of a new road, one of the alternatives to be evaluated on the basis of annual cost consists of a bridge at an estimated cost of **€ 1,350,000**, an embankment costing **€ 215,000**, and other earthworks at an estimated cost of **€ 38,000**. Maintenance on the earthwork and the embankment is estimated to reach an annual cost of **€ 30,000** over the first 4 years of its service and then drop to **€ 14,000** for every year thereafter. Maintenance on the bridge is expected to remain constant throughout its life at a figure of **€ 70,000** a year.

What is the total equivalent uniform annual cost of this alternative if the life of the bridge is estimated at 60 years, the life of the earthworks and the embankments is in perpetuity and the interest rate to be used is **15 %**?

8. A proposed highway project requires an initial investment of **€ 10 million** and a supplementary investment of **€ 5 million** at the end of the tenth year. The project will have a useful life of 50 years, counting from the date of the initial investment. The interest rate is 6 %. The cost of operation and maintenance is **€ 200,000** per year. The benefits of the project has been estimated to begin with **€ 1.0 million** per year for the first 15 years (at the end of each year), thereafter increasing at once to **€ 2.75 million** per year and remaining constant for the remaining 35 years. Determine the value of Benefit-cost (B/C) ratio, Net Present Value (B-C), and Internal Rate of Return (IRR).
9. In diverting river water for an irrigation project, two alternative schemes are prepared, as follows:  
Scheme 1. Open ditch and tunnel with a capital cost of **€ 2,500,000** and an annual maintenance cost of **€ 40,000** per year.  
Scheme 2. Pipework and open flume with a capital cost of **€ 1,750,000** and a maintenance cost of **€ 80,000** per year, with a major replacement cost of **€ 120,000** every 10 years.  
Either of the above schemes will provide the service required. If the current interest rate is **12 %**, compare the two schemes on the basis of capitalized cost ( $n$  is 100 = perpetuity).
10. In a remote wilderness in Africa a rich ore deposit has been discovered. It has been estimated that all ore can be mined during a period of 20 years. The most economical way to bring out the ore is by river. To make the river navigable there are two alternative projects:  
Plan A to regulate the river by training works, excavation and blasting of rock, with a total initial cost of **€ 10,000,000** and a cost of dredging of **€ 2,000,000** per year.  
Plan B to canalize the river by means of weirs and navigation locks: initial costs **€ 20,000,000** and cost of operation and maintenance of **€ 400,000** per year.  
Capital for both projects is available at **10 %** interest.  
The terminal value of the navigation works after 20 years is assumed to be nil.  
Question a: Make a cost comparison of annual costs.
- The cost of dredging of Plan A is now expected to be as follows:  
**€ 100,000** during the first year and then gradually increases by an amount of **€ 200,000** per year till it would reach a cost of **€ 3,900,000** during the twentieth year.  
Question b: Determine which of the two projects is more economic.

11. A new highway of 25 m wide is in the stage of being designed. A considerable portion of the highway has to be cut deeply (10 m) in the surrounding terrain of sandy soils. The problem is to determine the most economic side slope of the cut. If they are steep, they will require a lot of maintenance due to erosion during heavy rainfall. If they are flat, they require extra excavation during the construction of the highway. The capital cost of excavation and disposal of the soil is € 3.00 per m<sup>3</sup>.

Slope	Total excavation (m <sup>3</sup> ) per km	Annual slope maintenance (€)
1 : 1 (n = 1)	250,000 + 100,000 = 350,000	€ 80,000 per km
1 : 2 (n = 2)	250,000 + 200,000 = 450,000	€ 50,000 per km
1 : 3 (n = 3)	250,000 + 300,000 = 550,000	€ 34,000 per km
1 : 4 (n = 4)	250,000 + 400,000 = 650,000	€ 24,000 per km

The capital cost of the road deck is € 250,000 per km. The useful life of the project is 50 years. Annual maintenance of the road deck costs € 3,000 per km. The interest rate is 5%.

12. An appraisal of three alternatives, mutually exclusive projects, A, B, and C, is being made for a company that requires a return of at least 10 % on its invested capital. The estimated details of the investment are shown in the table below. Which investment should be recommended and why? Support your recommendation and reasoning by calculation.

<i>Euro</i>	<i>Project A</i>	<i>Project B</i>	<i>Project C</i>
Initial cost	100,000	160,000	280,000
Scrap value	nil	nil	40,000
Net annual receipts	18,400	30,600	42,300
Life, years	8	8	10

13. A decision has to be made with regard to the installation of automatic control equipment on a concrete batching plant installed at the construction site. Quotations for the equipment show its cost to be € 300,000, but its installation will have the effect of reducing annual labour cost from an estimated € 150,000 to € 45,000. Maintenance of the automatic plant is expected to amount to € 6,000 per year more than the manually controlled plant and only this excess cost need be considered in the analysis.

The automatic equipment, if installed, will have a salvage value of € 30,000 irrespectively of the length of time it is in use. The contractor carrying out the work state their rate of return on capital to be 10 %. Will the selection of the automatic equipment for the contract with a duration of 3.5 years be justified, and what is the minimum contract period that will do this?

14. A public agency has sufficient funds available for a number of projects. One of these projects can be executed in four ways (A, B, C or D). The investments and the net annual benefits of the 4 alternatives are listed in the following table:

<i>Alternative</i>	Investment	Net annual benefits
A	100	20
B	200	30
C	300	50
D	500	75

All amounts are given in thousands of euros.

Assume that all alternatives have an infinitely long service life and that the net annual benefits remain constant in the future. Questions:

- Which alternative has the highest rate of return ?
- Which alternative is to be preferred if unused funds can be invested in other projects with a rate of return of **10 %** ?
- Would you come to another conclusion than that given under b, if unused funds could be invested in projects with a rate of return of **14 %** ?

15. For the installation of a pipeline connection two different payment schedules are offered:

- an immediate payment of **€ 1,150** at the moment the connection is made, or
- 8 annual payments of **€ 231.50** with the additional condition that these payments have to be made at the beginning of each year.

Questions:

- Which payment proposal do you prefer if you can borrow **€ 1,150** now for 8 years at **12%** per year under normal conditions (payment at the end of the period).
- What is the effective annual interest rate in the case of 8 annual payments?

16. The first cost of a project is **€ 100,000**. The annual equivalent operation and maintenance costs are **€ 15,000**. The annual equivalent benefits are **€ 26,500**. The life of the investment is 25 years. Its net salvage value is zero.

Questions:

- Estimate the internal rate of return of the project.
- Could the investment be made economically if funds are available at an interest rate of **4%** per year? Explain your answer briefly.
- In how many years can a loan for the financing of this project be repaid, if the loan carries an annual interest rate of **4 %** and the annual surplus is initially used for this repayment ?

17. Water has to be transported by gravity by means of a canal. The canal has a useful life of 20 years and requires an investment of € 1,000,000. The interest rate is 10 % per year. The net salvage value of the canal after 20 years of operation is assumed to be zero. The annual equivalent maintenance and operation costs are estimated at € 100,000. Calculate the constant transportation cost (unit cost) in € per m<sup>3</sup> for the following cases:
- the annual transport is 15 million m<sup>3</sup> throughout the 20 years' period;
  - the annual transport is 13 million m<sup>3</sup> during the first period of 10 years and 17 million m<sup>3</sup> during the second period of 10 years.

18. The following loans were taken to finance the planning, design and construction of a project:

<i>Loan</i>	<i>Annual interest rate</i>	<i>date</i>
€ 1,000,000	10 %	31 <sup>st</sup> Dec.2002
€ 2,000,000	8 %	1 <sup>st</sup> Jan. 2004
€ 5,000,000	6 %	1 <sup>st</sup> Jan. 2005
€ 5,000,000	4 %	31 <sup>st</sup> Dec.2005

An additional loan will be needed for the final payment of € 1,000,000 due on the 1<sup>st</sup> of January 2007. All previous and future loans are consolidated ("refinanced") at an interest rate of 4 % per year on the 1<sup>st</sup> of January 2006, the day the project is put into operation. The expected annual equivalent operation and maintenance cost are € 1,000,000. The expected annual revenue (gross benefit) is € 3,250,000. The net salvage value after 20 years is expected to be € 1,500,000.

Questions:

- What is the first cost of this project and what is the total depreciation ?
  - What is the internal rate of return ?
  - What is the equivalent annual surplus (profit) of this project ?
  - What is the marginal rate of return of a proposed extension which will cost an additional € 1,500,000 , which will not raise the O & M costs and net salvage value but which will raise the annual revenue to € 3,400,000 ?
  - Will it be justified from an economic point of view to invest these € 1,500,000 in the proposed extension if this money can also be invested in another project which will have an internal rate of return of 12 % ?
19. A project according plan A requires an investment of € 4,000,000. Its useful service life is 15 years. The annual costs for maintenance and operation are € 200,000. The annual benefits are estimated at € 624,000. It is being considered to extend the project by an additional investment of € 1,000,000. This plan B (the extended version of plan A) requires a total investment of € 5,000,000. The total annual costs for maintenance and operation will increase to € 240,000 , whereas the total annual benefits are now estimated at € 722,000.

Questions:

- a. Determine the Benefit-Cost ratios of the plans A and B with interest at **6 %**.
  - b. Determine the rate of return of the plans A and B, as well as the marginal rate of return of plan B with respect to plan A.
  - c. Will it be worthwhile to execute plan A or plan B if unused funds can be invested in other projects having a rate of return of **5 %** ?
20. The useful life of an **€ 10 million** bridge depends on how often it is repaired and painted. Use the formula:  $y = x^2 + 20$ , in which  $y$  is the useful life in years, and  $x$  is the number of times per decade that the bridge gets a repair and paint job at a cost of **€ 250,000** each time. The interest rate is **5 %**. Determine the most economic frequency (in times per decade) of giving the bridge a repair and paint job.
21. In a country a new coal mine will be put into production; the total output will be exported. There are 2 options for the transportation of the coal to the port of export:
- a. Water transport  
The river on which the mine is situated has to be improved for navigation:
    - Length 400 km
    - Construction capacity 50 km/year
    - Start of construction 1st January 2002
    - Construction costs LC  $40 \times 10^6$  per 100 km, spread evenly over the construction period, payable at the end of each year
    - Maintenance costs LC  $2 \times 10^6$  per 100 km per year
    - Transportation costs LC 0.05 per ton per 100 km
  - b. Rail transport.  
A new railway line has to be constructed:
    - Length 375 km
    - Construction capacity 75 km/ year
    - Start of construction 1st January 2005
    - Construction costs LC  $32 \times 10^6$  per 100 km, spread evenly over the construction period, payable at the end of each year
    - Maintenance costs LC  $2.5 \times 10^6$  per 100 km per year
    - Transportation costs LC 0.07 per ton per 100 km.
- Other relevant data are:
- **LC** is one unit of Local Currency
  - Total production  $5 \times 10^6$  ton per year
  - For water transport start of construction: 1st January 2002
  - For rail transport start of construction: 1st January 2005
  - Both options have a life time of 50 years, without any residual value.
  - All costs and benefits occur at the end of the year.
  - Discount rate **10 %**.

Questions :

- a. If the project is financed from internal resources (local currency = L.C.), which of the two options is to be preferred?
- b. Investments will be provided partly from external resources (foreign currency **F.C.**), but all costs for maintenance and transportation will be financed from internal resources (L.C.). The local currency (L.C.) is overvalued by a factor 2; meaning foreign component costs (F.C.) is 2x expressed in local currency (L.C.).

Proportion of foreign currency in total investment costs:

- Water transport 20% F.C. (and 80 % L.C.)
- Rail transport 80% F.C. (and 20 % L.C.)

Which option is to be preferred now?

- c. Transportation time for the railway line is 5 hours less than for water transport, against a value of LC 0.02 per ton per hour.  
Which option is to be preferred for each of the cases 1. and 2. above?

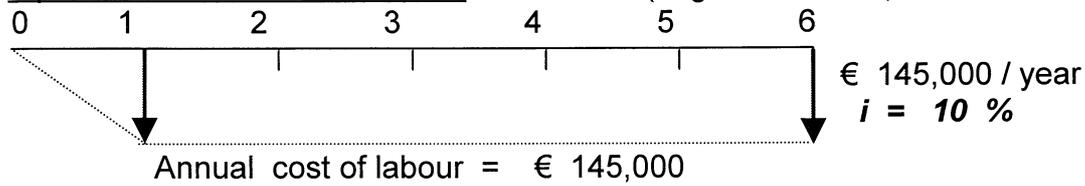
22. The purchase price for a piece of construction equipment is € **20,000** . The operating costs based on the annual average estimated hours of operation are: € **800** in the first year; € **1200** in the second year, € **1500** in the third year, € **1800** in the fourth year and € **2100** in the fifth year.  
The resale value of the plant can be assumed as follows: € **15,000** after 3 years, € **12,000** after 4 years and € **8,000** after 5 years .  
The cost of capital is **8 %** per year.  
Question: Calculate the optimum replacement age.

23. A reinforced concrete road pavement, including the base, is laid for € **100.00 per m<sup>2</sup>**.  
A flexible pavement to give the same service is laid for € **90.00 per m<sup>2</sup>**.  
The flexible pavement has major maintenance every 5 years, which costs the equivalent of € **3.25 per m<sup>2</sup> per year**. The concrete pavement has a first lifetime of 40 years, after which it is resurfaced with asphalt costing € **31.00 per m<sup>2</sup>**.  
Thereafter it is maintained at the same cost as a flexible pavement. In addition, both types of road require annual maintenance estimated to amount to € **0.67 per m<sup>2</sup>**.  
On the basis of both roads giving perpetual service, compare the capitalized costs of 2000 m<sup>2</sup> of road at an interest rate of **12 %**.

#### 4.7. Answers exercises

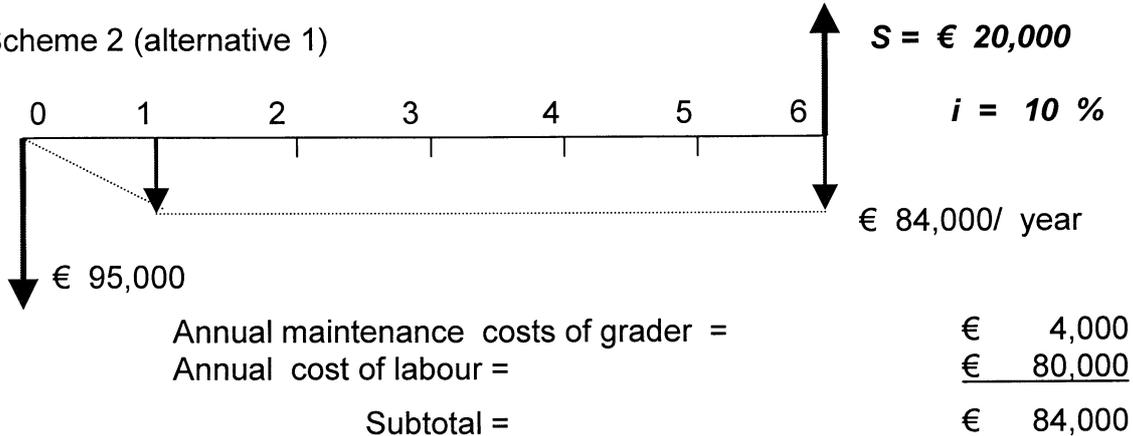
##### Problem 1.

a. equivalent annual cost method. Scheme 1 (original situation)



This is the sole annual outgoing and requires no conversion to annual payments.

Scheme 2 (alternative 1)

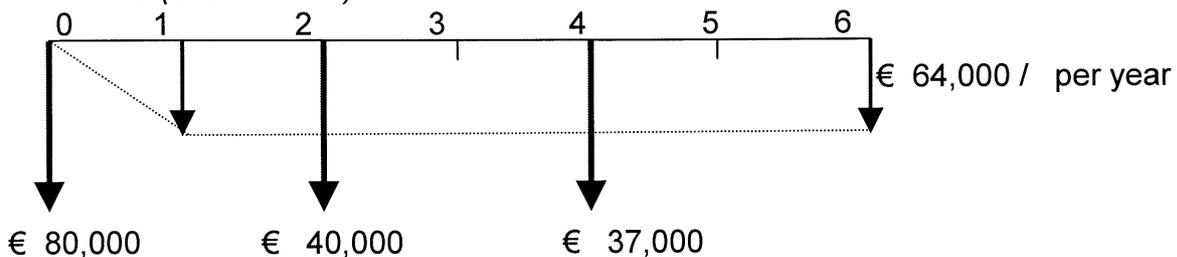


Annual capital recovery cost of the motor grader (where S is the salvage value of the grader):

$$(P - S) (A/P, 10\%, 6 \text{ years}) + S \cdot i = € 19,220$$

**Total equivalent annual cost = € 103,220**

Scheme 3 (alternative 2)



Annual capital recovery of initial cost:

$$80,000 (A/P, 10\%, 6 \text{ years}) = 80,000 (0.2296) = € 18,368$$

Annual capital recovery for capital cost at end of 2 years:

$$40,000 (P/F, 10\%, 2) (A/P, 10\%, 6) = 40,000 (0.8265) (0.2296) = € 7,591$$

Annual capital recovery for capital cost at end of 4 years:

$$40,000 (P/F, 10\%, 4) (A/P, 10\%, 6) = 40,000 (0.6830) (0.2296) = € 5,802$$

Annual labour costs =

Annual labour costs =	€ 64,000
<b>Total equivalent annual cost =</b>	<b>€ <u>95,761</u></b>

Scheme 3 is therefore the most economic on the basis of this evaluation because its equivalent annual cost is lower than those of the other two schemes.

There are a number of points to be noted. The first concerns the treatment of salvage values when computing annual capital recovery costs. The salvage value (€ 20,000) will become available from the sale of the grader at the end of 6 years. Therefore, the part of the cost which is invested over the 6 years of the grader's useful life, and which will not be recoverable as salvage, is the initial cost *less* the salvage value (€ 95,000 – € 20,000 = € 75,000). Since the salvage value will become available again at the end of 6 years it is only necessary to *charge* to each equivalent annual cost the interest on that amount. Treating each year separately, the salvage value can be looked on as being locked up or loaned for the initial purpose of the grader during each year and it is therefore not possible to earn interest or profit by investing the money elsewhere. Account is taken of this in the calculation.

In Scheme 3, each of the payments is converted to present value before being converted to an equivalent uniform series of payments over the 6 years of the comparison.

Finally, the only overriding assumption is that each of the three schemes considered will either give equally good service if put into operation and/or at least will provide the minimum service required. In making an economic choice between the alternatives, it is assumed that the technical merit of each alternative has been examined and found to be satisfactory. The only considerations that may now affect the ultimate decision are the *irreducible* factors.

One example of an irreducible factor might be that there is an ample supply of skilled labour in an area where unemployment is high. It therefore becomes a social obligation of the contractor to act beneficially as he is able towards the local community. There may, for the contractor, be other spinoffs in doing that, which though irreducible in themselves, create a better climate in which to work – a benefit that may well outweigh some of the other considerations.

In the above problem, the comparison between the schemes was made on the basis that each of them represented the annual cost for 6 years. The equivalent annual costs were therefore comparable because the lives of the alternatives were assumed to be the same. This may not always be the case, particularly where the construction of more permanent installations is under consideration.

## **b. Present value method**

### **Scheme 1**

Present value of annual labour cost over 6 years:  
 € 145,000 ( *P/A*, 10 %, 6 years ) = € 145,000 . (4.3552) = **€ 631,504**

### **Scheme 2**

Initial cost of motor grader =	€ 95,000
Present value of maintenance cost and labour cost	
€ 84,000 . ( <i>P/A</i> , 10 %, 6 years ) = € 84,000 . (4.3552) =	€ 365,837
Subtotal =	€ 460,837
<u>Less:</u> Present value of salvage value	
€ 20,000 . ( <i>P/F</i> , 10 %, 6 years ) = € 20,000 . (0.56448) =	€ 11,290
<b>Present value of total costs =</b>	<b>€ 449,547</b>

**Scheme 3**

Initial cost of first section of road =	€	80,000
Present value of second investment:		
€ 40,000 . ( P/F, 10 %, 2 years) = € 40,000 . (0.82645) =	€	33,058
Present value of third investment:		
€ 37,000 . ( P/F, 10 %, 4 years) = € 37,000 . (0.68302) =	€	25,272
Present value of annual labour cost over 6 years		
€ 64,000 . ( P/A, 10 %, 6 years) = € 64,000 . (4.3552) =	€	<u>278,733</u>
<b>Present value of total costs =</b>	<b>€</b>	<b><u>417,063</u></b>

Therefore, on the basis of the above present value evaluation the economic appraisal comes out in favour of Scheme 3, since, in effect, with the given interest rates, the whole scheme can be financed with a smaller lump sum than the other two.

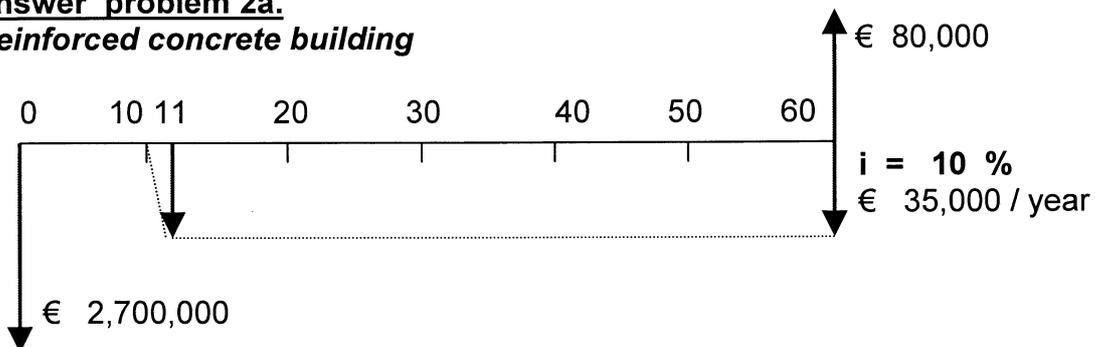
In the case of scheme 3, where there are several staged investments over the period under consideration, it will be noted that one step in the computation has been saved in considering present value rather than equivalent annual cost method for comparison purposes. On the other hand, all the payments for labour, for example, that are already convenient form for annual costs, need to be converted to a lump-sum present value.

Check

Scheme	Equivalent annual cost	%	Present value	%
1	€ 145,000	100 %	€ 631,504	100 %
2	€ 103,220	71.2 %	€ 449,547	71.2 %
3	€ 95,761	66.0 %	€ 417,063	66.0 %

**Answer problem 2a.**

**Reinforced concrete building**



Capital recovery (per year) =  $(P - S) \cdot (A/P, 10 \%, 60 \text{ years}) + S \cdot i =$   
 $(€ 2,700,000 - € 80,000) \cdot (0.1003) + € 80,000 (0.10) = € 270,786$

The sum of money at the end of year 10 equivalent to € 35,000 per year from years 11 to 60:

€ 35,000 . ( P/A, 10 %, 50 years) = € 35,000 . ( 9.9148 ) = € 347,018

Present value of € 347,018 at year 0:

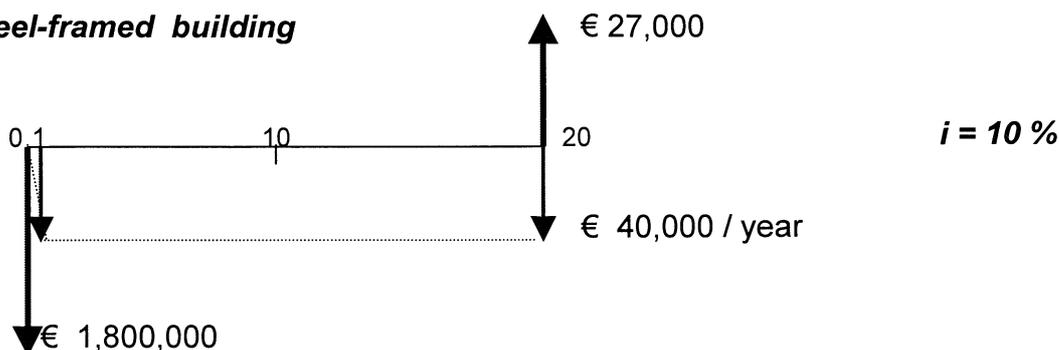
€ 347,018 ( P/F, 10 %, 10 years) = € 347,018 (0.3856) = € 133,810

Therefore, equivalent annual cost over 60 years of € 35,000 a year from years 11 to 60:

€ 133,810 . ( A/P, 10 %, 60 years) = € 133,810 (0.1003) = € 13,421

Therefore, **total equivalent annual cost** = 270,786 + 13,431 = **€ 284,207**

### Steel-framed building



$$\text{Capital recovery} = (\text{€ } 1,800,000 - \text{€ } 27,000) \cdot (A/P, 10\%, 20 \text{ years}) + 27,000 \cdot i$$

$$= 1,773,000 \cdot (0.1175) + 27,000 \cdot (0.10) = \text{€ } 211,028 / \text{per year}$$

Therefore, **total equivalent annual cost** = € 211,028 + € 27,000 = **€ 238,073**

The steel-framed building is therefore cheaper when the comparison is made on basis of annual costs.

This problem raises a number of points. A comparison has been made on the basis of annual cost and it is therefore implicit in the calculation that after 20 years the steel-framed building can be replaced at the same cost as the initial installation and that the replacement will continue at this cost at intervals of 20 years. Rising costs are inevitable in this context, though it is not unreasonable that such a method of comparison should be used because, in the majority of cases, the future cost increases, when discounted to the present time, quickly become a relatively small proportion of present costs.

In the case of the reinforced concrete building, the capital investment is being made now, and therefore no question of increased cost in the replacement situation arises.

If the replacement cost of the steel-framed building in 20 years' time is increased by 50 % over the present-day cost, that is, it becomes € 2,700,000, then the present value of the increase in cost under similar conditions of interest amounts to :

$$\text{€ } 900,000 \cdot (0.1486) = \text{€ } 133,779$$

If the second replacement cost in 40 years' time increases by 50 % over the first replacement value, that is, it becomes € 4,050,000, then the present value of the total increase amounts to:

$$\text{€ } 2,250,000 \cdot (0.02209) = \text{€ } 49,703$$

The two sums produce an equivalent uniform annual cost of

$$(\text{€ } 133,779 + \text{€ } 49,703) \cdot (0.10032) = \text{€ } 18,407$$

over the total life of 60 years under consideration.

The **total equivalent annual cost** now becomes: **€ 256,480**

The steel-framed building remains therefore to be cheaper when the comparison is made on basis of annual costs.

Quite apart from the financial aspects of the economical appraisal, there may be considerable advantages within many businesses from constructing buildings with a shorter life.

New developments in products and building materials may enable such a company to replace the building in 20 years' time with one that gives improved performance.

Replacement may well take place at a cost comparable to that of the original building or investment because of technical improvements. With the long-life building in such a situation it may be difficult to make good use of it in changed circumstances unless money is spent on its rehabilitation. This aspect becomes an irreducible factor in such a situation.

Alternatively, future costs can be estimated only by the interpretation of historic trends. Since, historically, costs have always risen continuously and steadily (with a few exceptions), it seems likely that they will continue to do so. A longer-life investment is clearly advantageous in this circumstance.

### **Answer Problem 2b.**

### **Present value method**

#### **Reinforced concrete building**

Initial cost of building =	€ 2,700,000
Less: Present value of salvage value +	
€ 80,000 . (P/F, 10%, 60 years) = € 80,000 . (0.0033) =	€ <u>264</u>
Subtotal =	€ 2,699,736

Equivalent capital value at the end of year 10 of annual maintenance of € 35,000 per year from years 11 to 60:

€ 35,000 . (P/A, 10 %, 50 years) = € 35,000 . (9.9148) =	€ 347,018
Present value of € 347,018 at year 0:	
€ 347,018 . (P/F, 10 %, 10 years) = € 347,018 . (0.3856) =	€ <u>133,810</u>
<b>Present value of total payments over 60 years =</b>	<b>€ <u>2,833,546</u></b>

#### ***Steel-framed building***

Initial cost of building =	€ 1,800,000
Present value of maintenance cost:	
€ 40,000 . (P/A, 10 %, 60 years) = € 40,000 . (9.9671) =	€ 398,684
Present value of renewal cost less salvage cost at the end of 20 years:	
(€ 1,800,000 – € 27,000) . (P/F, 10 %, 20 years) = € 1,773,000 . (0.14865)	= € 263,556
Present value of renewal cost less salvage cost at the end of 40 years:	
(€ 1,800,000 – € 27,000) . (P/F, 10 %, 40 years) = € 1,773,000 . (0.02210)	= € <u>39,183</u>
Subtotal =	€ 2,501,423

Less:

Present value of € 27,000 . (P/F, 10 %, 60 years):	
27,000 . (0.00328) =	€ <u>89</u>
<b>Present value of total payments over 60 years =</b>	<b>€ <u>2,500,334</u></b>

This confirms the result of the analysis made by the equivalent uniform annual cost method .

In the above problem, using the present value method where the buildings have different lives, it should be noted that the comparison has to be made over a period of time that is the lowest common multiplier of the lives of the alternatives.

It is therefore necessary in the case of the steel building to consider the replacement costs at the end of 20 and 40 years, together with salvage values at the end of 20, 40, and 60 years.

The present value of the series of maintenance payments for the concrete building could have been calculated in a different way. The payments did not commence until year 11 and they continue until the end of year 60. If the factor for conversion of an annual payment to present value for the first 10 years is subtracted from the similar factor over a 60-year period and is then multiplied by the annual amount, the same result will be obtained (note small arithmetical error due to the rounding of the factors).

$$\begin{aligned} &\text{Present value of payments for years 11 – 60:} \\ &\text{€ } 35,000 \cdot [ (P/A, 10\%, 60 \text{ years}) - (P/A, 10\%, 10 \text{ years}) ] = \\ &\text{€ } 35,000 \cdot ( 9.9671 - 6.1445 ) = \text{€ } 35,000 \cdot ( 3.8226 ) = \quad \underline{\underline{\text{€ } 133,791}} \end{aligned}$$

Having obtained either total equivalent annual costs or total present values, then either of these amounts can readily be converted into the other. For example, the total payments at total present value of the concrete building can be converted to total annual costs as follows:

$$\begin{aligned} &\text{Equivalent annual cost:} \\ &\text{€ } 2,833,546 \cdot (A/P, 10\%, 60 \text{ years}) = \text{€ } 2,833,546 \cdot (0.1003) = \quad \underline{\underline{\text{€ } 284,205}} \end{aligned}$$

### Answer problem 3

$$\begin{aligned} &\text{Annual capital recovery cost of the piling rig} \\ &(\text{€ } 75,000 - \text{€ } 37,500) \cdot (A/P, 10\%, 2 \text{ years}) + \text{€ } 37,500 \cdot i = \\ &= \text{€ } 37,500 \cdot (0.5762) + \text{€ } 3,750 = \text{€ } 21,608 + \text{€ } 3,750 = \quad \underline{\underline{\text{€ } 25,358}} \end{aligned}$$

### Answer problem 4

Calculate the present value of each scheme using 10 %

*Scheme A*

$$\begin{aligned} &\text{Present value of installation cost} = \quad \text{€ } 24,000 \\ &\text{Present value of maintenance costs:} \\ &\text{€ } 9,500 \cdot (P/A, 10\%, 20 \text{ years}) = \text{€ } 9,500 \cdot ( 8.5135 ) = \quad \underline{\underline{\text{€ } 80,878}} \\ &\quad \quad \quad \text{total present value} = \quad \underline{\underline{\text{€ } 104,878}} \end{aligned}$$

*Scheme B*

$$\begin{aligned} &\text{Present value of installation cost} = \quad \text{€ } 26,000 \\ &\text{Present value of maintenance costs:} \\ &\text{€ } 6,000 \cdot (P/A, 10\%, 20 \text{ years}) = \text{€ } 6,000 \cdot ( 8.5135 ) = \quad \underline{\underline{\text{€ } 51,081}} \\ &\quad \quad \quad \text{total present value} = \quad \underline{\underline{\text{€ } 77,081}} \end{aligned}$$

*Scheme C*

$$\begin{aligned} &\text{Present value of installation cost} = \quad \text{€ } 31,000 \\ &\text{Present value of maintenance costs:} \\ &6,000 \cdot (P/A, 10\%, 20 \text{ years}) = \text{€ } 5,200 \cdot ( 8.5135 ) = \quad \underline{\underline{\text{€ } 44,270}} \\ &\quad \quad \quad \text{total present value} = \quad \underline{\underline{\text{€ } 75,270}} \end{aligned}$$

At 10 % Scheme C is the most economical because it has the smallest present value.

Repeating the calculations at **6 %**

*Scheme A*

Present value of installation cost =	€	4,000
Present value of maintenance costs:		
€ 9,500 . (P/A, 6 %, 20 years) = € 9,500 . ( 11.4679 ) =	€	<u>108,945</u>
<b>total present value =</b>	<b>€</b>	<b><u>132,945</u></b>

*Scheme B*

Present value of installation cost =	€	26,000
Present value of maintenance costs:		
€ 6,000 . (P/A, 6 %, 20 years) = € 6,000 . ( 11.4679 ) =	€	<u>68,807</u>
<b>total present value =</b>	<b>€</b>	<b><u>94,807</u></b>

*Scheme C*

Present value of installation cost =	€	31,000
Present value of maintenance costs:		
€ 6,000 . (P/A, 6 %, 20 years) = € 5,200 . ( 11.4679 ) =	€	<u>59,633</u>
<b>total present value =</b>	<b>€</b>	<b><u>90,633</u></b>

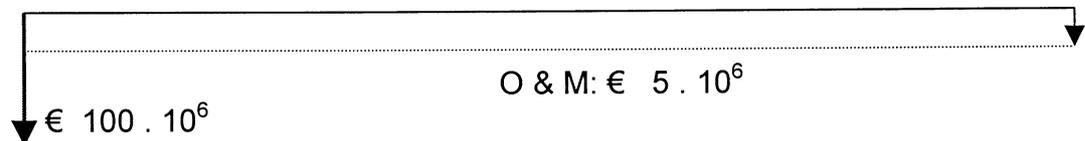
Scheme C is at 6 % the most economical; the difference has become larger due to the lower interest rate . Only for a certain interest rate higher than 10 % there will be a certain interest rate whereby Scheme B becomes more economical as the difference in maintenance costs has less weight.

### Answer problem 5

*Alternative 1*

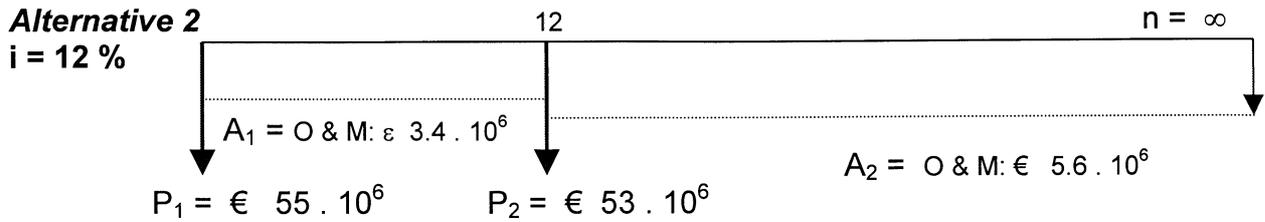
$i = 12 \%$

$n = \infty$



Annuity $A = P \cdot i$ (for $n = \infty$ ) = € 100 . 10 <sup>6</sup> . 0.12 =	€	12 . 10 <sup>6</sup>
Operation and Maintenance (O & M)	€	<u>5 . 10<sup>6</sup></u>
<b>Total equivalent annual costs:</b>	<b>€</b>	<b><u>17 . 10<sup>6</sup></u></b>

Or capitalized costs:  $P + \frac{5 \cdot 10^6}{0.12} = (\text{€ } 100 + \text{€ } 41.67) \cdot 10^6 =$  **€ 141.67 . 10<sup>6</sup>**



Present value (Capitalized costs):

$$P_1 + \frac{A_1}{i} + [P_2 + \frac{A_2}{i}] \cdot (P/F, i, 12) =$$

$$\{ 55 + \frac{3.4}{0.12} + [53 + \frac{2.2}{0.12}] \} \cdot 10^6 \cdot \frac{1}{1.12^{12}} =$$

$$\{ 55 + 28.33 + 71.33 \cdot 0.2567 \} \cdot 10^6 = \quad \underline{\underline{€ 101.64 \cdot 10^6}}$$

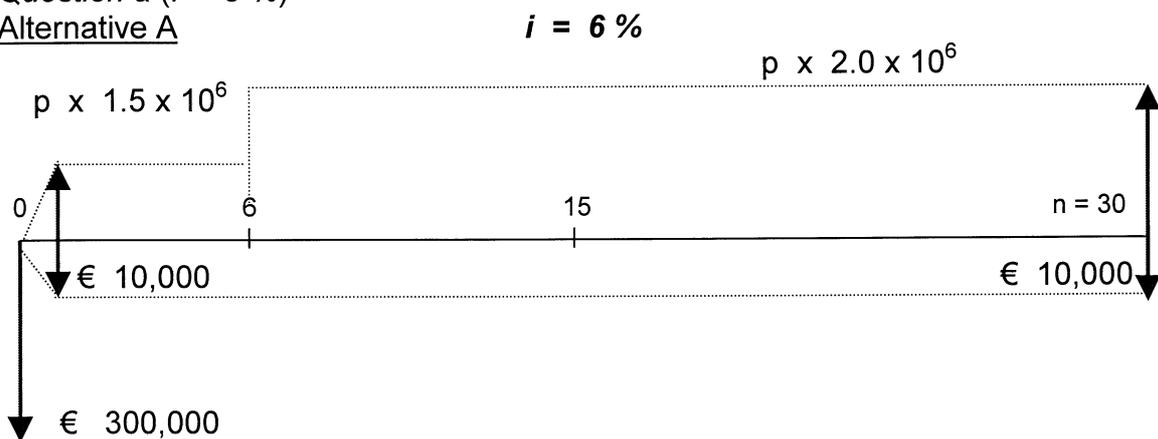
The second alternative is much cheaper.

The total equivalent annual costs of alternative 2:  $101.64 \cdot 10^6 \cdot 0.12 = € 12.2 \cdot 10^6$

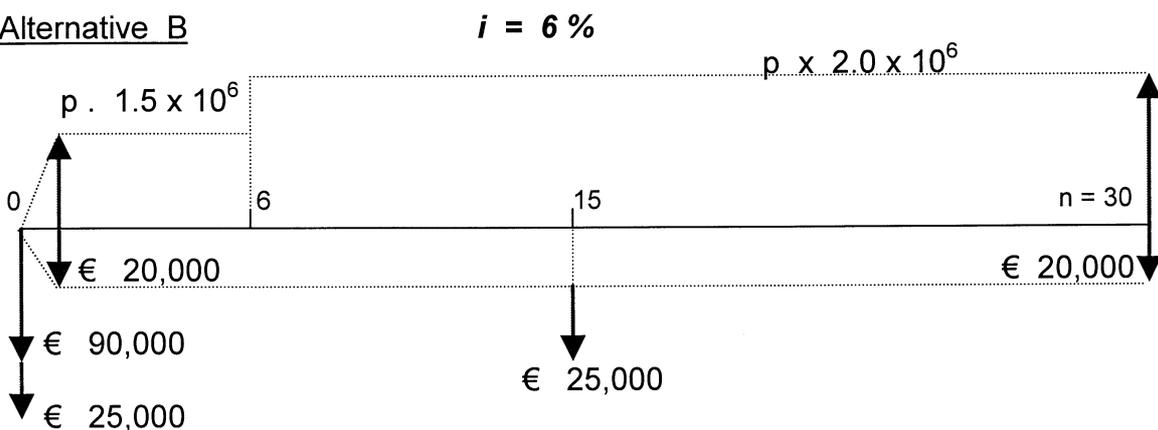
### Answer problem 6

Question a ( $i = 6\%$ )

Alternative A



Alternative B



**Equivalent annual costs**

Alternative A

$$€ 300,000 \cdot [A/P, 6\%, 30] + € 10,000 = € 300,000 \cdot 0.07265 + € 10,000$$

$$= \underline{\underline{€ 31,795}} \quad (P.V. = € 437.6 \times 10^3)$$

Alternative B

$$\begin{aligned} & \{ \text{€ } 115,000 + \text{€ } 25,000 \cdot [P/F, 6\%, 15] \} \cdot [A/P, 6\%, 30] + \text{€ } 20,000 = \\ & \{ \text{€ } 115,000 + \text{€ } 25,000 \times 0.4172 \} \cdot 0.07265 + \text{€ } 20,000 = \\ & \text{€ } 125,430 \times 0.07265 + \text{€ } 20,000 = \underline{\underline{\text{€ } 29,112}} \\ & (\text{P.V.} = \text{€ } 400.7 \times 10^3) \end{aligned}$$

Conclusion: **Alternative B is the most economic alternative.**

Or:  $\text{€ } 90,000 \cdot [A/P, 6\%, 30] + \text{€ } 25,000 [A/P, 6\%, 15] + \text{€ } 20,000 =$   
 $\text{€ } 90,000 \cdot 0.0727 + \text{€ } 25,000 \cdot 0.103 + \text{€ } 20,000 =$   
 $\text{€ } 6,543 + \text{€ } 2,575 + \text{€ } 20,000 = \underline{\underline{\text{€ } 29,118}}$

Answer Question 6b. (i = 4 %)

**Equivalent annual costs**

Alternative A

$$\begin{aligned} & \text{€ } 300,000 \cdot [A/P, 4\%, 30] + \text{€ } 10,000 = \text{€ } 300,000 \cdot 0.05783 + \text{€ } 10,000 \\ & = \\ & \underline{\underline{\text{€ } 27,349}} \quad (\text{P.V.} = \text{€ } 472.9 \times 10^3) \end{aligned}$$

Alternative B

$$\begin{aligned} & [ \text{€ } 115,000 + \text{€ } 25,000 \cdot (P/F, 4\%, 15) ] \cdot (A/P, 4\%, 30) + \text{€ } 20,000 = \\ & [ \text{€ } 115,000 + \text{€ } 25,000 \cdot 0.5552 ] \cdot 0.05783 + \text{€ } 20,000 = \\ & 128,881 \cdot 0.05783 + 20,000 = \underline{\underline{\text{€ } 27,453}} \\ & (\text{P.V.} = \text{€ } 474.7 \times 10^3) \end{aligned}$$

Conclusion: **Alternative A is the most economic alternative (just).**

Question c.

P.V. annual benefit (for both alternatives) for  $p$  = unit cost per  $m^3$  :

$$\begin{aligned} & (p \cdot 1.5 \cdot 10^6) \cdot (P/A, 6\%, 6) + (p \cdot 2.0 \cdot 10^6) \cdot (P/A, 6\%, 24) \cdot (P/F, 6\%, 6) \\ & [ (1.5 \cdot 4.9164) + (2.0 \cdot 12.5502 \cdot 0.7050) ] \cdot p \cdot 10^6 = \\ & [ 7.3746 + 17.6958 ] \cdot p \cdot 10^6 = 25.07 \cdot p \cdot 10^6 \end{aligned}$$

**Cost per  $m^3$**  (alternative B):

$$\text{€ } 400.7 \times 10^3 = 25.07 \times p \times 10^6 \longrightarrow \underline{\underline{p = \text{€ } 0.016 / m^3}}$$

Answer problem 7

**i = 15 %**

	Investment	Maintenance (per year)	Lifetime
Bridge	€ 1,350,000	€ 70,000	n = 60 years
Embankment	€ 215,000	€ 30,000 for the first 4 years ; € 14,000 thereafter	n = ∞
Other earthworks	€ 38,000		

**Embankment & other earthworks:**

$$\begin{aligned} \text{Present value: } & 215,000 + 38,000 + 30,000 \cdot (P/A, i, 4) + \frac{14,000}{i} \cdot (P/F, i, 4) = \\ & 215,000 + 38,000 + 30,000 \cdot \frac{1.15^4 - 1}{0.15 \times 1.15^4} + \frac{14,000}{0.15} \cdot \frac{1}{1.15^4} = \\ & 263,000 + 30,000 \cdot 2.855 + 14,000 \cdot 3.8117 = \\ & 263,000 + 85,649 + 53,364 = \text{€ } 402,013 \end{aligned}$$

Equivalent annual costs:  $P \cdot i = \text{€ } 402,013 \cdot 0.15 = \text{€ } 60,302$

**Bridge:**

Annuity (Capital recovery):  $\text{€ } 1,350,000 \cdot (A/P, i, 60) =$   
 $\text{€ } 1,350,000 \cdot \frac{0.15 \times 1.15^{60}}{1.15^{60} - 1} = \text{€ } 1,350,000 \cdot 0.15003 = \text{€ } 202,546$

Annual maintenance : € 70,000

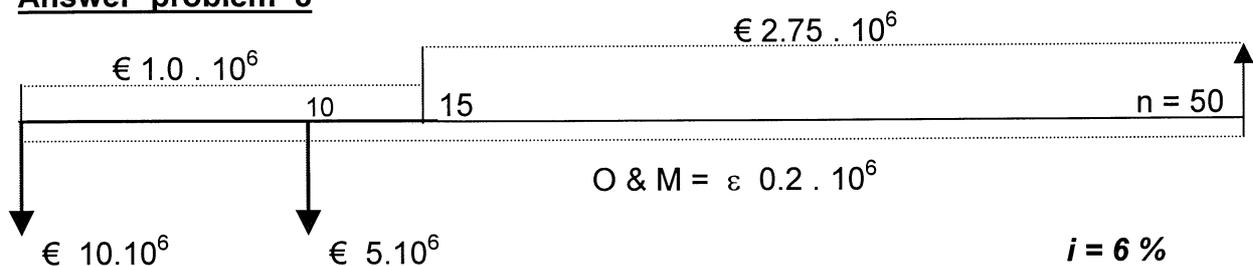
**Total equivalent uniform annual costs € 332,848**

Summary:

Capital recovery for the bridge:	€ 1,350,000 · 0,15003 =	€ 202,546
Annual maintenance of bridge:		€ 70,000
Interest for embankment and earthworks:	€ 263,000 · 0.15 =	€ 39,450
Basic annual maintenance on embankment and earthworks		€ 14,000
Equivalent annual cost of extra maintenance during first 4 years:	€ 16,000 · (P/A, i, 4) · 0.15 =	€ <u>6,852</u>

**Total equivalent uniform annual costs: € 332,848**

**Answer problem 8**



$$\begin{aligned} \Sigma \text{ Costs : } & \text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 (P/F, i, 10) + \text{€ } 0.2 \cdot 10^6 \cdot (P/A, i, 50) = \\ & \text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 \cdot \frac{1}{1.06^{10}} + \text{€ } 0.2 \cdot 10^6 \cdot \frac{1.06^{50} - 1}{0.06 \cdot 1.06^{50}} = \\ & \text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 \cdot 0.5584 + \text{€ } 0.2 \cdot 10^6 \cdot 15.76 = \\ & (10 + 2.792 + 3.152) \cdot 10^6 = \text{€ } 15.944 \cdot 10^6 \end{aligned}$$

$$\begin{aligned} \Sigma \text{ Benefits} &: \text{€ } 1.0 \cdot 10^6 \cdot (P/A, i, 15) + \text{€ } 2.75 \cdot 10^6 (P/A, i, 35) \cdot (P/F, i, 15) = \\ & (\text{€ } 1.0 \cdot \frac{1.06^{15} - 1}{0.06 \cdot 1.06^{15}} + \text{€ } 2.75 \cdot 10^6 \cdot \frac{1.06^{35} - 1}{0.06 \cdot 1.06^{35}} \cdot \frac{1}{1.06^{15}}) \cdot 10^6 = \\ & (\text{€ } 1.0 \cdot 9.7122 + \text{€ } 2.75 \cdot 14.4925 \cdot 0.417) \cdot 10^6 = \\ & (\text{€ } 9.7122 + \text{€ } 16.6498) \cdot 10^6 = \text{€ } \mathbf{26.342 \cdot 10^6} \end{aligned}$$

**B/C ratio:**  $\Sigma \text{ Benefits} / \Sigma \text{ Costs} = \text{€ } 26.342 / \text{€ } 15.944 = \mathbf{1.65}$   
**NPV**  $\Sigma \text{ Benefits} - \Sigma \text{ Costs} = (\text{€ } 26.342 - \text{€ } 15.944) \cdot 10^6 =$   
 $\text{€ } \mathbf{10.399 \cdot 10^6}$

**IRR** Try  $i = 10\%$  (as  $6\%$  gives a positive NPV)  
 $\Sigma \text{ Costs} : \text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 (P/F, i, 10) + \text{€ } 0.2 \cdot 10^6 (P/A, i, 50) =$   
 $\text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 \cdot \frac{1}{1.10^{10}} + \text{€ } 0.2 \cdot 10^6 \cdot \frac{1.10^{50} - 1}{0.10 \cdot 1.10^{50}} =$   
 $\text{€ } 10 \cdot 10^6 + \text{€ } 5 \cdot 10^6 \cdot 0.3855 + \text{€ } 0.2 \cdot 10^6 \cdot 9.92 =$   
 $(\text{€ } 10 + \text{€ } 1.93 + \text{€ } 1.98) \cdot 10^6 = \text{€ } \mathbf{13.91 \cdot 10^6}$

$$\begin{aligned} \Sigma \text{ Benefits} &: \text{€ } 1.0 \cdot 10^6 \cdot (P/A, i, 15) + \text{€ } 2.75 \cdot 10^6 (P/A, i, 35) \cdot (P/F, i, 15) = \\ & (\text{€ } 1.0 \cdot \frac{1.10^{15} - 1}{0.10 \cdot 1.10^{15}} + \text{€ } 2.75 \cdot 10^6 \cdot \frac{1.10^{35} - 1}{0.10 \cdot 1.10^{35}} \cdot \frac{1}{1.06^{15}}) \cdot 10^6 = \\ & (\text{€ } 1.0 \cdot 7.606 + \text{€ } 2.75 \cdot 9.644 \cdot 0.239) \cdot 10^6 = \\ & (\text{€ } 7.606 + \text{€ } 6.35) \cdot 10^6 = \text{€ } \mathbf{13.95 \cdot 10^6} \end{aligned}$$

**The Internal rate of return is 10 %.**

### Answer problem 9

#### **Scheme 1**

Capitalized cost:

$$\text{€ } 2,500,000 + \text{€ } 40,000 / i = \text{€ } 2,500,000 + \text{€ } 333,333 = \text{€ } \mathbf{2.833}$$

million

#### **Scheme 2**

Capitalized cost

$$\text{€ } 1,750,000 + \text{€ } 80,000 / i + \text{€ } 120,000 (P/F, 12\%, 10 \text{ years}) + \text{€ } 120,000$$

$$\cdot (P/F, 12\%, 20 \text{ years}) + \text{€ } 120,000 \cdot (P/F, 12\%, 30 \text{ years}) + \text{€ } 120,000 \cdot$$

$$(P/F, 12\%, 40 \text{ years}) + \text{etc.} =$$

$$\text{€ } 1,750,000 + \text{€ } 666,667 + \text{€ } 38,637 + \text{€ } 12,440 + \text{€ } 4,005 + \text{€ } 1,290 +$$

...

$$= \text{€ } \mathbf{2.474 \text{ million}}$$

**Remark:** The replacement cost of € 120,000 every 10 years can be considered as an equivalent 'annual' cost, whereby annual is now 10 years and the compounded interest rate for 10 years is  $1.12^{10} = 3.10585 - 1 = 2.10585$

**Scheme 2**

Capitalized cost

$$\begin{aligned} & \text{€ } 1,750,000 + \text{€ } 80,000 / 0.12 + \text{€ } 120,000 / 2.10585 = \\ & \text{€ } 1,750,000 + \text{€ } 666,667 + \text{€ } 56,984 = \text{€ } \mathbf{2.474 \text{ million}} \end{aligned}$$

**Answer problem 10**

Question a

Annual costs

**Plan A** Capital recovery cost:  $\text{€ } 10,000,000 \cdot [A/P, i, 20] = \text{€ } 1,175,000$   
 Operation and maintenance :  $\text{€ } \underline{2,000,000}$   
 total  $\text{€ } \mathbf{3,175,000}$

**Plan B** Capital recovery cost:  $\text{€ } 20,000,000 \cdot [A/P, i, 20] = \text{€ } 2,350,000$   
 Operation and maintenance :  $\text{€ } \underline{400,000}$   
 total  $\text{€ } \mathbf{2,750,000}$

Plan B is less costly than Plan A.

Question b

Present value

**Plan A** Initial costs  $\text{€ } 10,000,000$   
 Dredging  $\text{€ } 100,000 \cdot [P/A, i, 20] + \text{€ } 200,000 [P/C, i, 20]$   
 $= \text{€ } 100,000 \cdot 8.5136 + 200,000 \cdot 55.41 = \text{€ } \underline{11,953,000}$   
 total  $\text{€ } \mathbf{21,953,000}$   
 or  $\text{€ } \mathbf{2,580,000 / year}$

**Plan B** Initial costs  $\text{€ } 20,000,000$   
 Operation and maintenance:  
 $\text{€ } 400,000 \cdot [P/A, i, 20] = \text{€ } 400,000 \cdot 8.5136 = \text{€ } \underline{3,405,000}$   
 total  $\text{€ } \mathbf{23,405,000}$

or  $\text{€ } \mathbf{2,750,000 / year}$

**Plan A is less costly than Plan B.**

**Answer problem 11**

$i = 5 \%$ ,  $n = 50$ , annuity factor = 0.0548

Slope	Capital recovery costs of excavation (€)	Annual cost slope maintenance	Total annual cost (€)
1 : 1 (n = 1)	$350.000 \cdot \text{€ } 3.00 \cdot 0.0548 = 57,540$	$\text{€ } 80,000$	$\text{€ } 137,540$
1 : 2 (n = 2)	$450.000 \cdot \text{€ } 3.00 \cdot 0.0548 = 73,980$	$\text{€ } 50,000$	$\text{€ } 123,980$
1 : 3 (n = 3)	$550.000 \cdot \text{€ } 3.00 \cdot 0.0548 = 90,420$	$\text{€ } 34,000$	$\text{€ } 124,000$
1 : 4 (n = 4)	$650.000 \cdot \text{€ } 3.00 \cdot 0.0548 = 106,860$	$\text{€ } 24,000$	$\text{€ } 130,860$

The most economical slope will be around 1 : 2.5 ( n = 2.5 ).

**Answer problem 12**

**i = 10 %**

	Project A	Project B	Project C
n (life)	8 years	8 years	10 years
[ P / A]	5.335	5.335	6.145
P.V. benefits	18,400 . 5.335 = € 98,163	30,600 . 5.335 = € 163,251	42,300 . 6.145 = € 259,915
Initial cost	€ 100,000	€ 160,000	€ 280,000
P.V. of scrap value	nil	nil	€ 40,000 . 0.3856 = € 15,422
NPV	- € 1.837	+ € 3,251	- € 4,663
Recommendation	return < 10 % <b>rejected</b>	return > 10 % <b>acceptable</b>	return < 10 % <b>rejected</b>

**Answer problem 13**

i = 10 %, n = 3,5 years

Costs: Capital recovery cost:  $(€ 300,000 - € 30,000) \cdot (A/P, i, n) =$   
 $€ 270,000 \cdot (A/P, 10\%, n)$

Annuity for 3.5 years: 0.353; capital recovery:

€ 270,000 . 0.353 = € 95,200

Interest: € 30,000 . 0.10 = € 3,000

Extra maintenance costs: € 6,000

Total costs € 104,200

Benefits: Nett annual cost savings:  $(€ 150,000 - € 45,000) = € 105,000$

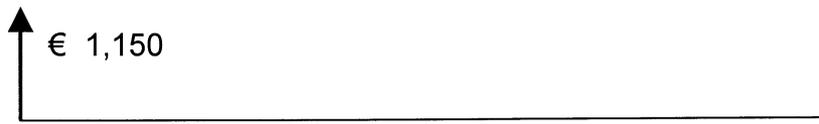
As benefits exceeds costs (just) the investment is justified (for i = 10 %)

### Answer problem 14

$n = \infty$

	Alternative A	Alternative B	Alternative C	Alternative D
Investment	100	200	300	500
Annual benefits	20	30	50	75
PV benefits	$20 / i$	$30 / i$	$50 / i$	$75 / i$
NPV	$20 / i - 100$	$30 / i - 200$	$50 / i - 300$	$75 / i - 500$
IRR	$20 / 100 = 0.2$ <b>(20 %)</b>	$30 / 200 = 0.15$ (15 %)	$50 / 300 = 0.167$ (16.7 %)	$75 / 500 = 0.15$ (15 %)
Conclusion:	<b>Highest rate Of return</b>			
Unused funds: ( <b><math>i = 10\%</math></b> )				
NPV	$20 / 0.10 - 100 = 200 - 100 = 100$	$30 / 0.10 - 200 = 300 - 200 = 100$	$50 / 0.10 - 300 = 500 - 300 = 200$	$75 / 0.10 - 500 = 750 - 500 = 250$
or Unused funds	$500 - 100 = 400$	$500 - 200 = 300$	$500 - 300 = 200$	nil
Annual benefits	$400 \cdot 0.10 + 20 = 40 + 20 = 60$	$300 \cdot 0.10 + 30 = 30 + 30 = 60$	$200 \cdot 0.10 + 50 = 20 + 50 = 70$	$0 + 75 = 75$
Conclusion:				<b>Preferred Alternative</b>
Unused funds: ( <b><math>i = 14\%</math></b> )				
NPV	$400 \cdot 0.14 + 20 = 56 + 20 = 76$	$300 \cdot 0.14 + 30 = 42 + 30 = 72$	$200 \cdot 0.14 + 50 = 28 + 50 = 78$	$0 + 75 = 75$
Conclusion:			<b>Preferred Alternative</b>	

**Answer problem 15**  
**Payment schedule a.**



Annual payment for  $i = 12\%$ :  $\text{€ } 1150 \cdot (A/P, 12\%, 8) = 1150 \cdot 0.2013 = \text{€ } 231.50$ . This payment is done at the end of the year!



Present Value (PV) of payment schedule B ( $i = 12\%$ ):  
 $\text{€ } 231.5 + \text{€ } 231.5 \cdot (P/A, 12\%, 7 \text{ years}) = 231.5 + 231.5 \cdot 4.564 = \text{€ } 231.5 (1 + 4.564) = \text{€ } 231.5 \cdot 5.564 = \text{€ } 1,288$

The same payment of  $\text{€ } 231.50$  is done at the beginning of the year.

Schedule A is the better schedule for the party that is paying; schedule B is the better schedule for the receiving part; the difference is  $\text{€ } 231.50 \cdot 0.12 = \text{€ } 27.78$  per year. For payment schedule b one only needs to borrow  $\text{€ } 1,150 - \text{€ } 231,5 = \text{€ } 918.50$ .

$n = 7$  years:

$$\begin{aligned} \text{€ } 918.50 \cdot (A/P, i, 7) &= \text{€ } 231, 50 \\ (A/P, i, 7) &= 231,50 / 918.50 = \mathbf{0.2520} \\ i = 16\%: \text{ Annuity} &= 0.2476; \quad i = 18\%: \text{ Annuity} = 0.2624 \\ \text{Interpolation gives an effective annual interest of } &16.6\%. \end{aligned}$$

**Answer problem 16**

**Question a**

Compare annual costs against annual benefits (NPV = 0)

Capital recovery:  $\text{€ } 100,000 \cdot (A/P, i, 25)$

Annuity factors:  $i = 10\% : 0.1102; i = 12\% : 0.1275$

$$\begin{aligned} \text{NPV} = 0: \quad \text{€ } 100,000 \cdot (A/P, i, 25) + \text{€ } 15,000 &= \text{€ } 26,500 \\ (A/P, i, 25) &= (26,500 - 15,000) / 100,000 = \mathbf{0.1150} \end{aligned}$$

By interpolation one finds the **IRR = approx. 10.5 %**

**Question b :  $i = 4\%$**

Yes, investments can be made economically because the cost of money at  $4\%$  will result in a positive NPV (NPV = 0 for  $i = 10.5\%$ ).

**Question c**

Annual surplus:  $\text{€ } 26,500 - \text{€ } 15,000 = \text{€ } 11,500$

This surplus is being used to repay the loan, which carries an annual interest of  $4\%$ .

So  $\text{€ } 100,000 \cdot (A/P, 4\%, n) = 11,500$  ( $n = ?$ )

$$(A/P, 4\%, n) = 0.1150$$

$n = 10$  years : annuity = 0.1233

$n = 12$  years: annuity = 0.1066

By interpolation one finds  **$n = 11$  years.**

### Answer problem 17

#### Question a , $i = 10\%$

Annual capital recovery cost:

$$€ 1,000,000 \cdot (A/P, i, 20) = € 1,000,000 \cdot 0.1175 = € 117,500$$

Annual equivalent maintenance & operation costs: € 100,000

Total annual costs: € 217,500

Annual benefit:

$$15 \cdot 10^6 \times \text{unit cost}$$

Transportation cost (unit cost) per  $m^3$ :  $217,000 / 15,000,000 = € \underline{\underline{0.0145}}$

#### Question b

Present value of all costs:  $PV = 217,500 \cdot (P/A, 10\%, 20 \text{ years}) = € 1,851,000$

Present value of benefits, whereby  $X = \text{unit cost}$ :

$$13 \cdot 10^6 \cdot X \cdot (P/A, 10\%, 10) + 17 \cdot 10^6 \cdot X \cdot (P/A, 10\%, 10) \cdot (P/F, 10\%, 10)$$

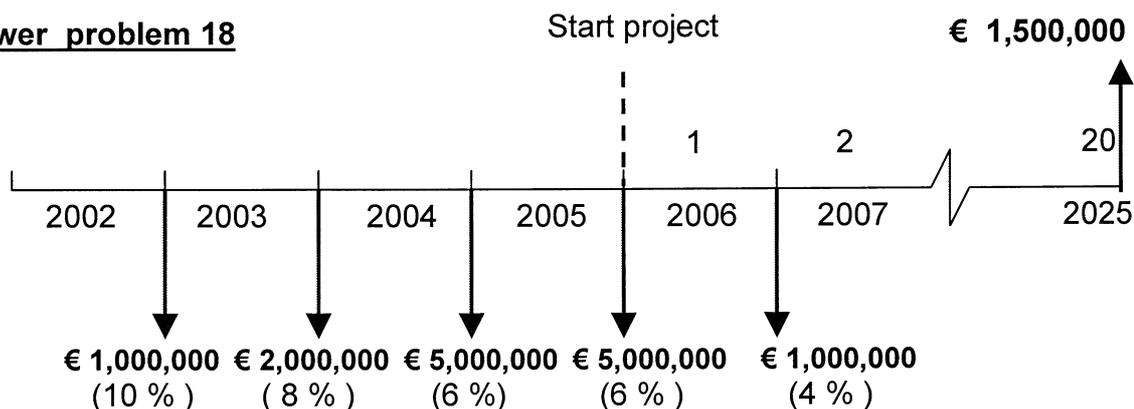
=

$$13 \cdot 10^6 \cdot X \cdot 6.1446 + 17 \cdot 10^6 \cdot X \cdot 6.1446 \cdot 0.3855 =$$

$$(79.88 + 40.27) \cdot 10^6 \cdot X = 120.15 \cdot 10^6 \cdot X$$

Transportation cost  $X$  (unit cost) per  $m^3$ :  $1,851,000 / 120,150,000 = € \underline{\underline{0.0154}}$

### Answer problem 18



#### a. First costs

Is compounded costs on the day the project is put into operation (1-1-2006) =

$$1,000,000 \cdot (F/P, 10\%, 3) + 2,000,000 \cdot (F/P, 8\%, 2) + 5,000,000 \cdot$$

$$(F/P, 6\%, 1) + 5,000,000 + 1,000,000 \cdot (P/F, 4\%, 1) =$$

$$1,000,000 \cdot 1.331 + 2,000,000 \cdot 1.1411 + 5,000,000 \cdot 1.06 + 5,000,000 + 1,000,000 / 1.04 =$$

$$1,331,000 + 2,282,200 + 5,300,000 + 5,000,000 + 961,500 =$$

$$€ \underline{\underline{14,874,700}} \quad \text{Say } € \underline{\underline{14.9 \text{ million}}}.$$

#### b. Total depreciation

First cost – salvage value = € 14.9 million - € 1.5 million =

€ **13.4 million**

#### c. Internal rate of return

NPV = 0 or  $\Sigma$  all costs =  $\Sigma$  all benefits

$$€ 13,374,000 \cdot (A/P, i\%, 20) + € 1,500,000 \cdot i + € 1,000,000 =$$

$$€ 3,250,000$$

Find  $i$  by trail and error.

For  $i = 10\%$ , annuity factor = 0.1175  
 $13,374,000 \cdot 0.1175 + 1,500,000 \cdot 0.10 + 1,000,000 = \text{€ } 2,725,000$   
 For  $i = 12\%$ , annuity factor = 0.1339  
 $13,374,000 \cdot 0.1339 + 1,500,000 \cdot 0.12 + 1,000,000 = \text{€ } 2,975,000$   
 For  $i = 14\%$ , annuity factor = 0.1510  
 $13,374,000 \cdot 0.1510 + 1,500,000 \cdot 0.14 + 1,000,000 = \text{€ } 3,235,000$   
**IRR = 14 % (slightly more).**

d. Equivalent annual surplus

Actual 'costs' of money is **4 %**.

Annual surplus = annual revenue - annual costs =

$\text{€ } 3,250,000 - \text{€ } 1,000,000 - \text{€ } 1,500,000 \cdot 0.04 - \text{€ } 13,374,000 \cdot (A/P, 4\%, 20) = \text{€ } 3,250,000 - \text{€ } 1,000,000 - \text{€ } 60,000 - \text{€ } 13,374,000 \cdot 0.0736$

$= \text{€ } 3,250,000 - \text{€ } 1,000,000 - \text{€ } 60,000 - \text{€ } 986,000 =$

**€ 1,204,000**

e. Marginal rate of return

Annual capital recovery cost of additional initial cost of  $\text{€ } 1,500,000 = \text{€ } 1,500,000 \cdot (A/P, i, 20)$

Additional revenues:  $\text{€ } 3,400,000 - \text{€ } 3,250,000 = \text{€ } 150,000$ .

$(A/P, i, 20) = 150,000 / 1,500,000 = 0.100$

for  $i = 6\%$  annuity = 0.0872; for  $i = 8\%$  annuity = 0.1019; **so  $i = 7.9\%$  (approx.)**

f. Justification

The investment of  $\text{€ } 1,500,000$  is not justified because these amount can yield 12 % in another project against about 8 % in this project.

**Answer problem 19**

Question a ( $i = 6\%$ ,  $n = 15$  years)

**Plan A**

$\Sigma$  Present Value Benefits:  $\text{€ } 624,000 \cdot (P/A, 6\%, 15) = 624,000 \cdot 9.7122 = \text{€ } 6,060,500$

$\Sigma$  Present Value All Costs:

$\text{€ } 4,000,000 + \text{€ } 200,000 \cdot (P/A, 6\%, 15) =$

$\text{€ } 4,000,000 + \text{€ } 200,000 \cdot 9.7122 = \text{€ } 5,942,000$

**B / C – factor :**  $\text{€ } 6,060,500 / \text{€ } 5,942,000 = \underline{\underline{1.02}}$

**Plan B**

$\Sigma$  Present Value Benefits:  $\text{€ } 722,000 \cdot (P/A, 6\%, 15) = \text{€ } 722,000 \cdot 9.7122 = \text{€ } 7,012,000$

$\Sigma$  Present Value All Costs:

$\text{€ } 5,000,000 + \text{€ } 240,000 \cdot (P/A, 6\%, 15) = \text{€ } 5,000,000 + \text{€ } 240,000 \cdot 9.7122 = \text{€ } 7,330,900$

**B / C – factor :**  $\text{€ } 7,012,000 / \text{€ } 7,330,900 = \underline{\underline{0.96}}$

Question b

**Plan A**

The IRR > 6 % as B / C- factor > 1 for i = 6 %.

Try 7 %: discounting factor : 9.1079 and B / C = 0.98 → IRR = 6.5 %

**Plan B**

The IRR < 6 % as B / C – factor < 1 for i = 6 %.

Try 5 %: discounting factor : 10.38 and B / C = 1.02

Try 5.5 %: discounting factor : 10.0376 and B / C = 0.98 → IRR = 5.25 %

**Marginal rate of return of Plan B with respect to Plan A:**

Plan B – Plan A (= actual extension)

Σ Present Value Benefits:

$$(\text{€ } 722,000 - \text{€ } 624,000) \cdot (P/A, i, 15) = \text{€ } 98,000 \cdot (P/A, i, 15)$$

Σ Present Value All Costs:

$$(\text{€ } 5,000,000 - \text{€ } 4,000,000) + (\text{€ } 240,000 - \text{€ } 200,000) \cdot (P/A, i, 15)$$

$$\text{For } i = 5 \% \text{ B / C – factor} = (98,000 \cdot 10.38) / (1,000,000 + 40,000 \cdot 10.38) = 0.72$$

$$\text{For } i = 2 \% \text{ B / C – factor} = (98,000 \cdot 12.85) / (1,000,000 + 40,000 \cdot 12.85) = 0.83$$

$$\text{For } i = 0 \% \text{ B / C – factor} = (98,000 \cdot 15) / (1,000,000 + 40,000 \cdot 15) = 0.92$$

**the marginal rate of return of the extension is negative !**

Question c

Unused funds are defined as the difference in investment of Plan A and Plan B:

€ 1,000,000 (the additional investment). As the marginal rate of return of the additional investment is lower than 5 % (and even negative) the unused funds should be invested in other projects (with a rate of return of 5 %).

**Answer problem 20 i = 5 %**

Frequency x (times/ decade)	Useful life y $y = x^2 + 20$ (in years)	Annual capital recovery cost (depreciation) $= 10 \cdot 10^6 \cdot$ annuity	Annual repair & maintenance cost at € 250,000 / time	Total annual costs
<b>x = 0</b> (no painting)	y = 20 years	$10 \cdot 10^6 \cdot 0.0802$ = 820,000	0	820,000
<b>x = 5 / decade</b> (every 2 years)	y = 45 years	$10 \cdot 10^6 \cdot 0.0563$ = 563,000	$5 \times 250,000 / 10$ = 125,000	688,000
<b>x = 6 / decade</b>	y = 56 years	$10 \cdot 10^6 \cdot 0.0535$ = 535,000	$6 \times 250,000 / 10$ = 150,000	<b>685,000</b>
<b>x = 7 / decade</b>	y = 69 years	$10 \cdot 10^6 \cdot 0.0518$ = 518,000	$7 \times 250,000 / 10$ = 175,000	693,000
<b>x = 10</b> (every year)	y = 120 years	$10 \cdot 10^6 \cdot 0.05$ = 500,000	= 250,000	750,000

Most economic frequency: **6 times / decade**

## Answer problem 21

### Question a

#### **Water transport**

Construction time:  $\frac{400}{8} = 8$  years

Start of construction: 1<sup>st</sup> January 2002

Construction cost/ year:  $\frac{(40 \times 10^6)}{8} \times \frac{400}{100} = \text{LC } 20 \cdot 10^6 / \text{year}$

End of construction: 31<sup>st</sup> December 2009

Compounding factor:  $(F/A, 10\%, 8 \text{ years}) = \frac{(1+0.10)^8 - 1}{0.10} = \mathbf{11.44}$

Construction cost at the end of the project:  $11.44 \cdot (20 \cdot 10^6) = \text{LC } 228.8 \cdot 10^6$

Annuity (10 %, 50 years):  $\frac{0.10 \times (1+0.10)^{50} - 1}{(1+0.10)^{50} - 1} = \mathbf{0.101}$

Annual cost of construction costs:  $\text{LC } (228.8 \cdot 10^6) \cdot 0.101 = \text{LC } 23.11 \cdot 10^6$

Annual maintenance costs:  $\text{LC } (2 \cdot 10^6) \cdot \frac{400}{100} = \text{LC } 8.00 \cdot 10^6$

Transportation costs:  $\text{LC } 0.05 \cdot \frac{400}{100} \cdot 5 \cdot 10^6 = \underline{\text{LC } 1.00 \cdot 10^6}$

**Total annual costs** **LC 32.11 .10<sup>6</sup>**

#### **Rail transport**

Construction time:  $\frac{375}{5} = 5$  years

Start of construction: 1<sup>st</sup> January 2005

Construction cost/ year:  $\frac{(32 \times 10^6)}{5} \times \frac{375}{100} = \text{LC } 24 \cdot 10^6 / \text{year}$

End of construction: 31<sup>st</sup> December 2009

Compounding factor:  $(F/A, 10\%, 5 \text{ years}) = \frac{(1+0.10)^5 - 1}{0.10} = \mathbf{6.11}$

Construction cost at the end of the project:  $6.11 \cdot (24 \cdot 10^6) = \text{LC } 146.64 \cdot 10^6$

Annuity (10 %, 50 years):  $\frac{0.10 \times (1+0.10)^{50} - 1}{(1+0.10)^{50} - 1} = \mathbf{0.101}$

Annual cost of construction costs:  $\text{LC } (146.64 \cdot 10^6) \cdot 0.101 = \text{LC } 14.81 \cdot 10^6$

Annual maintenance costs:  $\text{LC } (2.5 \cdot 10^6) \cdot \frac{375}{100} = \text{LC } 9.83 \cdot 10^6$

Transportation costs:  $\text{LC } (0.07 \cdot \frac{375}{100}) \cdot 5 \cdot 10^6 = \underline{\text{LC } 1.31 \cdot 10^6}$

**Total annual costs** **LC 25.50 .10<sup>6</sup>**

So, **Project B is preferred.**

**Question b**

**Water transport (20 % FC)**

Construction cost / year: + 20 %

Annual cost of construction costs: + 20 % =  $1.2 \cdot \text{LC } 23.11 \cdot 10^6 = \text{LC } 27.73 \cdot 10^6$

Annual maintenance costs (same as question a):  $\text{LC } 8.00 \cdot 10^6$

Transportation costs (same as question a):  $\text{LC } 1.00 \cdot 10^6$

**Total annual costs**  $\text{LC } 36.73 \cdot 10^6$

**Rail transport (80 % FC)**

Construction cost / year: + 80 %

Annual cost of construction costs: + 80 % =  $1.8 \cdot \text{LC } 14.81 \cdot 10^6 = \text{LC } 26.66 \cdot 10^6$

Annual maintenance costs costs (same as question a):  $\text{LC } 9.38 \cdot 10^6$

Transportation costs (same as question a):  $\text{LC } 1.31 \cdot 10^6$

**Total annual costs**  $\text{LC } 37.35 \cdot 10^6$

In this case, **Project A is preferred.**

**Question c**

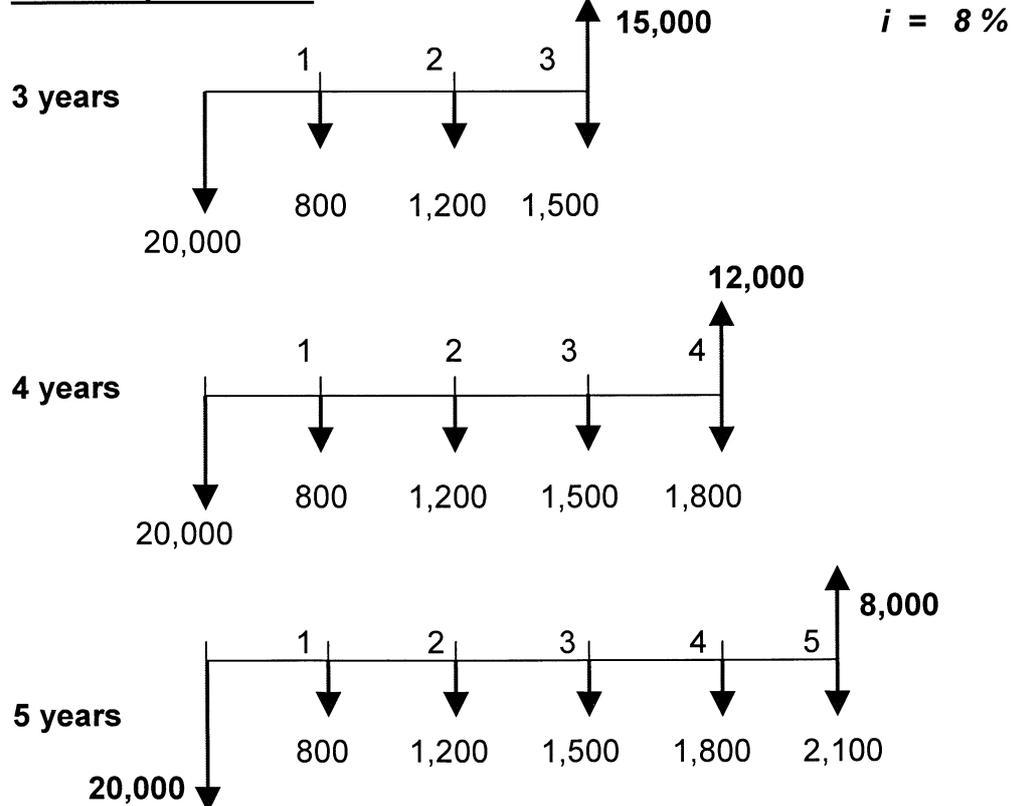
Compare the difference between the two projects.

The 'costs' of the transportation time is relative: the railway time is faster and therefore cheaper, by 5 hours x LC 0.02 / ton x  $5 \times 10^6$  ton per year =  $\text{LC } 0.5 \cdot 10^6$

For the first case the difference between the two alternatives becomes larger;

For the second case the difference between the two alternatives becomes smaller and the two alternatives are about the same.

**Answer problem 22**



	Depreciation (annual)	Interest (8 %) on resale value	Equivalent annual cost of maintenance costs	Total annual costs
<b>3</b> years	$5,000 \cdot (A/P, i, 3) =$ $5,000 \cdot 0.3880 =$ 1,940	$15,000 \cdot 0.08$ $= 1,200$	$(800/1.08 + 1200/1.08^2 + 1500/1.08^3) \cdot (A/P, i, 3)$ $= 2,960 \cdot 0.3880 = 1,149$	<b>4,289</b>
<b>4</b> years	$8,000 \cdot (A/P, i, 4) =$ $8,000 \cdot 0.3019 =$ 2,415	$12,000 \cdot 0.08$ $= 960$	$(800/1.08 + 1200/1.08^2 + 1500/1.08^3 + 1800/1.08^4) \cdot (A/P, i, 4) =$ $4,283 \cdot 0.3019 = 1,293$	4,668
<b>5</b> years	$12,000 \cdot (A/P, i, 5)$ $= 12,000 \cdot 0.2505 =$ 3,006	$8,000 \cdot 0.08$ $= 640$	$\{800/1.08 + 1200/1.08^2 + 1500/1.08^3 + 1800/1.08^4 + 2100/1.08^5\} \cdot [A/P, i, 5]$ $= 5,712 \cdot 0.2505 = 1,496$	5,142

**Sell the equipment after 3 years !**

### Answer problem 23

$i = 12\%$ ,  $n = \infty$

#### **Reinforced concrete road pavement per m<sup>2</sup>**

Σ Present value costs:

$$\begin{aligned}
 & \text{€ } 100 + 0.67 / 0.12 + 31 \cdot (P/F, 12\%, 40 \text{ years}) + \\
 & 3.25 / 0.12 \cdot (P/F, 12\%, 40 \text{ years}) = \\
 & 100 + 5.583 + 31 \cdot 0.0107 + 27.08 \cdot 0.0107 = \\
 & 100 + 5.583 + 0.624 = \text{€ } 106.207 / \text{m}^2 \\
 & \text{per } 2,000 \text{ m}^2 : 2,000 \cdot \text{€ } 106.207 / \text{m}^2 = \quad \quad \quad \text{€ } \underline{\underline{212,414}}
 \end{aligned}$$

#### **Flexible pavement per m<sup>2</sup>**

Σ Present value costs:

$$\begin{aligned}
 & 90 + 3.25 / 0.12 + 0.67 / 0.12 = \\
 & 90 + 3.92 / 0.12 = \text{€ } 122.667 / \text{m}^2 \\
 & \text{per } 2,000 \text{ m}^2 : 2,000 \cdot \text{€ } 122.667 / \text{m}^2 = \quad \quad \quad \text{€ } \underline{\underline{245,333}}
 \end{aligned}$$

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